

# Global identification of spring balancer, dynamic parameters and drive gains of heavy industrial robots

A. Jubien and M. Gautier

**Abstract**—In this paper, the global identification of spring balancer, dynamic parameters and joint drive gains of a 6 Degrees Of Freedom (*DOF*) robot is performed. Off-line identification method is based on the use of the Inverse Dynamic Identification Model (*IDIM*) which takes into account a spring balancer for gravity compensation and linear Least Squares (*LS*) technique to estimate the parameters from the positions and joint torques. It is key to get accurate values of joint drive gains to get accurate identification because the joint torques are calculated as the product of the current reference by the joint drive gains. Recently a new method validated on small payload robots (less than 10 Kg) allows to identify simultaneously all joint drive gains and dynamic parameters. This method is based on the Total Least Squares (*TLS*) solution of an over-determined linear system obtained with the inverse dynamic model calculated while the robot is tracking reference trajectories without load and trajectories with a known payload fixed on the robot. This method is used to identify accurately the heavy industrial robot Kuka KR270 (270Kg payload) with its spring balancer. This is a new step to promote a practical and easy to use method for global dynamic identification of any small or heavy gravity compensated industrial robots that does not need any *a priori* data, which are too often missing from manufacturer's data sheet.

## I. INTRODUCTION

Accurate dynamic robots models are needed to control and simulate their motions with precision and reliability. Identification of robots has been widely investigated in the last decades. The usual identification process is based on the Inverse Dynamic Model (*IDM*) and Least Squares (*LS*) estimation. This method, called *IDIM-LS* (Inverse Dynamic Identification Model with Least Squares), has been performed on several prototypes and industrial robots with accurate results [1].

In most case, heavy duty handling industrial robots have spring balancer (gravity compensator) to avoid to require a big motor power to compensate gravity joint torques caused by the weight of robot links. Then to get accurate identification, it is necessary to take into account its modeling in dynamic model of the robot before performing the estimation of parameters.

Another point to consider is the calculation of joint torques. Accurate values of joint drive gains must be known

to calculate the joint torques as the product of the known the current references by the joint drive gains [2]. This needs to calibrate the drive train constituted by a current source amplifier gain which supplies a permanent magnet DC or a brushless motor with torque constant coupled to the link through direct or gear train with gear ratio. Because of large values of the gear ratio for industrial robots, joint drive gain is very sensitive to errors in current source amplifier gain and torque constant which must be accurately measured from special, time consuming, heavy tests, on the drive chain [2][3].

Recently, new method for the global identification of the joint drive gains data was validated on two small payload robots (3 (Kg) and 10 (Kg)) [4][5][6]. This method uses the current reference and the position sampled data while the robot is tracking one reference trajectory without load fixed on the robot and one trajectory with a known payload fixed on the robot whose inertial parameters are measured. Contrary to the previous works [2], all drive gains are calculated in the same solving loop by the Total *LS* solution of an over-determined system in order to take into account the dynamic coupling between the robot axes.

In this paper the identification of dynamic parameters and joint drive gains of a 6 *DOF* Kuka KR270 (270 (Kg) maximum payload) industrial robot is performed with a payload of 175 (Kg). This robot has a spring balancer attached to the second joint. The spring balancer induces a torque on the joint 2. A modeling of the spring balancer is proposed and its parameters are identified simultaneously with the usual dynamic parameters of the robot.

This paper is divided into 7 sections. Section II describes the modeling of serial robots. Section III presents the usual method for dynamic identification of robots, based on *IDIM-LS* method. Section IV presents the new modeling and identification for robot drive gains parameters. Section V is devoted to the modeling of Kuka KR270 industrial robot and its spring balancer. Section VI presents the experimental global identification of the robot. Finally, the last section gives the conclusion.

## II. MODELING

### A. Modified Denavit and Hartenberg notation

The kinematics of serial robots is defined using the Modified Denavit and Hartenberg (*MDH*) notation [7]. In this notation, the link  $j$  fixed frame is defined such that: the  $z_j$  axis is taken along joint  $j$  axis; the  $x_j$  axis is along the common normal between  $z_j$  and  $z_{j+1}$ ;  $\alpha_j$  and  $d_j$  parameterize the angle and distance between  $z_{j-1}$  and  $z_j$

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along  $x_{j-1}$ , respectively;  $\theta_j$  and  $r_j$  parameterize the angle and distance between  $x_{j-1}$  and  $x_j$  along  $z_j$ , respectively.

### B. Inverse Dynamic Model

The *IDM* of a robot calculates the joint torques  $\tau_{idm}$  as a function the joint positions, velocities and accelerations. It can be obtained from the Newton-Euler or the Lagrangian equations [7]. It is given by the following relation:

$$\tau_{idm} = M(q)\ddot{q} + N(q, \dot{q}) \quad (1)$$

Where  $q$ ,  $\dot{q}$  and  $\ddot{q}$  are respectively the  $(n \times 1)$  vectors of joint positions, velocities and accelerations;  $M(q)$  is the  $(n \times n)$  robot inertia matrix;  $N(q, \dot{q})$  is the  $(n \times 1)$  vector of centrifugal and frictions forces/torques.  $n$  is the number of moving links.

The choice of the modified Denavit and Hartenberg frames attached to each link allows a dynamic model that is linear in relation to a set of standard dynamic parameters  $\chi_{st}$  [8]:

$$\tau_{idm} = IDM_{st}(q, \dot{q}, \ddot{q})\chi_{st} \quad (2)$$

$$\text{with } \chi_{st} = [\chi_{st1}^T \quad \chi_{st2}^T \quad \dots \quad \chi_{stm}^T]^T$$

Where  $IDM_{st}(q, \dot{q}, \ddot{q})$  is the  $(n \times N_s)$  Jacobian matrix of  $\tau_{idm}$ , with respect to the  $(N_s \times 1)$  vector  $\chi_{st}$  of the standard parameters.  $\chi_{stj}$  is composed of standard dynamic parameters of axis  $j$ :

$$\chi_{stj} = [XX_j \quad XY_j \quad XZ_j \quad YY_j \quad YZ_j \quad ZZ_j \quad MX_j \quad MY_j \quad MZ_j \quad M_j \quad Ia_j \quad Fv_j \quad Fc_j \quad \tau_{off_j}]^T \quad (3)$$

Where:

-  $XX_j, XY_j, XZ_j, YY_j, YZ_j, ZZ_j$  are the six components of the robot inertia matrix of link  $j$ ;  $MX_j, MY_j, MZ_j$  are the components of the first moments of link  $j$ ;  $M_j$  is the mass of link  $j$ ;  $Ia_j$  is a total inertia moment for rotor and gears of actuator of link  $j$ ;  $Fv_j$  and  $Fc_j$  are the viscous and Coulomb friction parameters of joint  $j$ ;  $\tau_{off_j}$  is an offset parameter which take into account of the dissymmetry of Coulomb friction of joint  $j$  and of the motor current amplifier offset of joint  $j$ ;  $N_s = 14 \times n$  is the number of standard parameters.

### III. IDIM-LS: INVERSE DYNAMIC IDENTIFICATION MODEL WITH LEAST SQUARES METHOD

Because of perturbations due to noise measurement and modeling errors, the actual force/torque  $\tau$  differs from  $\tau_{idm}$  by an error  $e$ , such that:

$$\tau = \tau_{idm} + e = IDM_{st}(q, \dot{q}, \ddot{q})\chi_{st} + e \quad (4)$$

Where:

$$\tau = v_\tau g_\tau = \begin{bmatrix} v_\tau^1 & 0 & \dots & 0 \\ 0 & v_\tau^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & v_\tau^n \end{bmatrix} \begin{bmatrix} g_\tau^1 \\ g_\tau^1 \\ \vdots \\ g_\tau^n \end{bmatrix} \quad (5)$$

Where  $v_\tau$  is the  $(n \times n)$  matrix of the actual references of the current amplifiers ( $v_\tau^j$  corresponds to actuator  $j$ ).  $g_\tau$  is the  $(n \times 1)$  vector of the joint drive gains ( $g_\tau^j$  corresponds to actuator  $j$ ) with:

$$g_\tau = NG_i K_t \quad (6)$$

Where  $N, G_i$  and  $K_t$  are the  $(n \times n)$  matrix of gear ratios, the  $(n \times n)$  matrix of current source amplifier gains and the  $(n \times n)$  matrix of torque constants respectively.

The minimal model which contains only the  $b$  base parameters [8][9] is defined from (4):

$$\tau = \tau_{idm} + e = IDM(q, \dot{q}, \ddot{q})\chi + e \quad (7)$$

Where a subset  $IDM(q, \dot{q}, \ddot{q})$  of  $IDM_{st}(q, \dot{q}, \ddot{q})$  defines the vector  $\chi$  of the  $b$  base parameters. They can be obtained from the standard inertial parameters by regrouping some of them by means of linear relation.

The vector  $\hat{\chi}$  is the least squares (*LS*) solution of an over determined system built from the sampling of (7), while the robot is tracking exciting trajectories [10]:

$$Y = W\chi + \rho \quad (8)$$

Where:  $Y$  is the  $(r \times 1)$  measurement vector,  $W$  the  $(r \times b)$  observation matrix, and  $\rho$  is the  $(r \times 1)$  vector of errors. The number of rows is  $r = n * n_e$ , where the number of recorded samples is  $n_e$ .

Calculating the *LS* solution of (8) from perturbed data in  $W$  and  $Y$  may lead to bias if  $W$  is correlated to  $\rho$ . Then, it is essential to filter data in  $Y$  and  $W$  before computing the *LS* solution. Velocities and accelerations are estimated by means of a band-pass filtering of the positions. To eliminate high frequency noises and torque ripples, low pass filtering and a downsampling is performed on  $Y$  and on each column of  $W$  with decimate filter. More details about data filtering can be found in [11] and [12].

## IV. IDENTIFICATION OF JOINT DRIVES GAINS

### A. IDIM Including a Payload

The payload is considered as a link  $n+1$  fixed to the link  $n$  of the robot [13]. Only  $n_L$  of its parameters are considered known. The model (7) becomes:

$$\tau = v_\tau g_\tau = [IDM \quad IDM_{ul} \quad IDM_{kl}] [\chi^T \quad \chi_{ul}^T \quad \chi_{kl}^T]^T + e \quad (9)$$

Where:  $\chi_{kl}$  is the  $(n_L \times 1)$  vector of the known inertial parameters of the payload;  $\chi_{ul}$  is the  $((10-n_L) \times 1)$  vector of the unknown inertial parameters of the payload;  $IDM_{kl}$  is the  $(n \times n_L)$  jacobian matrix of  $\tau_{idm}$ , with respect to the

vector  $\chi_{kl}$ ;  $IDM_{ul}$  is the  $(n \times (10 - n_l))$  jacobian matrix of  $\tau_{idm}$ , with respect to the vector  $\chi_{ul}$ .

### B. Total Least Squares identification of robot dynamic parameters and the drive gains

Details on the *TLS* identification method can be found in [14] and many papers of the same authors. This method has been applied in [4][5][6] for the global identification of the drive gains and the dynamic parameters on a two 6 *DOF* small payload robots. The scaling of parameters uses the accurate value of an additional payload mass. In order to identify the payload parameters, it is necessary that the robot carried out two sets of trajectories: without the payload and with the payload fixed to the end-effector [13]. The over determined system built from the sampling of (9) is the following:

$$Y = \begin{bmatrix} V_{\tau a} \\ V_{\tau b} \end{bmatrix} g_{\tau} = \begin{bmatrix} W_a & 0 & 0 \\ W_b & W_{ul} & W_{kl} \end{bmatrix} \begin{bmatrix} \chi^T & \chi_{ul}^T & \chi_{kl}^T \end{bmatrix}^T + \rho \quad (10)$$

Where:  $V_{\tau a}$  is the matrix of  $v_{\tau}$  samples in the unloaded case;  $V_{\tau b}$  is the matrix of  $v_{\tau}$  samples in the loaded case;

$$V_{\tau i} = \begin{bmatrix} V_{\tau i}^1 & 0 & \dots & 0 \\ 0 & V_{\tau i}^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & V_{\tau i}^n \end{bmatrix}, \quad V_{\tau i}^j = \begin{bmatrix} v_{\tau i,1}^j \\ v_{\tau i,2}^j \\ \vdots \\ v_{\tau i,r/n}^j \end{bmatrix} \text{ with } i = a, b \quad (11)$$

$v_{\tau i,k}^j$  is the  $k$ -th sample of current reference for actuator  $j$ ,  $W_a$  is the observation matrix of the robot in the unloaded case,  $W_b$  is the observation matrix of the robot in the loaded case,  $W_{ul}$  is the observation matrix of the robot corresponding to the unknown payload inertial parameters,  $W_{kl}$  is the observation matrix of the robot corresponding to the known payload inertial parameters.

But owing to the fact that  $W_{kl}(q, \dot{q}, \ddot{q})$  and  $W_{ul}(q, \dot{q}, \ddot{q})$  are correlated by the same noisy data  $(q, \dot{q}, \ddot{q})$ , the total least squares (*TLS*) solution is more adapted [14], rewriting (10) as:

$$W_{tot} \chi_{tot} = \rho \quad (12)$$

Where:

$$W_{tot} = \begin{bmatrix} V_{\tau a} & -W_a & 0 & 0 \\ V_{\tau b} & -W_b & -W_{ul} & -W_{kl} \chi_{kl} \end{bmatrix} \quad (13)$$

$W_{tot}$  is a  $r \times c$  with  $(c = n + n_b + n_{ul} + 1)$  matrix and  $\chi_{tot} = [g_{\tau}^T \quad \chi^T \quad \chi_{ul}^T \quad 1]^T$  is a  $(c \times 1)$  vector.

Without perturbation,  $\rho = 0$  and  $W_{tot}$  must be rank deficient to get the non-null solutions  $\hat{\chi}_{tot} = \lambda \hat{\chi}_{tot}^n \neq 0$  (where  $\hat{\chi}_{tot}^n$  is a vector of unit norm, i.e.  $\|\hat{\chi}_{tot}^n\| = 1$ ) depending on a scale coefficient  $\lambda$ . However because of the measurement perturbations,  $W_{tot}$  is a full rank matrix. Therefore, the system (12) is replaced by the compatible system closest with respect the Frobenius norm:

$$\hat{W}_{tot} \hat{\chi}_{tot} = 0 \text{ with } \hat{\chi}_{tot} = [g_{\tau}^T \quad \chi^T \quad \chi_{ul}^T \quad 1]^T \quad (14)$$

Where  $\hat{W}_{tot}$  is the rank deficient matrix, with the same dimension as  $W_{tot}$ , which minimizes the Frobenius norm:

$$\|W_{tot} - \hat{W}_{tot}\|_F \quad (15)$$

$\hat{\chi}_{tot}$  is the solution of the compatible system (14) closest to (12).  $\hat{W}_{tot}$  is calculated with the singular value decomposition (*SVD*) of  $W_{tot}$ :

$$W_{tot} = U S V^T \quad (16)$$

where  $U$  and  $V$  are orthonormal matrices, and  $S = \text{diag}(s_i)$  is a diagonal matrix with singular values  $s_i$  of  $W_{tot}$  sorted in decreasing order.

$\hat{W}_{tot}$  is given by:

$$\hat{W}_{tot} = W_{tot} - s_c U_c V_c^T \quad (17)$$

where  $s_c$  is the smallest singular value of  $W_{tot}$  and  $U_c$  ( $V_c$ , resp.) the last columns of  $U$  ( $V$ , resp.) corresponding to  $s_c$ . Then, the normalized optimal solution  $\hat{\chi}_{tot}^n$  is given by the last column  $V_c$  of  $V$ ,  $\hat{\chi}_{tot}^n = V_c$ , which belongs to the kernel of  $\hat{W}_{tot}$ .

There is infinity of vectors  $\hat{\chi}_{tot} = \lambda \hat{\chi}_{tot}^n$  which are solutions of (14) depending on a scale factor  $\lambda$ . The unique solution  $\hat{\chi}_{tot}^* = \hat{\lambda} \hat{\chi}_{tot}^n$  for the robot can be found by taking into account that the last value  $\hat{\chi}_{tot_c}^*$  of  $\hat{\chi}_{tot}^*$  must be equal to 1 according to (14), i.e.  $\hat{\lambda} = 1 / \hat{\chi}_{tot_c}^n$ , with  $\hat{\chi}_{tot_c}^n$  the last value of  $\hat{\chi}_{tot}^n$ .

### C. Statistical Analysis

Standard deviations  $\sigma_{\hat{\chi}_i}$ , are estimated assuming that all errors in data matrix  $W_{tot}$  are independently and identically distributed with zero mean and common covariance matrix  $C_{WW}$  such that:

$$C_{WW} = \hat{\sigma}_W^2 I_{r_W} \quad (18)$$

where  $I_{r_W}$  is the identity matrix of dimension  $(r.c.r.c)$ .

An unbiased estimation of the standard deviation  $\hat{\sigma}_W$  is [14]:

$$\hat{\sigma}_W = s_c / \sqrt{r - c} \quad (19)$$

The covariance matrix of the estimation error is approximated by [14]:

$$C_{\hat{\chi}\hat{\chi}} \approx \hat{\sigma}_W^2 \left( 1 + \|\hat{\chi}_{1:c-1}\|_2^2 \right) \left( \hat{W}_{tot_{1:c-1}}^T \hat{W}_{tot_{1:c-1}} \right)^{-1} \quad (20)$$

with  $\hat{\chi}_{1:c-1}$  the vector containing the  $c-1$  first coefficients of  $\hat{\chi}_{tot}^*$  and  $\hat{W}_{tot_{1:c-1}}$  a matrix composed of the  $c-1$  first columns of  $\hat{W}_{tot}$ . Finally,  $\sigma_{\hat{\chi}_i}^2 = C_{\hat{\chi}\hat{\chi}}(i, i)$  is the  $i^{\text{th}}$  diagonal coefficient of  $C_{\hat{\chi}\hat{\chi}}$  and the relative standard deviation  $\% \sigma_{\hat{\chi}_i}$  is given by:  $\% \sigma_{\hat{\chi}_i} = 100 \sigma_{\hat{\chi}_i} / |\hat{\chi}_i|$ , for  $|\hat{\chi}_i| \neq 0$ .

$$\% \sigma_{\hat{\chi}_i} = 100 \sigma_{\hat{\chi}_i} / |\hat{\chi}_i| \text{ for } |\hat{\chi}_i| \neq 0 \quad (21)$$

## V. MODELING THE KUKA KR270 AND ITS SPRING BALANCER

### A. Modeling of the Kuka KR270

The Kuka KR270 (see fig. 1) robot has a serial structure with 6 rotational joints. Its kinematics is defined using the *MDH* notation described in section II-A. The geometric parameters defining the robot frames are given in table I.

All mechanical variables are given in SI unit in joint side. The robot is characterized by a kinematic coupling effect between the joint 4,5 and 6:

$$\begin{bmatrix} \dot{q}_{m_4} \\ \dot{q}_{m_5} \\ \dot{q}_{m_6} \end{bmatrix} = \begin{bmatrix} K_4 & 0 & 0 \\ C_{54} & K_5 & 0 \\ C_{64} & C_{65} & K_6 \end{bmatrix} \begin{bmatrix} \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \end{bmatrix} \quad (22)$$

With  $\dot{q}_{mj}$  is the motor velocity of joint  $j$  and  $\dot{q}_j$  is the joint velocity of joint  $j$ . The values ( $C_{54}, C_{64}, C_{65}$ ) are very low (factor 100) compared to the values ( $K_4, K_5, K_6$ ), therefore the kinematic coupling effect is not considered in the dynamic modeling.

The Kuka KR270 has  $b = 66$  base parameters with spring balancer parameters.

TABLE I. MDH PARAMETERS OF THE KR270 ROBOT

$j$	$\sigma_j$	$\alpha_j$	$d_j$	$\theta_j$	$r_j$
1	0	$\pi$	0	$q_1$	$rl_1 (= -0.750m)$
2	0	$\pi/2$	$d_2 (= 0.350m)$	$q_2$	0
3	0	0	$d_3 (= 1.250m)$	$q_3 + \pi/2$	0
4	0	$-\pi/2$	$d_4 (= 0.055m)$	$q_4$	$rl_4 (= -1.100m)$
5	0	$\pi/2$	0	$q_5$	0
6	0	$-\pi/2$	0	$q_6 - \pi/2$	$rl_6 (= -0.230m)$
$L$	2	0	0	0	0

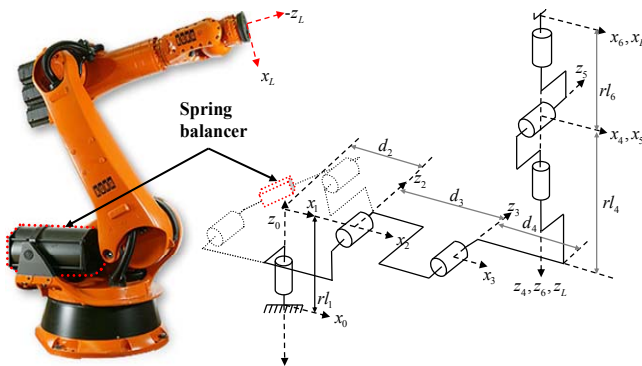


Figure 1. Link frame of the KR270 robot and picture of robot

The support of payload and payload are shown on fig. 2

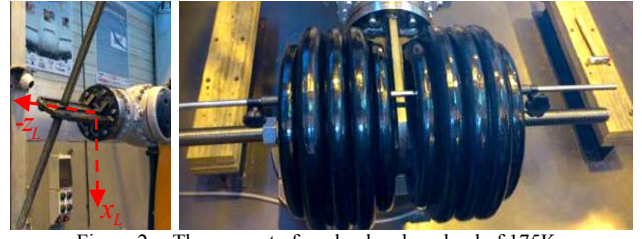


Figure 2. The support of payload and payload of 175Kg

### B. Modeling of the spring balancer

This robot has a spring balancer attached to the second joint. It is composed by a spring and induces a torque on the joint 2. This system compensates the gravity joint torques caused by the weight of robot links 2 to 6. The location of the system on the robots is shown in figure 1.

The torque induced must be taken into account in the dynamic model of the robot to identify all parameters accurately. The kinematics of the spring balancer uses the *MDH* notation in order to compute the compensation torque applied on joint 2. The *MDH* parameters defining the spring balancer frames are given in table II and its kinematics are detailed in figure 3.

TABLE II. MDH PARAMETERS OF THE SPRING BALANCER

$j$	$a(j)$	$\sigma_j$	$\alpha_j$	$d_j$	$\theta_j$	$r_j$
21	20	0	0	$d_{21} (= 0.6964m)$	$q_{21}$	0
22	21	0	0	$d_{22} (= 0.1847m)$	0	0
23	20	0	0	0	$q_{23}$	0
24	23	1	$\pi/2$	0	0	$rl_{24}$

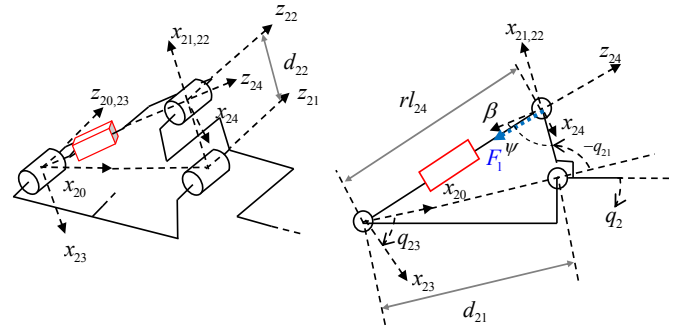


Figure 3. Frames of spring balancer of the KR270 robot

The spring balancer applies a force  $F_1$  on the body 2 in the direction  $-\bar{z}_{24}$ , this force depend on the value  $rl_{24}(q_2)$ :

$$\begin{aligned} F_1 &= K_r (rl_{24}(q_2) - rl_{24\min}) + K_{off} \\ &= K_r rl_{24}(q_2) + K_0 \text{ with } K_0 = K_{off} - K_r rl_{4\min} \end{aligned} \quad (23)$$

Where  $K_r$  ( $N/m$ ) is the stiffness of the spring,  $K_0$  ( $N$ ) is the force applied by the spring when  $rl_4 = rl_{4\min}$  and  $K_{off}$  ( $N$ ) is an offset term.

The compensation torque  $\tau_{sb}$  applied on joint 2 depends on the force  $F_1$ :

$$\begin{aligned}
\tau_{sb} &= F_1 \cos(\beta) d_{22} \\
&= F_1 \cos(\psi(q_2) - 90) d_{22} \\
&= F_1 \sin(\psi(q_2)) d_{22}
\end{aligned} \tag{24}$$

With:

$$\psi(q_2) = q_{23}(q_2) - q_{21}(q_2) - 90 \tag{25}$$

Consequently, the values  $q_{21}(q_2)$ ,  $rl_4(q_2)$  and  $q_{23}(q_2)$  must be expressed function of  $q_2$ .

The angle  $q_{23}(q_2)$  depends directly of  $q_2$ :

$$q_{21} = \phi + q_2 \text{ with } \phi = -1.6148 \text{ (rd)} \tag{26}$$

With  $\phi$  is the value  $q_{21}$  when  $q_2 = 0$ .

To compute  $rl_4(q_2)$  and  $q_{23}(q_2)$ , it is necessary to compute  ${}^{20}T_{22}$  and  ${}^{20}T_{24}$ :

$${}^{20}T_{22} = \begin{bmatrix} {}^{20}A_{22} & {}^{20}P_{22} \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ with } {}^{20}P_{22} = \begin{bmatrix} d_{21} + d_{22} \cos(q_{21}) \\ d_{22} \sin(q_{21}) \\ 0 \end{bmatrix} \tag{27}$$

$${}^{20}T_{24} = \begin{bmatrix} {}^{20}A_{24} & {}^{20}P_{24} \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ with } {}^{20}P_{24} = \begin{bmatrix} rl_{24} \sin(q_{23}) \\ -rl_{24} \cos(q_{23}) \\ 0 \end{bmatrix} \tag{28}$$

The mark position  $R_{22}$  is identical to the position of the marker  $R_{24}$  relative to the reference  $R_{20}$ , so:

$${}^{20}P_{22} = {}^{20}P_{24}, \quad \left\| {}^{20}P_{22} \right\| = \left\| {}^{20}P_{24} \right\| \tag{29}$$

From equations (27), (28) and (29), :

$$rl_{24}^2 = (d_{21} + d_{22} \cos(q_{21}))^2 + (d_{22} \sin(q_{21}))^2 \tag{30}$$

$$\tan(q_{23}) = \frac{\sin(q_{23})}{\cos(q_{23})} = -\frac{d_{21} + d_{22} \cos(q_{21})}{d_{22} \sin(q_{21})} \tag{31}$$

$rl_4(q_2)$  and  $q_{23}(q_2)$  are calculated from equations (26), (30) and (31):

$$rl_{24}(q_2) = (d_{21}^2 + d_{22}^2 + 2d_{21}d_{22} \cos(\phi + q_2))^{1/2} \tag{32}$$

$$q_{23}(q_2) = \text{atan} \left( -\frac{d_{21} + d_{22} \cos(\phi + q_2)}{d_{22} \sin(\phi + q_2)} \right) \tag{33}$$

The gravity compensation torque  $\tau_{sb}$  is expressed in terms of  $K_r$  and  $K_0$ :

$$\tau_{sb} = K_r rl_{24}(q_2) \sin(\psi(q_2)) d_{22} + K_0 \sin(\psi(q_2)) d_{22} \tag{34}$$

The gravity compensation torque  $\tau_{sb}$  can be expressed linearly to the parameters  $K_r$  and  $K_0$ :

$$\begin{aligned}
\tau_{sb} &= IDM_{sb}(q_2) \chi_{sb} \\
&= \begin{bmatrix} rl_{24}(q_2) \sin(\psi(q_2)) d_{22} \\ \sin(\psi(q_2)) d_{22} \end{bmatrix}^T \begin{bmatrix} K_r \\ K_0 \end{bmatrix}
\end{aligned} \tag{35}$$

This torque is included in the  $IDM$  (2) and  $\chi_{st}$  is composed of standard dynamic parameters plus  $K_r$  and  $K_0$ :

$$\begin{aligned}
\tau_{idm} &= \begin{bmatrix} IDM_{1..2st} & IDM_{sb} & IDM_{3..n st} \end{bmatrix} \chi_{st} \\
\text{with } \chi_{st} &= \begin{bmatrix} \chi_{st1}^T & \chi_{st2}^T & \chi_{sb}^T & \chi_{st3}^T & \cdots & \chi_{stm}^T \end{bmatrix}^T
\end{aligned} \tag{36}$$

## VI. EXPERIMENTAL VALIDATION

The global identification of spring balancer, dynamic parameters and joint drive gains of the Kuka KR270 robot is performed. The sample acquisition frequency for joint positions and current reference is 500Hz. The cut-off frequency of the decimate filter is fixed at 20Hz. The mass  $ML$  has been weighed to 175(Kg).

Only the relevant parameters are given in table III, the other parameters are not significant because their relative standard deviations are large. Identified parameters (SI Units)

Parameter $s$	$\hat{\chi}^3$	$\sigma_{\hat{\chi}_i}$ (%)	$\chi^{op}$	$ e(\%) $
$g\tau_1$	<b>-2.76 10<sup>2</sup></b>	1.15	<b>-2.99 10<sup>2</sup></b>	<b>8.33</b>
$g\tau_2$	<b>-4.27 10<sup>2</sup></b>	1.35	<b>-3.89 10<sup>2</sup></b>	<b>9.77</b>
$g\tau_3$	<b>-4.02 10<sup>2</sup></b>	1.23	<b>-3.67 10<sup>2</sup></b>	<b>9.54</b>
$g\tau_4$	<b>-2.95 10<sup>2</sup></b>	1.42	<b>-2.58 10<sup>2</sup></b>	<b>14.3</b>
$g\tau_5$	<b>-3.06 10<sup>2</sup></b>	1.25	<b>-2.58 10<sup>2</sup></b>	<b>18.6</b>
$g\tau_6$	<b>2.15 10<sup>2</sup></b>	1.32	<b>1.83 10<sup>2</sup></b>	<b>17.4</b>
$ZZ_{1R}$	$1.19 \cdot 10^3$	0.63		
$Fv_1$	$1.09 \cdot 10^3$	0.75		
$Fc_1$	$3.01 \cdot 10^2$	1.42		
$XX_{2R}$	$-6.23 \cdot 10^2$	1.38		
$ZZ_{2R}$	$1.51 \cdot 10^3$	0.54		
$MX_{2R}$	$3.44 \cdot 10^2$	2.15		
$Fv_2$	$2.63 \cdot 10^3$	1.28		
$Fc_2$	$6.42 \cdot 10^2$	3.01		
$Off_2$	$-1.53 \cdot 10^2$	1.98		
$K_r$	<b>5.04 10<sup>4</sup></b>	3.9		
$K_0$	<b>-7.85 10<sup>4</sup></b>	2.1		
$XX_{3R}$	$9.96 \cdot 10^1$	5.6		
$ZZ_{3R}$	$1.20 \cdot 10^2$	3.2		
$MX_{3R}$	$1.45 \cdot 10^1$	3.3		
$MY_{3R}$	$-1.02 \cdot 10^2$	0.72		
$Ia_3$	$6.80 \cdot 10^2$	0.78		
$Fv_3$	$1.02 \cdot 10^2$	0.51		
$Fc_3$	$4.02 \cdot 10^2$	1.98		
$Ia_4$	$1.70 \cdot 10^2$	1.95		
$Fv_4$	$5.26 \cdot 10^2$	1.62		
$Fc_4$	$1.85 \cdot 10^2$	4.22		
$Ia_5$	$2.09 \cdot 10^2$	1.43		
$Fv_5$	$8.45 \cdot 10^2$	1.21		
$Fs_5$	$1.99 \cdot 10^2$	4.32		
$Ia_6$	$1.31 \cdot 10^2$	2.42		
$Fv_6$	$7.82 \cdot 10^2$	2.10		
$Fc_6$	$3.05 \cdot 10^3$	5.56		
$MZL$	$-3.83 \cdot 10^1$	1.81		
$ML$	<b>1.75 10<sup>2</sup></b>	-		

The mean relative error  $|\bar{e}|$  between the identified and manufacturer's values of  $\hat{g}_r$  is 13%. It is important error and it shows the importance of the identification of joint drive gains.

The spring balancer parameters are well identified with low relative standard deviation with  $K_r = 5.04 \cdot 10^4$  (N/m) and  $K_0 = -7.85 \cdot 10^4$  (N).

The identification of drive gains and spring balancer parameters allows to perform an accurate dynamic identification of robot.

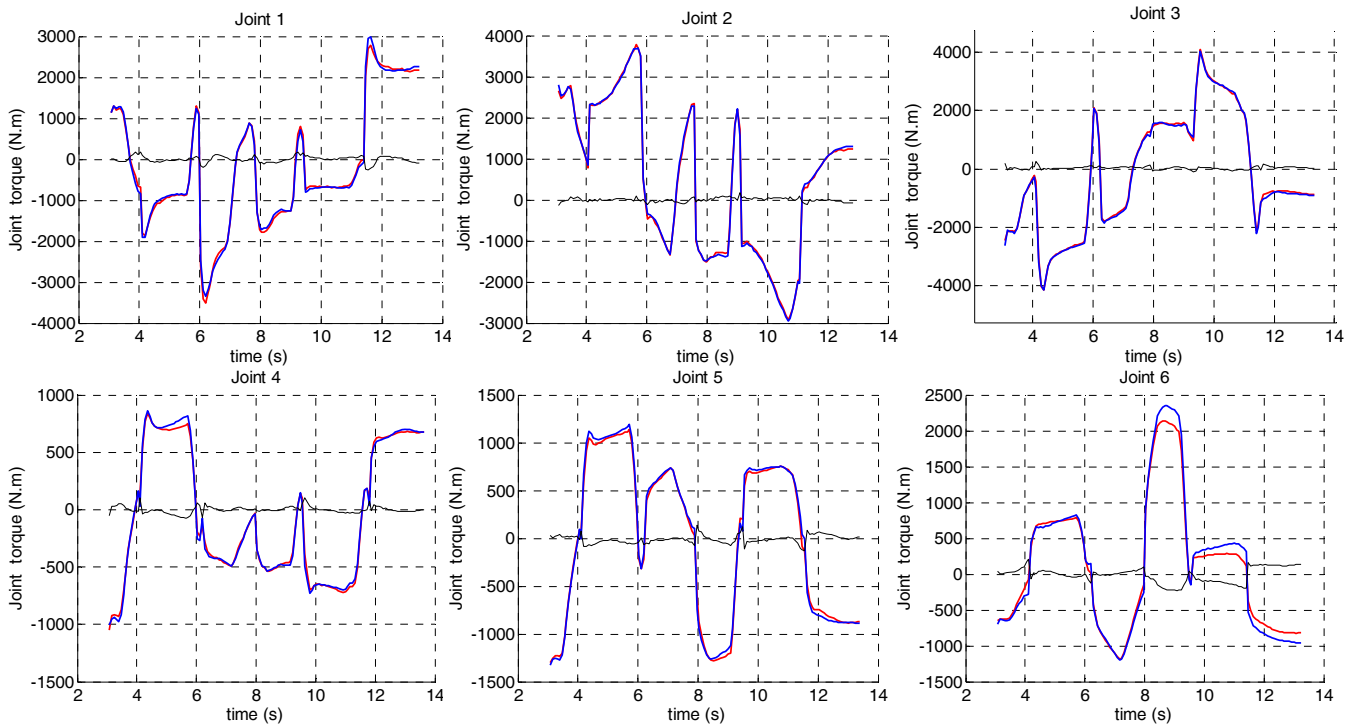
## VII. CONCLUSION

In this paper, the global identification of a 6 *DOF* heavy industrial robot was performed. The identification of drive gains, spring balancer parameters, dynamic parameters was performed. A modeling of its spring balancer was proposed and it is relevant for identification. The *IDIM-TLS* method can be used on heavy industrial robot without manufacturer

modeling and parameters of spring balancer to get accurate identification of joint drive gains and dynamic parameters.

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Cross-test validation (red: measure  $Y$ , blue: estimation  $W\chi$ , black: error  $Y - W\chi$ )

## REFERENCES

- [1] J. Hollerbach, W. Khalil, and M. Gautier, "Model Identification," in *Springer Handbook of Robotics*, Springer, 2008.
- [2] P. P. Restrepo and M. Gautier, "Calibration of drive chain of robot joints," *IEEE Conference on Control Applications*, 1995, pp. 526–531.
- [3] B. Corke, "In situ measurement of motor electrical constants," *Robotica*, vol. 14, no. 4, pp. 433–436, 1996.
- [4] M. Gautier and S. Briot, "New method for global identification of the joint drive gains of robots using a known inertial payload," *IEEE Conference on Decision and Control and European Control*, 2011, pp. 1393–1398.
- [5] M. Gautier and S. Briot, "New method for global identification of the joint drive gains of robots using a known payload mass," *IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2011, pp. 3728–3733.
- [6] M. Gautier and S. Briot, "Global identification of drive gains parameters of robots using a known payload," *IEEE International Conference on Robotics and Automation*, 2012, pp. 2812–2817.
- [7] W. Khalil and E. Dombre, *Modeling, Identification and Control of Robots*, 3rd edition. New York: Taylor and Francis Group, 2002.
- [8] M. Gautier and W. Khalil, "Direct calculation of minimum set of inertial parameters of serial robots," *IEEE Transactions on Robotics and Automation*, pp. 368–373, 1990.
- [9] H. Mayeda, K. Yoshida, and K. Osuka, "Base parameters of manipulator dynamic models," *IEEE Transactions on Robotics and Automation*, pp. 312–321, 1990.
- [10] M. Gautier and W. Khalil, "Exciting trajectories for the identification of base inertial parameters of robots," *IEEE Conference on Decision and Control*, 1991, pp. 494–499.
- [11] M. Gautier, "Dynamic identification of robots with power model," *IEEE International Conference on Robotics and Automation*, 1997, vol. 3, pp. 1922–1927.
- [12] M. T. Pham, M. Gautier, and P. Poignet, "Identification of joint stiffness with bandpass filtering," *IEEE International Conference on Robotics and Automation*, 2001, vol. 3, pp. 2867–2872.
- [13] W. Khalil, M. Gautier, and P. Lemoine, "Identification of the payload inertial parameters of industrial manipulators," *IEEE International Conference on Robotics and Automation*, 2007, pp. 4943–4948.
- [14] S. Van Huffel and J. Vandewalle, *The Total Least Squares Problem: Computational Aspects and Analysis*. Philadelphia, Pennsylvania: SIAM, 1991.