A16. Tangents Relating to Sets of Integers

This appendix briefly summarizes some other avenues explored to compute permanents efficiently (Chapter 5). One avenue led to an analysis of unique characterizations of a set of positive integers, e.g., in terms of its sum and product $\{\Sigma, \Pi\}$ and, slightly more generally, in terms of $\{\Sigma, \Pi, n\}$ with n being the number of integers in the set.

A related topic that was investigated began with a definition of the set of U-Primes (unique primes): A number $q > 1$ is a U-Prime if and only if it cannot be formed as the product of a subset of smaller U-Primes. This definition distinguishes U-Primes from ordinary primes, or O-Primes, by the use of the word "subset." For example, the number 4 is not an O-Prime, but it is a U-Prime because 4 cannot be expressed as the product of a subset of smaller U-Primes.

It turns out that the set of U-Primes can be generated as the set $\{p_j^{2^i}; i \geq 0, j \geq 1\}$, where p_j is the jth ordinary prime. This makes sense intuitively because the number of factors n of p_j in the prime factorization of a given integer can be expressed as the sum of the powers of two as in the binary number system.

The value of U-Primes is that they form a new binary number representation system. Specifically, the number represented by a string of n binary digits in this system is determined by taking the product of the U-Primes corresponding to the positions of the nonzero digits. U-Primes can be multiplied efficiently on a digit-by-digit basis: If the digits in position i of both multiplicands are zero, then the digit in position i of their product is zero. If the digits differ, then the digit in position i of the product is 1. And if the digits in position i of both multiplicands are 1, then the digit in position i of the product is zero and the digit in position j corresponding to the square of the U-Prime associated with position i is 1. If the digits in position j of the multiplicands differ, then the digit in position j is reset to zero and the position corresponding to the square of the U-Prime associated with position j is set to 1. This general quadratic carry rule is straightforward to determine for all other cases. It is also similarly strightforward to perform division by carrying in the direction of the square root of U-Primes.