Optimal Guaranteed Cost Control of a Biomimetic Robot Arm

Francesco Amato †, Domenico Colacino †, Carlo Cosentino †, Alessio Merola †

Abstract—In this paper, an optimal control problem for uncertain bilinear systems is formulated via a guaranteed cost approach and then applied to the design of a stabilizing controller for a robot arm actuated by Pneumatic Artificial Muscles (PAMs). The results show that the contributed methodology is suitable for efficiently designing control systems which can match the requirements both on safety and on energy efficiency for PAMs-driven robots during human-robot interactions. The performances of the state-feedback control system are evaluated on the basis of some numerical simulations.

Index Terms—Human-robot interaction, safe robot, pneumatic artificial muscle, guaranteed cost control

I. INTRODUCTION

In recent years, several applications in the fields of rehabilitation robotics, prosthetics, and surgical robotics have motivated the growing interest in the design of compliant and safe robots.

In conventional applications of industrial robots, the stiffness of the robot joints is a must to guarantee, e.g., the precise positioning of the end effector both for pick and place operations and for trajectory tracking. However, robots with stiff joints cannot share their workspace with humans, since inflexible movements can be harmful for the human operator in the case of an unexpected interaction, such as an impact between the robot and the human operator.

In contrast to the conventional actuators of industrial robots, McKibben artificial muscles, or Pneumatic Artificial Muscles (PAMs) actuators, have the natural compliance which is useful to absorb large position error, while avoiding hazards during human-robot interactions. PAMs consist of an air bladder which contracts when inflated. Such actuators, which can have similar features compared to human skeletal muscle (see [1]), possess high power\weight and power\volume ratios [2]–[3]. The inherent (and adjustable) compliance of PAMs is fundamental both to guarantee the confort of the patient in rehabilitation robots [4]–[5] and to design energy-efficient walking robots [6]. High energy efficiency and limited energy consumption are also important issues, e.g., in wearable exoskeleton robots [7], where energy-efficient PAMs actuators allow to design lightweight robots.

It should be noted that the highly nonlinear dynamics of PAMs limits the control possibilities for PAMs driven robots. In the context of the modelling of PAMs, some static length-tension relations were proposed in [1], [3]. To gain deeper insight into the dynamical features of PAMs, a phenomenological model was proposed in [8]. In a subsequent model, the stiff and viscous dynamical behaviour was characterized in [9] by adopting a parallel configuration of a spring and a damping element. Finally, in [10] the phenomenological model was augmented by an active force element.

A pair of PAMs can be arranged in an antagonistic configuration to drive a revolute joint. Taking the bladder pressure as control input and the kinematic variables of the joint (angular position and velocity) as state variables, the resulting control system is a bilinear system, once the model in [10] is adopted. The complexity of the models of PAMs actuators mainly derives from the nonlinear relation of the contracting force to the internal bladder pressure. Moreover, PAMs exhibit an hysteretic behaviour during inflation/deflation cycles. The hysteresis effects are mainly due to the nonlinear friction within the bladder wall. Few models focused on hysteresis [1], [11], [12].

The lack of a comprehensive description of the dynamical behaviour of PAMs gives rise to model uncertainties which pose some robustness issues for the control of PAMs actuators.

In the field of the design of the control system for PAMs, research efforts was devoted in [9], where a gain scheduling method was applied to an experimental model. Sliding Mode Control (SMC) [13], adaptive backstepping [14] and variable structure control [15] were also investigated. In some case, such methodologies provide some control laws which are not robust while, in other cases, the designed controller is nonlinear and complex.

The position control of robots actuated by PAMs can benefit from robustness of SMC. However, SMC can suffer from the chattering problem which is detrimental to the positioning performance.

Inherently robust control laws are expected to overcome the issues on the model uncertainty, without loss of the control performance due to unmodelled variability of the system parameters.

In this respect, a novel approach to the control of PAMs actuators, i.e. the optimal guaranteed cost control of bilinear systems, is proposed here. Generally speaking, the guaranteed cost approach allows to cope with optimal control problems within the framework of the linear state feedback control. The proposed approach derives from the methodology conceived in [16] and, more recently, in the context of linear uncertain systems in [17].

This paper investigates for the first time a robust and optimal control approach with safety and energy constraints for PAMs driven robots. The advantages of the proposed
approach are multiple:
1) Robustness of the control performance against parametric uncertainties;
2) Energy-efficient control, since the control law minimizes the energy of the input pressure;
3) Safety enhancement during human-robot interactions through adjustable joint compliance;
4) Ease of implementation in real control systems thanks to a linear state feedback control law.

The remainder of the paper is organized as follows. Section II presents the model of a robot with single link actuated by a pair of PAMs in antagonistic configuration. A characterization of the joint compliance is also made. Section III describes the optimal control methodology for uncertain bilinear systems, whereas Section IV presents the application of the methodology to the stabilization of the upright stance of the robot arm against simulated collisions. The simulation results show the tracking performance of the closed loop system, along with the safety and energy-saving features.

II. Dynamic Model of a Robot Arm Actuated by Pneumatic Artificial Muscles

The robot arm dealt with here is actuated by a pair of PAMs in an antagonistic configuration. The dynamic model of PAMs is adapted from [10]. The force actuated by each PAM is due to the contribution of an active force generator, a nonlinear spring and a nonlinear damper, as in

\[
\phi = f(p) - b(p)s - k(p)s,
\]

where \( \phi \) is the total force generated by the PAM, \( p \) denotes the pressure at which the PAM is inflated, \( s \) is the contraction length with respect to the resting position. The constitutive relations of the PAM dynamics are

\[
f(p) = f_0 + f_p|N|,
\]

\[
b(p) = b_0 + b_p|N/m/s|,
\]

\[
k(p) = k_0 + k_p|N/m|.
\]

The values of the parameters of the damping function \( b(p) \) vary depending on the phase of inflation\( \backslash \)deflation. The hysteresis effects due to the friction in the PMs bladder walls can be embodied as additional source of variability in the damping parameters \( b_0 \) and \( b_1 \).

Assuming the antagonistic configuration of Figure 1, in analogy with the human anatomy, the PAMs of the pair are denoted as \textit{biceps} and \textit{triceps}, respectively. To drive a revolute joint it is required to use a pair of PAMs, since each actuator only works in the contraction direction. Both PAMs are tied together around a pulley of radius \( r \). The torque imparted by the PAMs pair to the robot link is

\[
\tau = \tau_b - \tau_t = (\phi_b - \phi_t)r, \tag{1}
\]

where \( r \) is the pulley radius and \( \tau_b \) and \( \tau_t \) are the torques exerted by each PAM, as in

\[
\tau_b = (f_b - b_b s_b - k_b s_b)r, \tag{2a}
\]

\[
\tau_t = (f_t - b_t s_t - k_t s_t)r. \tag{2b}
\]

In (2), \( s_b \) and \( s_t \) denote the contraction length of the biceps and of the triceps, respectively.

The total torque can be expressed in terms of the joint angle \( \theta \), using the kinematic relations

\[
s_b = r\left(\theta + \frac{\pi}{2}\right), \quad s_t = r\left(\frac{\pi}{2} - \theta\right).
\]

The robot link is actuated by the pressure difference \( \Delta p \) from a nominal pressure at each muscle. In particular,

\[
p_b = p_0_b + \Delta p, \quad p_t = p_0_t - \Delta p,
\]

where \( p_0_b \) and \( p_0_t \) are positive nominal pressures and \( \Delta p \) is the input pressure which is assumed as the control action within the closed loop system.

By adjusting both the nominal pressure \( p_0 \) and the input pressure \( \Delta p \), it is possible to calibrate the minimum compliance of the robot joint in order to satisfy the safety requirements during human-robot interactions. In this respect, it is useful to derive the expression of the joint compliance.

Taking into account the spring terms from (1)-(2), the joint compliance \( c \) is defined as the ratio between the joint angle and the spring torque \( \tau_s \), as in

\[
c = \theta/\tau_s = \theta/[2r^2(k_0\theta + \pi k_1 \Delta p + k_1 p_0 \theta)]. \tag{3}
\]

In the above equation, it is assumed that the PAMs are inflated at the same nominal pressure \( p_0 = p_0_b = p_0_t \). Moreover, since the two PAMs are identical, it is licit to let \( k_0_b = k_0_t = k_0 \) and \( k_1_b = k_1_t = k_1 \).

III. Guaranteed Cost Control of Uncertain Bilinear Systems

The class of uncertain nonlinear systems dealt with here can be described by the following state-space representation

\[
\dot{x}(t) = (A + \Delta A)x(t) + (B + \Delta B)u(t) + N(x(t), u(t)) + \Delta N(x(t), u(t)) \tag{4}
\]

\[
x(0) = x_0
\]

where \( x(t) \in \mathbb{R}^n \) is the system state, \( u(t) \in \mathbb{R}^m \) is the control input, \( x_0 \) is the initial state. The nonlinearities are described by the bilinear matrix function

\[
N(x, u) = (N_1^T x \quad N_2^T x \quad \ldots \quad N_n^T x)^T u, \tag{5}
\]
with known constant matrices $N_i \in \mathbb{R}^{n \times m}$, $i = 1, \ldots, n$. The matrices $\Delta A$ and $\Delta B$ describe the parametric uncertainties of the linear part of system (4).

The uncertainties in the nonlinear term (5) are represented by

$$\Delta N(x, u) = (\Delta N_1^T x \quad \Delta N_2^T x \quad \ldots \quad \Delta N_n^T x) u,$$

(6)

where $\Delta N_i$, $i = 1, \ldots, n$ denote parameter-varying matrices of appropriate dimensions.

It is assumed that system (4) is subject to norm-bounded parametric uncertainties of the form

$$[\Delta A \quad \Delta B \quad \Delta N_1 \quad \ldots \quad \Delta N_n] = DF(t) \begin{bmatrix} E_1 & E_2 & S_1 & \ldots & S_n \end{bmatrix},$$

where $F(t)$ a is Lebesgue measurable matrix function satisfying

$$F^T(t) F(t) \leq I,$$

with $I$ identity matrix and $D, E_1, E_2, S_1, \ldots, S_n$ are known constant matrices of appropriate dimensions.

Furthermore, the following set of constraints on the control input of system (4) are specified

$$|u_i(t)| \leq u_{i,\text{max}},$$

(7)

where $u_{i,\text{max}}, \ i = 1, \ldots, n$ denote prescribed peak bounds on each component of $u(t)$.

For the uncertain system (4), consider the static state feedback control law

$$u(t) = K x(t),$$

(8)

where $K \in \mathbb{R}^{m \times n}$ is the control gain matrix.

The resulting closed loop system is defined as

$$\dot{x} = (A + DFE_1 + BK + DFE_2 K)x + \left(K^T (N_1 + DFS_1) T x \ldots K^T (N_n + DFS_n) T x\right) x,$$

(9)

Associated to the closed loop system (9) is the quadratic cost function

$$J := \int_0^\infty \left(x^T(t)Q x(t) + u^T(t)Ru(t)\right) dt,$$

(10)

$Q$ and $R$ are symmetric positive matrices which weight the error index function $\int_0^\infty x^T(t)x(t)dt$ and the energy consumption index function $\int_0^\infty u^T(t)u(t)dt$, respectively.

Now, the definition of robust guaranteed cost controller with input constraints can be stated for the class of uncertain nonlinear systems under consideration.

Definition 1: Given the constraints (7) and the cost function (10), the static state feedback controller (8) is said to be a Robust Guaranteed Cost Controller (RGCC) for the uncertain system (4), for all admissible uncertainties, if the following hold:

i) The cost index (10) for the closed loop system (9) satisfies

$$J \leq \bar{J},$$

with $\bar{J} > 0$, for an admissible set of initial conditions $\mathcal{D}, 0 \in \mathcal{D}$. The upper bound $\bar{J}$ is likely to be optimized.

ii) The input constraints (7) are satisfied whenever $x_0 \in \mathcal{D}$.

iii) The admissible set $\mathcal{D}$ is included into the Domain of Attraction (DA) of the closed loop system (9)\(^1\).

The methodological contribution of this paper concerns the design of a RGCC for the class of uncertain nonlinear systems (4) according to Definition 1.

As preparatory results, two lemmas are presented.

Lemma 1 ([18]): Let $\Omega_1, \Omega_2, \Omega_3$ be given constant matrices of appropriate dimensions and the matrix function $M(t)$ satisfying $M^T(t)M(t) \leq I$. Then, for any scalar $\epsilon > 0$,

$$\Omega_1 M(t) \Omega_2 + \Omega_2^T M^T(t) \Omega_1^T \leq \epsilon \Omega_1 \Omega_1^T + \epsilon^{-1} \Omega_2^T \Omega_2.$$

Lemma 2: Given a set of initial conditions $\mathcal{D}, 0 \in \mathcal{D}$, and the cost index (10), assume there exist an invariant set $\mathcal{E} \subset \mathbb{R}^n$ for system (9), $\mathcal{E} \supset \mathcal{D}$, a symmetric positive definite matrix $P$, a matrix $K$ and a quadratic Lyapunov function $v(x) = x^T P x$ such that, for all allowable uncertainties,

$$x^T \left(Q + \frac{K^T R K}{2} + P [A + BK + DFE_1 + DFE_2 K] \right. \left. \begin{bmatrix} x^T \left(N_1 + DFS_1 K \right) & \vdots & x^T \left(N_n + DFS_n K \right) \end{bmatrix} \right. \left. + \left(\left(A + BK + DFE_1 + DFE_2 K \right)^T + \left(K^T \left(N_1 + DFS_1 \right) T x \ldots K^T \left(N_n + DFS_n \right) T x \right) P \right) x \right) < 0, \ \forall x \in \mathcal{E}$$

(11)

Then, the state feedback controller (8) is a RGCC for uncertain system (4). Moreover, the upper bound of the performance cost of the closed loop system is given by $\bar{J} = x_0^T P x_0$ for all $x_0 \in \mathcal{E}$ (hence $x_0 \in \mathcal{D}$).

Proof: See the Appendix.

The definition of polytopic sets is recalled here.

Definition 2: A polytope $\mathcal{P} \subset \mathbb{R}^n$ can be described as follows:

$$\mathcal{P} = \text{conv} \left\{ x(1), x(2), \ldots, x(p) \right\} = \{ x \in \mathbb{R}^n : a_k^T x \leq 1, k = 1, 2, \ldots, q \},$$

(12a)

(12b)

where $p$ and $r$ are suitable integers, $x(i)$ denotes the $i$-th vertex of the polytope $\mathcal{P}$, $a_k \in \mathbb{R}^n$ and $\text{conv}\{\}$ denotes the operation of taking the convex hull of the argument.

Remark 1: In most of the engineering applications, the operative range of a nonlinear system, which is assigned in terms of the variations of the state variables, can be described by polytopic sets.

The main result of this paper resides in the next theorem where it assumed that the set of initial conditions $\mathcal{D} = \mathcal{P}$ has a polytopic structure.

Theorem 1: For the uncertain system (4), given a polytopic set of initial conditions $\mathcal{P}$ and the performance cost (10), if there exist scalars $\bar{J}, \bar{J} > 0$, $\gamma$ and $\epsilon_1, \epsilon_2 > 0$, a

\(^1\)For the sake of simplicity, we adopt the statement “the DA of the closed loop system” in place of “the DA of the zero equilibrium point of the closed loop system”.

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matrix $Y$ and symmetric positive definite matrices $X$ and $Z$ such that
\begin{equation}
0 < \gamma < 1
\end{equation}
holds for all $x \in \rho \mathcal{P}$. Pre- and post-multiplying the left-hand side of (14) by $J^{1/2}P^{-1}$ and letting $X = JP^{-1}$ and $Y = KK, (14)$ can be rewritten as
\begin{equation}
\begin{align*}
J^{-1}XQX + J^{-1}Y^TRY + AX + BY \\
+ DF(E_1X + E_2Y) + \left( x^T(N_1 + DFS_1)Y \right) \\
+ (AX + BY)^T + [DF(E_1X + E_2Y)]^T \\
+ (Y^T(N_1 + DFS_1)x \ldots Y^T(N_n + DFS_n)x) \\
< 0, \hspace{1cm} \forall x \in \rho \mathcal{P}
\end{align*}
\end{equation}
According to the Schur complements (see [19], p.7), the above inequality can be rearranged as
\begin{equation}
\begin{align*}
\Omega(x) & := AX + BY + (AX + BY)^T \\
+ \left( \begin{array}{c}
\begin{array}{c}
\vdots \\
x_{(i)}^T \end{array}
\end{array} \right) \begin{array}{c}
\vdots \\
\vdots
\end{array} + \left( \begin{array}{c}
\vdots \\
x_{(i)}^T \end{array} \right) \\
\Theta(x) & := \begin{array}{c}
\vdots \\
x_{(i)}^T \end{array} \begin{array}{c}
\vdots \\
\vdots
\end{array} \begin{array}{c}
\vdots \\
x_{(i)}^T \end{array} \\
\vdots \\
\vdots
\end{equation}
By applying Lemma 1, inequality (16) is satisfied for all $F$, with $F^T F \leq I$ if there exist some positive scalars $\epsilon_1, \epsilon_2$ such that
\begin{equation}
\begin{align*}
DF(E_1X + E_2Y) + [DF(E_1X + E_2Y)]^T & \leq \epsilon_1 DD^T + \epsilon_1^{-1} (E_1X + E_2Y)^T (E_1X + E_2Y), \\
\Theta(x) + \Theta^T(x) & \leq \epsilon_2 \begin{array}{c}
\vdots \\
x_{(i)}^T \end{array} \begin{array}{c}
\vdots \\
x_{(i)}^T \end{array} \begin{array}{c}
\vdots \\
x_{(i)}^T \end{array} \\
\vdots \\
\vdots \\
\vdots
\end{align*}
\end{equation}
From the above inequalities, by applying the Schur complements again, (16) becomes
\begin{equation}
\begin{align*}
\Xi(x) & := AX + BY + (AX + BY)^T \\
& + \left( \begin{array}{c}
\begin{array}{c}
\vdots \\
x_{(i)}^T \end{array}
\end{array} \begin{array}{c}
\vdots \\
x_{(i)}^T \end{array} \\
\vdots \\
\vdots
\end{align*}
\end{equation}
with

\[ \Xi(x) := \Omega(x) + \epsilon_1 DDT, \]
\[ \Pi(x) := \epsilon_2 \begin{pmatrix} x^T D & \ldots & 0 \\vdots & \ddots & \vdots \\ldots & \ldots & \ldots \\ldots & \ldots & \ldots \\ldots & \ldots & \ldots \\ldots & \ldots & \ldots \\ldots & \ldots & \ldots \end{pmatrix}^T. \]

The matrix function at the left-hand side of (18) is an affine function of the state variables, therefore it is negative definite on \( \rho P \) iff the property holds at the vertices of the polytope (see [20], Ch. 3.), then

\[
\begin{pmatrix}
\Xi(\rho x(i)) & X & Y^T & W^T & M^T & \Pi^T(\rho x(i)) \\
X & -\bar{J}Q^{-1} & 0 & 0 & 0 & 0 \\
Y & 0 & -\bar{J}R^{-1} & 0 & 0 & 0 \\
W & 0 & 0 & -\epsilon_1 I & 0 & 0 \\
M & 0 & 0 & 0 & -\epsilon_2 I & 0 \\
\Pi(\rho x(i)) & 0 & 0 & 0 & 0 & -\epsilon_2 I
\end{pmatrix}
< 0, \quad i = 1, 2, \ldots, r. \tag{19}
\]

By multiplying both the sides of (19) by \( \gamma \), it follows that (19) is equivalent to (13f).

Now the proof proceeds through the following steps.

i) Letting \( X = \bar{J}P^{-1} \) and \( Y = KX \), it follows that (13e) is equivalent to

\[
\begin{pmatrix} Z \\bar{J}P^{-1} \end{pmatrix} \geq 0, \tag{20}
\]

Using the Schur complements, it is straightforward to verify that (20) yields

\[ JKP^{-1}K^T \leq Z \]

By denoting the \( i \)-th row of the matrix \( K \) by \( K_i \), the following hold

\[
|u_i|^2 = |K_ix(t)|^2 = |K_iP^{-1/2}P^{1/2}x(t)|^2 \\
\leq \|K_iP^{-1/2}\| \|K_iP^{1/2}\|^2 \\
= K_iP^{-1}K_i^T x^T(t)Px(t) \\
\leq K_iP^{-1}K_i^T J \leq (Z)_{ii} \tag{21}
\]

From condition (13e), (21) allows to conclude that the control law (8) satisfies the input constraint.

ii) From the Schur complements and \( X = \bar{J}P^{-1} \), it is straightforward to recognize that condition (13c) guarantees that the ellipsoid

\[ \mathcal{E} = \{ x \in \mathbb{R}^n, x^T Px \leq \bar{J} \}. \tag{22} \]

contains the polytope \( P \) (see [19], p. 69). Therefore, for all \( x_0 \in \mathcal{E} \) (hence \( x_0 \in P \) \( v(x) \leq \bar{J} \).

iii) By the application of the Schur complements, letting \( X = \bar{J}P^{-1} \) and \( \gamma = 1/\rho \), (13b) is equivalent to

\[
\frac{a_k^T}{\rho} \bar{J}P^{-1} x_{k} \leq 1, \quad k = 1, 2, \ldots, q, \tag{23}
\]

which implies \( \rho P \supset \mathcal{E} \) (see [19], p. 70). Such an inclusion guarantees that \( \hat{v}(x) \) is negative definite on the invariant set \( \mathcal{E} \).

The point i) and ii) allow to apply Lemma 2, along with the fact \( \mathcal{E} \) is an invariant set. The inclusion \( P \subset \mathcal{E} \) completes the proof.

For a given \( \gamma \in (0, 1) \), the conditions of Theorem 1 are a set of Linear Matrix Inequalities (LMIs) [19] in the variables \( \bar{J}, \epsilon_1, \epsilon_2, X, Y, Z \), which can be solved via available software [21]. A one parameter search for \( \gamma \) over the interval \((0, 1)\) is necessary to solve the optimization problem.

In particular, for a given scalar \( \gamma \in (0, 1) \), the design problem for the optimal guaranteed cost controller can be formalized through the following convex optimization problem

**Problem 1:**

\[
\begin{array}{c}
\min \\
J_{\epsilon_1, \epsilon_2, X, Y, Z, \bar{J}} \\
s.t. \quad X > 0, \quad Z > 0, \quad (13b), \quad (13c), \quad (13d), \quad (13e), \quad (13f)
\end{array}
\]

If Problem 1 has an optimal solution, then \( u(t) = YX^{-1}x(t) \) a RGCC satisfying the input constraint (7) for system (4). For all admissible uncertainties and initial conditions, the worst-case performance cost of the closed loop system (9) is \( \bar{J} \).

**IV. APPLICATION OF A GUARANTEED COST CONTROL SCHEME TO THE BIO-MIMETIC ROBOT ARM:**

**SIMULATION RESULTS**

In the context of the application to biomimetic robots interacting with humans, the objective of the guaranteed cost control strategy is to keep the upright stance of the robot arm during a collision between the robot and a human operator, while reducing the risk of injuries for the human operator. It is assumed that these collisions, which occur in an impulsive fashion, alter the zero initial condition (of position and of velocity) of the robot arm in upright stance. Whenever the perturbed initial state of the robot arm is within some admissible bounds, i.e. into a prescribed polytope, the guaranteed cost controller takes some corrective actions to recover the vertical position, achieving an optimal and safe control performance. The optimality of the control performance resides in the fact that the upper bound \( \bar{J} \) of the cost (10) for the closed loop system is minimized; thus guaranteeing that the angular deviation of the arm is regulated to zero against perturbations, with satisfactory dynamical performance and minimum control effort.

The dynamics of the robot arm is described as

\[
\ddot{\theta} = \alpha \theta + (\beta - \zeta) \dot{\theta} + \delta \Delta p + \sigma \hat{\theta} \Delta p + \tau_g,
\]

with

\[
\alpha = - (2r^2/I) (K_0 + K_1 p_0), \\
\beta = - (r^2/I) [b_{kl} + b_{01} + p_0 (b_{kl} + b_{11})], \\
\delta = (2rf_1 - k_1 \pi r^2) / I, \\
\sigma = r^2 (b_{kl} - b_{11}) / I.
\]

\( \tau_g \) is the gravity torque which, within the closed loop system, can be compensated from the measurement of the joint angle. \( I \) denotes the link inertia and \( \zeta \) is the viscous friction coefficient at the joint. Taking into account that the closed
loop system is under state feedback control, the closed loop state can be defined as

\[ x(t) = \begin{pmatrix} \theta \\ \dot{\theta}(t) \int (\theta_r - \theta(t)) \, dt \end{pmatrix} \]

where \( \theta_r \) is the reference value of the angular position which is taken as \( \theta = 0 \). Note that the integral state is essential to the robust control of the upright stance of the robot arm. The values of the PAMs parameters are drawn from [10], \( I=0.1815 \, [\text{kgm}^2], \) \( \xi=0.1464 \), whereas for the uncertain damping parameters it is assumed that \( b_{\text{hi}} \in [0.5, 1.1], b_{\text{lo}} \in [0.5, 1.1], b_{1} \in [-0.001, 0.008] \) and \( b_{1} \in [-0.001, 0.008] \).

Letting \( b = \bar{b} \pm \Delta b^2 \) and \( u = \Delta p \), it is possible to write the closed loop system in the form (4), with

\[
A = \begin{pmatrix} 0 & 1 & 0 \\ \alpha & \beta - \zeta/I & 0 \\ -1 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ \delta \end{pmatrix}, \quad N_2 = \begin{pmatrix} 0 \\ \sigma \end{pmatrix},
\]

\[
E_1 = \begin{pmatrix} 0 & -r^2 \Delta b_{\text{hi}}/I & 0 \\ 0 & -r^2 \Delta b_{1,\text{hi}}/I & 0 \\ 0 & -r^2 \Delta b_{1,\text{lo}}/I & 0 \end{pmatrix}, \quad S_2 = \begin{pmatrix} 0 \\ -r^2/I \Delta b_{\text{hi}} \end{pmatrix}, \quad \sigma = \begin{pmatrix} 0 \\ 0 \end{pmatrix}
\]

\[
D = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \quad N_1 = N_3 = 0_{3 \times 1}, \quad S_1 = S_3 = 0_{4 \times 1}.
\]

Let define the set of admissible perturbations \( \mathcal{P} \), expressed in terms of the variations of the state variables with respect to the zero equilibrium

\[
\mathcal{P} := [-\pi/3, \pi/3] \times [-0.5, 0.5] \times [-0.5, 0.5].
\]

The dimensions of the set \( \mathcal{P} \) can be defined on the basis of the possible human-robot interactions within a known workspace.

To fulfill the safety requirements during human-robot collisions, a constraint on the peak bound of the control input is included into the optimization problem 1. Indeed, from the compliance function (3), it is evident that prescribing a minimum compliance is equivalent to give an upper bound on the control input pressure \( \Delta p \). Given the values of the model parameter and the set \( \mathcal{P} \), a prescribed minimum compliance of 5 [rad/Nm] is achieved for \( |u(t)| \leq 2.5 \) [kPa].

According to Problem 1, an optimal solution is found, with the controller gain given by

\[
K = \begin{pmatrix} -0.6881 \\ -0.6171 \\ 0.6842 \end{pmatrix}.
\]

For all admissible uncertainties and state perturbations, the estimated worst-case performance cost is \( J = 20.8027 \).

Besides robustness and optimality, another advantage of the designed control law is the ease of implementation in a real control system.

Figure 2 represents the behaviour of the arm in response to simulated collisions. The designed controller allows to recover the upright stance after moderate oscillations about the vertical position. From Figure 3, it is clear that the robust stabilization of the link is achieved satisfying the prescribed input constraint, thus conferring the desired compliant behaviour to the robot arm. In addition, the moderate control effort of the designed RGCC gives rise to an energy saving performance of the closed loop system.

V. Conclusions

A fundamental requirement in human-robot interactions is the reduction of the risk of injuries for the human operator. In this context, the robot safety can be achieved by adjusting the compliance of the robot joints. Safe robots with adaptable compliance are of growing interest in the applications both of prosthetics and of rehabilitation, as well as in wearable assistive devices, where another important issue is the energy efficiency. In such applications, PAMs are adopted to realize joints with adjustable compliance. The complex and nonlinear behaviour of PAMs poses some challenging problems in the design of the position control systems of safe robots.

In this paper, the problem of controlling the position of a robotic arm driven by PAMs has been settled in the framework of the robust and optimal control of bilinear
systems. The position control system of a PAMs-driven robot is formulated as a bilinear system encompassing the model uncertainty due to nonlinear friction effects. Through a guaranteed cost approach, an optimal control problem, which allows to address both the safety constraints and the energy efficiency requirements, is formulated for the closed loop system.

The simulation results show that the proposed control methodology can be effectively applied to the stabilization of the upright stance of a PAMs driven arm which undergoes some perturbed initial conditions (of position and of velocity) after an impact with an unknown obstacle. Under the guaranteed cost controller, the closed loop system exhibits good tracking performance, also satisfying the constraints on safety and input energy. The proposed approach provides a linear state-feedback control law which is not affected by some drawbacks of other control methodologies, e.g., chattering in SMC.

As future work, the proposed approach can be extended to the trajectory tracking with compliance control.

APPENDIX

A. Proof of Lemma 2

Consider the candidate Lyapunov function \( v(x) = x^T P x \).

The time derivative of \( v(x) \) along the trajectories of the closed loop system reads

\[
\dot{v}(x) = x^T \{ P[A + BK + DFE_1 + DFE_2 K] + \left( x^T(N_1 K + DFS_1) \right) \ldots \left( x^T(N_n K + DFS_n K) \right) \} x
\]

Taking into account (24) into (11) gives

\[
\dot{v}(x) < -x^T (Q + K^T R K) x < 0 \quad \forall x \in \mathcal{E} \quad (25)
\]

From standard Lyapunov arguments (see [22], Ch. 4), the equilibrium point \( x = 0 \) is asymptotically stable and the invariant set \( \mathcal{E} \) is contained into the DA of the zero equilibrium point, since \( \dot{v}(x) \) is negative definite over \( \mathcal{E} \).

Integrating both sides of (25) from 0 and +\( \infty \), it is readily obtained

\[
x^T(\infty) P x(\infty) - x^T(0) P x(0) < -\int_0^\infty \left( x^T(t) Q x(t) + u^T(t) R u(t) \right) dt < 0
\]

Since \( \mathcal{E} \) is contained into the DA of the closed loop system, every trajectory starting from \( x_0 \in \mathcal{E} \) implies that

\[
x^T(\infty) P x(\infty) \rightarrow 0,
\]

Therefore,

\[
J < v(x(0)).
\]

For the closed loop system (9), the control law (8) satisfies the upper bound \( \dot{J} = x_0^T P x_0 \) for all \( x_0 \in \mathcal{E} \) (hence \( x_0 \in D \)).

REFERENCES

[20] F. Amato, Robust Control of Linear Systems Subject to Uncertain Time-Varying Parameters. Springer Verlag, 2006.