Robustness and Efficiency of a Variable-Leg-Spring Hopper

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Abstract— In a previous study, stability of a spring-mass hopping-model with variable rest length and stiffness during stance has been investigated. Here, robustness of the stable hopping solutions is addressed. For this, the basin of attraction of the stable fixed points is determined. Results show a trade-off between maximum stability and maximum robustness. Additional velocity-dependent damping only slightly increases robustness. However, the areas of a given robustness level are enlarged.

As a third criterion for successful locomotion efficiency is investigated. To do so, the work-based cost of movement as well as the work ratio between elastic and total work are estimated. Similarly to robustness, the areas of a given maximum cost of movement grow for increasing additional damping. At the same time the maximum cost of transport only slightly increases. However, the work ratio decreases drastically, i.e. less work is done passively by the spring and hopping becomes less efficient.

I. Introduction

Given their complexity, legs operate amazingly spring-like [1]. This motivated the spring-mass model of terrestrial locomotion, also called spring-loaded inverted pendulum (SLIP, [2], [3], [4]). As the models presented in these studies are energy-conservative, they only exhibit neutral stability with respect to energy perturbations. Biological systems however show the ability to compensate various energy losses or perturbations. Additionally, biological limbs do not possess constant spring parameters. Experimental data suggest an increasing rest length and decreasing leg stiffness during ground contact for hopping ([5], [6]) and running ([7], [8], [9]).

The effects of leg parameter adjustments on the dynamics of bouncing gaits have been investigated based on the spring-mass model. However, most of these studies consider either variable rest-length ([10], [11]) or variable stiffness ([12], [13], [14]) while keeping the other parameter fixed. As these parameter adjustments inject energy into the system, some sort of dissipation (most commonly viscous damping) has to be included to ensure periodic gaits. Only recently, simultaneous changes of rest length and stiffness have been addressed [15]. Within this approach additional damping is not required because of the interaction between energy injection via one leg parameter and energy withdrawal via the other one: Concurrent increases of rest length and decreases of stiffness (subsequently referred to as functional damping via leg softening) are sufficient for stable periodic hopping.

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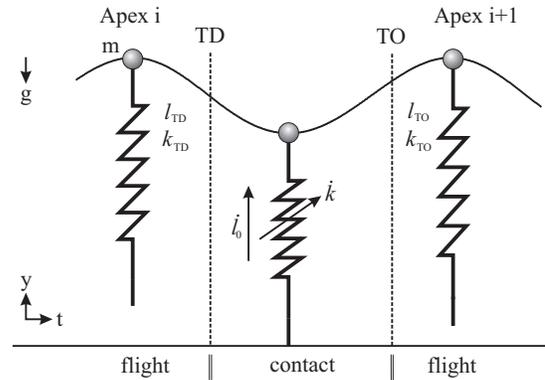
Fig. 1. Variable-leg-spring hopper. The model consists of a point mass with an attached massless spring leg. Spring parameters, rest length l_0 and stiffness k, change linearly with time during contact phase (Eq. 2) and are held constant during flight phases. Reset of spring parameters to their respective touchdown value, l_{TD} and k_{TD}, takes place at apex, i.e. the highest point of center-of-mass trajectory.

Furthermore, the simulated stable hopping patterns (without additional damping) closely resemble the afore mentioned human data with respect to center-of-mass trajectories and ground-reaction forces.

Following [17] and [18], the work presented here complements the notion of cyclic stability (how fast small perturbations are compensated, cf. [19]) with the notion of robustness (how large perturbations can be). Similarly to the results for walking [18], a trade-off between stability and robustness is expected for hopping. Additional damping is expected to further increase robustness, as damping increases the tolerance for perturbations in apex height and thus, the size of the basin of attraction. Furthermore, damping is expected to enlarge the area of a given level of robustness, as damping also increases the region of stable solutions (cf. Fig. 6 in [15]).

To further compare functional damping via leg softening on the one hand and additional velocity-dependent damping on the other, energy efficiency for increasing damping coefficient is investigated. In order to do so, a work-based cost of movement inspired by [16] and [18] as well as the ratio between elastic and total work are calculated. As additional damping increases the non-elastic properties of the hopper for a given parameter setup, less efficient hopping for increasing damping is expected.

II. Methods

A. Model

The spring-mass model as used to describe hopping and running consists of a point mass m on top of a massless

spring with rest length l_0 and stiffness k ([2], [3]). The spring contributes to the system dynamics only during ground contact, i.e. for $y \leq l_0$. Additionally, velocity-dependent damping is included during stance. With gravity opposing the spring force, the equation of motion is

$$m\ddot{y} = F_{spring} + F_{damp} - mg, \tag{1a}$$

$$F_{spring} = \begin{cases} 0, \text{ flight phase} \\ k(l_0 - y), \text{ contact phase,} \end{cases} \tag{1b}$$

$$F_{damp} = \begin{cases} 0, \text{ flight phase} \\ -\delta\dot{y}, \text{ contact phase.} \end{cases} \tag{1c}$$

As a first-order approximation of variable leg parameters, rest length and stiffness are assumed to change linearly with time between touchdown (TD) and take-off (TO),

$$l_0(t) = l_{TD} + \dot{l}_0(t - t_{TD}), \tag{2a}$$

$$k(t) = k_{TD} + \dot{k}(t - t_{TD}). \tag{2b}$$

During flight phases rest length and stiffness are kept constant and are reset at each apex to l_{TD} or k_{TD}, respectively.

In order to simplify system analysis, dimensionless parameters and state variables are introduced: stiffness $K = k_{TD} l_{TD} (mg)^{-1}$, stiffness rate $K' = \dot{k}(l_{TD}/g)^{3/2} m^{-1}$, rest-length rate $L_0' = \dot{l}_0 (g l_{TD})^{-1/2}$, damping coefficient $D = \delta (l_{TD}/g)^{1/2} m^{-1}$, vertical position $Y = y/l_{TD}$ and time $\tau = \sqrt{g/l_{TD}}\,(t - t_{TD})$. Thus, the (dimensionless) equation of motion during ground contact reads

$$Y'' = (K + K'\tau)(1 + L_0'\tau - Y) - DY' - 1, \tag{3}$$

where $'$ denotes the time derivative with respect to τ.

B. Robustness

Depending on the chosen parameter set and the initial conditions, periodic hopping solutions Y^* may occur. In [15] these periodic solutions have been identified numerically for $L_0' \in [-0.2, 0.2]$ and $K' \in [-30, 30]$ and stability has been investigated. Here, robustness is assessed by determining the basin of attraction for the stable hopping solutions Y_s^*.

Within the basin of attraction initial conditions will converge towards a periodic solution. In principle, the basin of attraction is confined between the touchdown condition, $Y_0 > 1$, and the unstable fixed point (Fig. 2). For initial apex heights $Y_0 \leq 1$ the leg will not be initialized and the mass point follows a free-fall trajectory until ground contact, whereas beyond the unstable fixed point Y_u^* all initial conditions diverge.

However, two effects may cause the hopper to fall down (ground contact of the point mass) before encountering the theoretical boundaries of the basin of attraction. Damping, whether functional or velocity-dependent, may prevent take-off for sufficiently small initial apex heights and a given choice of L_0', as the energy withdrawal may be not compensable with this actuation. For sufficiently large initial apex heights the model may hit the upper falling-down barrier Y_{GC}. Beyond Y_{GC} the spring cannot store sufficient initial energy

and thus, properly support the point mass, resulting in total leg compression and ground contact (GC) of the point mass.

To determine the effective boundaries of the basin of attraction, a bisection method with the initial interval $[1; Y^*]$, and $[Y^*; K]$ respectively, is used. For the motivation of the latter see App. A). The bisections terminate for an interval size below 10^{-9}.

Robustness is defined here as the largest step the model could either take up or down without ground contact of the point mass, i.e. the minimum distance from the fixed point to the boundaries of the effective basin of attraction.

C. Efficiency

Two notions of efficiency are used in this study: (a) the work-based cost of movement, C_{Y_0}, and (b) the ratio of elastic and total work, $\eta = W_{ext}/W_{int}$. The first notion describes hopping performance with respect to hopping height, whereas the second notion is a measure for the relation between elastic and non-elastic properties of the hopper.

The calculation of the work done by spring and damper is straight-forward using Eq. 1. To determine the total mechanical work performed by the system due to variable leg parameters the time derivative of the system energy

$$E(\tau) = Y + \frac{1}{2}Y'^2 + \frac{1}{2}(K + K'\tau)(1 + L_0'\tau - Y)^2. \tag{4}$$

is required. Using Eq. 3 for simplification the time derivative yields

$$E' = \frac{1}{2}(1 + L_0'\tau - Y)^2 K' + (K + K'\tau)(1 + L_0'\tau - Y)L_0'. \tag{5}$$

Thus, positive and negative contributions to the work due to variable leg parameters are solely determined by the signs of the parameter rates, L_0' and K', as during ground contact the conditions $K > K'\tau$ and $Y \leq 1 + L_0'\tau$ have to be satisfied.
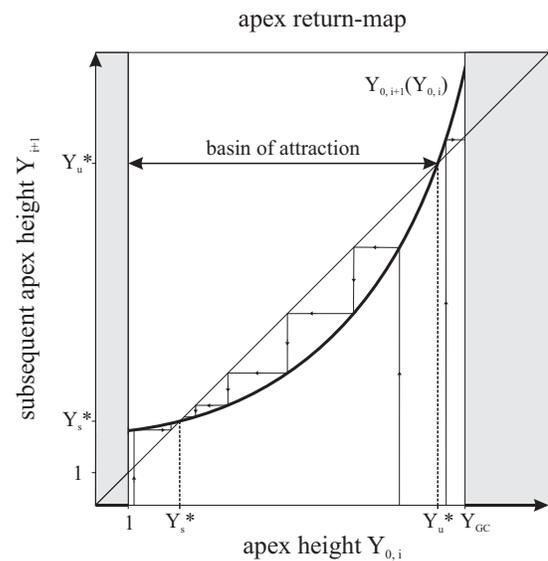

Fig. 2. Apex return map. Stable and unstable fixed points are shown (Y_s^* and Y_u^* respectively). Initial conditions with $1 < Y_{0,i} < Y_u^*$ converge towards Y_s^*. Within the areas shaded gray the point mass hits the ground, i.e. $Y_{0,i+1} \equiv 0$.
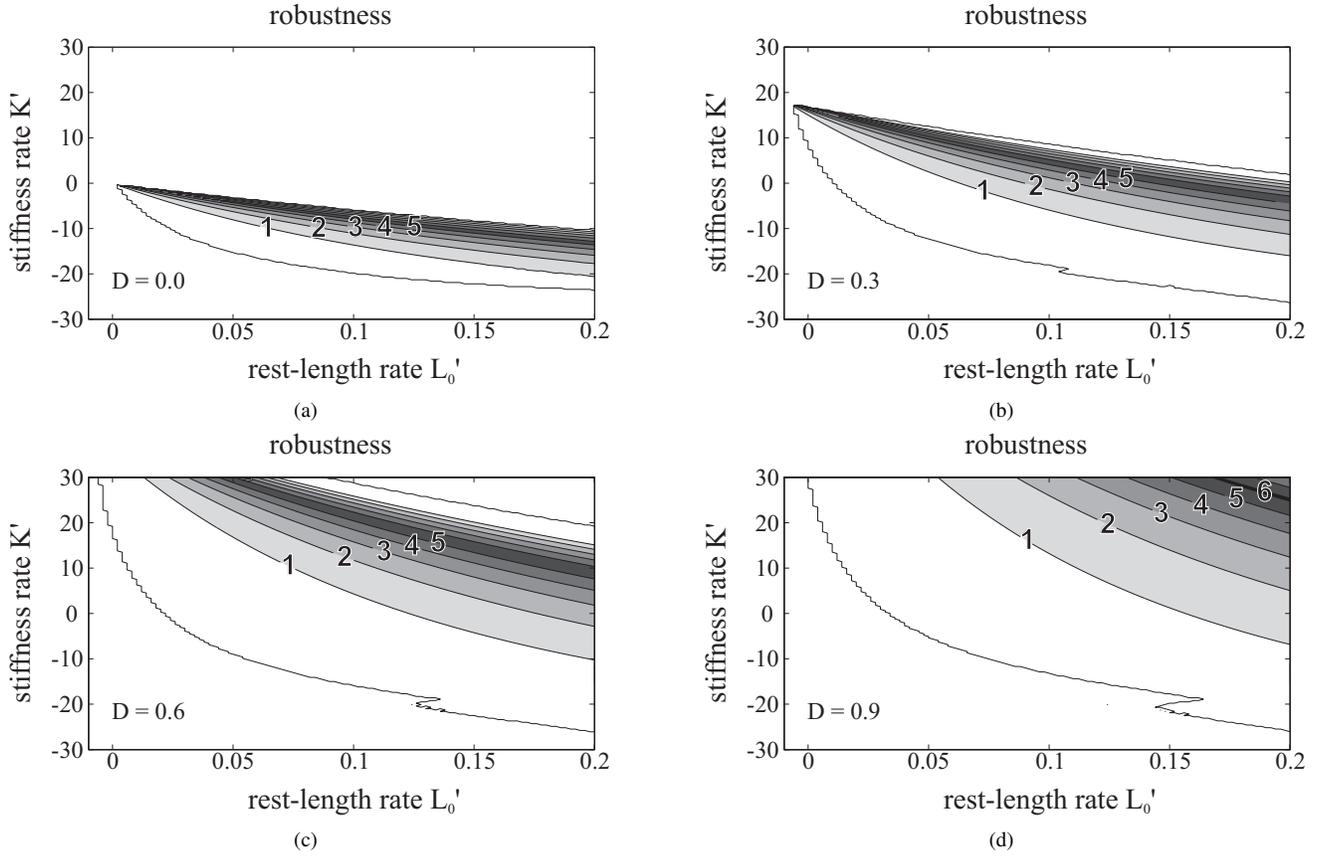
Fig. 3. Influence of damping coefficient D on the robustness of stable hopping solutions. Robustness is mapped with respect to dimensionless stiffness rate K' and rest-length rate L_0' for stiffness $K = 25$ ($k \approx 19.6\,\mathrm{kN\,m^{-1}}$ for human dimensions, $m = 80\,\mathrm{kg}$ and $l_0 = 1\,\mathrm{m}$). Increments of 0.3 for K' and 0.002 for L_0' were used (corresponding to $\Delta\dot{k} \approx 0.7\,\mathrm{kN\,m^{-1}\,s^{-1}}$ and $\Delta\dot{l_0} \approx 0.7\,\mathrm{cm\,s^{-1}}$ in dimensional parameters).

Following [16], [18], but simplifying the notation used therein, the work-based cost of movement is defined as

$$C_{Y_0} := \frac{1}{Y_0} \int_0^{\tau_s} \left(|P_{\text{spring}}| + |P_{\text{damp}}| + |P_{L_0'}| + |P_{K'}| \right) d\tau, \quad (6)$$

where because of Eqs. 1 and 5 the individual contributions are given by

$$P_{\text{spring}} = (K + K'\tau)(1 + L_0'\tau - Y)Y', \quad (7a)$$

$$P_{\text{damp}} = -DY'^2 \quad (7b)$$

$$P_{L_0'} = (K + K'\tau)(1 + L_0'\tau - Y)L_0', \quad (7c)$$

$$P_{K'} = \frac{1}{2}(1 + L_0'\tau - Y)^2 K', \quad (7d)$$

and τ_s denotes stance time. Integration is done numerically using the setup described in Section II-D.

Similarly, the work ratio of elastic and total work is defined as

$$\eta := \frac{W_{\text{spring}}}{W_{\text{total}}} = \frac{\int_0^{\tau_s} |P_{\text{spring}}|\,d\tau}{\int_0^{\tau_s} \left(|P_{\text{spring}}| + |P_{L_0'}| + |P_{K'}| + |P_{\text{damp}}| \right) d\tau}. \quad (8)$$

D. Simulation Protocol

Numerical integration is done in MATLAB/SIMULINK (R2010a, The MathWorks Inc., Natick, MA, USA) using the implemented Runge-Kutta variable step integrator (ode45) with a maximum time-step size of 10^{-2} and relative and absolute tolerance $\leq 10^{-12}$.

Solutions are mapped with respect to dimensionless stiffness rate $K' \in [-30, 30]$ and rest-length rate $L_0' \in [-0.2, 0.2]$. Increments of 0.3 for K' and 0.002 for L_0' are used.

In order to avoid solutions with negative leg stiffness or rest length, termination conditions for vanishing stiffness and rest length are implemented. The simulation also terminates for ground contact of the point mass, i.e. when falling down.

III. RESULTS

A. Robustness

Results show a trade-off between maximum stability (Fig. 6 in [15]) and maximum robustness (Fig. 3 in this study). Robustness is only marginally increased if additional damping is added. Nonetheless, the areas of a given robustness level enlarge considerably for increasing damping.

If additional damping is included the regions confining the area of stable solutions at the lower boundary (regions IIIb and V in [15]) do not merge smoothly anymore. Within a certain interval of rest-length rates L_0' stiffness vanishes at the
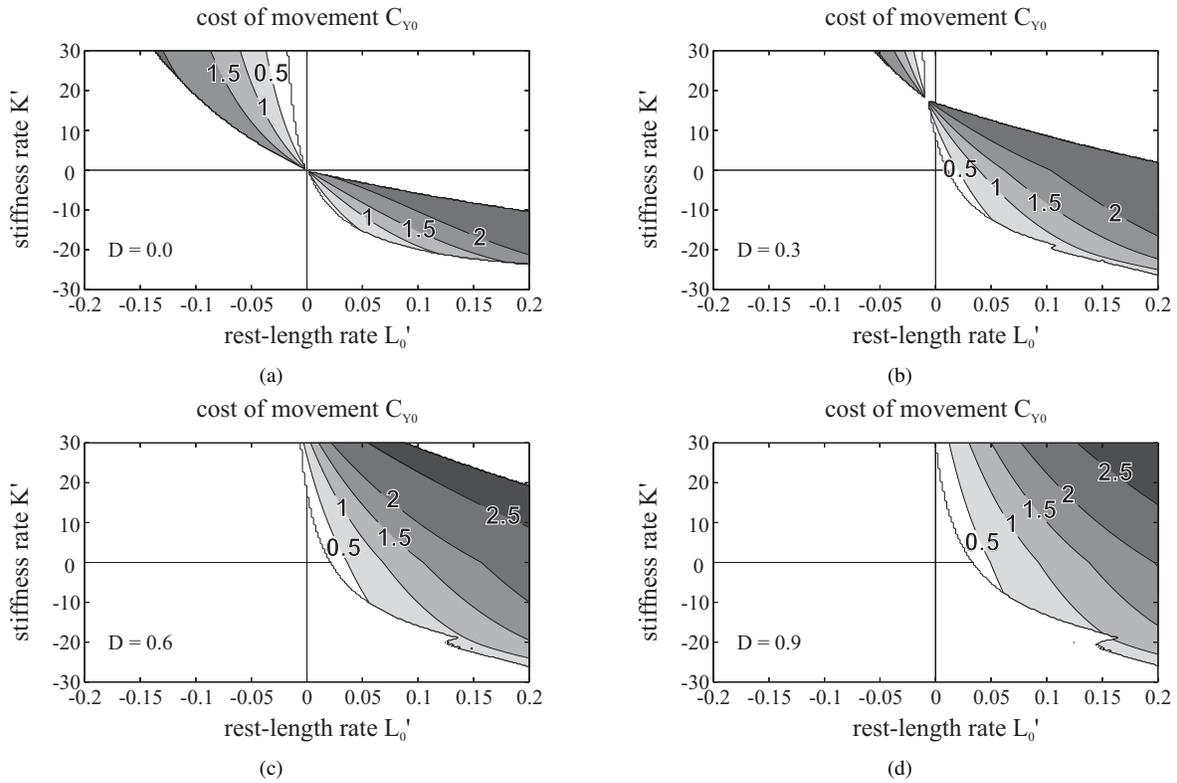
Fig. 4. Influence of damping coefficient D on work-based cost of movement. Cost of transport is mapped with respect to dimensionless stiffness rate K' and rest-length rate L_0' for stiffness $K = 25$ ($k \approx 19.6\,\mathrm{kN\,m^{-1}}$ for human dimensions, $m = 80\,\mathrm{kg}$ and $l_0 = 1\,\mathrm{m}$). Increments of 0.3 for K' and 0.002 for L_0' were used (corresponding to $\Delta \dot{k} \approx 0.7\,\mathrm{kN\,m^{-1}\,s^{-1}}$ and $\Delta \dot{l}_0 \approx 0.7\,\mathrm{cm\,s^{-1}}$ in dimensional parameters).
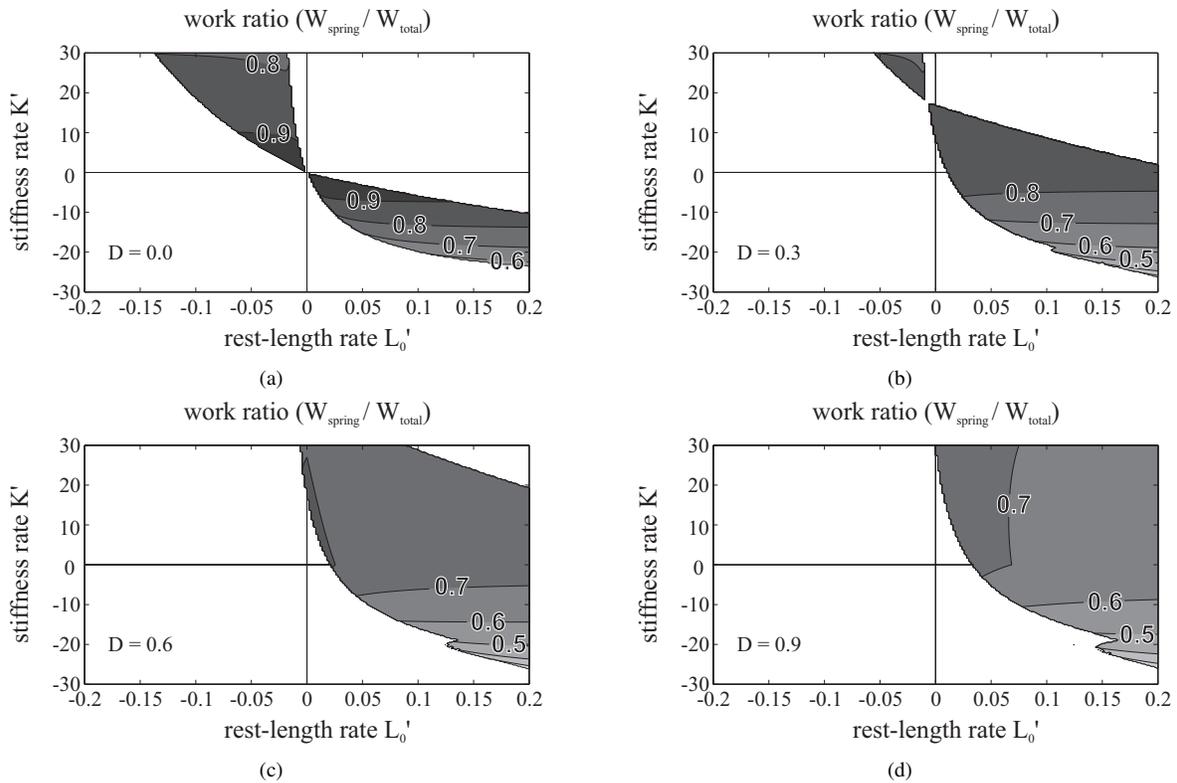

Fig. 5. Influence of damping coefficient D on the ratio of elastic and total work. Work ratio is mapped with respect to dimensionless stiffness rate K' and rest-length rate L_0' for stiffness $K = 25$ ($k \approx 19.6\,\mathrm{kN\,m^{-1}}$ for human dimensions, $m = 80\,\mathrm{kg}$ and $l_0 = 1\,\mathrm{m}$). Increments of 0.3 for K' and 0.002 for L_0' were used (corresponding to $\Delta \dot{k} \approx 0.7\,\mathrm{kN\,m^{-1}\,s^{-1}}$ and $\Delta \dot{l}_0 \approx 0.7\,\mathrm{cm\,s^{-1}}$ in dimensional parameters).

boundary for two distinct stiffness rates K'. Between these points stiffness remains non-negative and stable hopping is possible. Thus, a bulge forms, growing more and more prominent for increasing damping.

B. Efficiency

The work-based cost of movement C_{Y_0} does only slightly increase with additional damping (Fig. 4). However, the regions of a given C_{Y_0}-level spread out.

At the same time, less work is done elastically. The work ratio η clearly decreases (Fig. 5).

IV. DISCUSSION

A. Robustness

As in running [18] there is a trade-off between stability and robustness. Stability is maximal for small hopping heights [15], while robustness is maximal for medium hopping heights. This is due to the symmetric definition of robustness as the maximum step-up or step-down perturbation the hopper can compensate. Thus, robustness is the minimum distance of the stable fixed point to either boundary of the basin of attraction. For medium hopping heights the stable fixed point is located in the middle of the basin of attraction rather than towards either end, maximizing the distance to both boundaries and therefore robustness.

If reasonable constraints are considered, e.g. restricting apex height to $y_0 \leq 1.5\,l_{\text{TD}}$ and rest-length change to $l_{\text{TO}} \leq 1.1\,l_{\text{TD}}$ (Figs. 3(a)+(e) in [15]), the hopping solutions correspond rather to optimized stability than robustness. However, robustness may still be considerable (up to one half of leg length at touchdown, Fig. 3).

Additional velocity-dependent damping shifts the upper falling-down barrier outwards. The hopper is able to achieve larger hopping heights. At the same time, the stable fixed point Y_s^* is not noticeably shifted towards the middle of the basin of attraction. The stable fixed point remains closer to one of the boundaries. Thus, robustness only is marginally increased. Nonetheless, one benefit of additional damping is that the areas for a given level of robustness increase considerably.

B. Efficiency

As the model used in this study is quite conceptional, it was not attempted to estimate a metabolic or specific cost of transport (e.g. [20], [21]). Rather the work-based cost of movement C_{Y_0} was applied to calculate the mechanical requirements to achieve the periodic hopping height Y_0 within the variable-leg-spring concept and with additional damping. Following an argument of [22], this simplified approach is valid as long as it is used for comparisons within one model.

Additional velocity-dependent damping only marginally increases the work-based cost of movement with respect to hopping height, while the regions for a given cost level enlarge considerably (Fig. 4). This suggests a benefit of additional damping with respect to hopping performance or efficiency.

However, the work ratio decreases drastically with additional damping (Fig. 5). The work done passively by the spring, so to say "for free", reduces, whereas the contribution of the damper to support the point mass against gravity increases. Hopping becomes more visco-elastic and less efficient. Thus, from an efficiency point-of-view, the functional damping via leg softening is more beneficial for hopping than the commonly used velocity-dependent damping.

V. CONCLUSION

The variable-leg-spring concept allows for periodic hopping that is stable, robust and efficient (see [15] and the results presented here). The key feature of this control scheme is the functional damping via leg softening.

Such a control may be encoded in biological musculo-skeletal systems via neural programs, e.g. the λ-model describing actuation of the ankle joint [23]. Experimental data suggest that the visco-elasticity of bouncy gaits is not fully described by a parallel arrangement of linear elasticity and additional velocity-dependent damping. For instance, velocity-dependent damping predicts non-zero landing forces at the instant of touch-down. These are not observed in human hopping (e.g. [6]). Secondly, takeoff would take place while the leg is still compressed. Functional damping avoids these discrepancies with respect to experimental data [15].

Recent developments in the field of robotics also reflect the variable-leg-spring concept. For instance, the 1-DOF hopping robot Chobino1D features an improved MACCEPA design to actuate the knee joint [24]. In this system joint stiffness increases with knee deflection, resembling the dynamic properties of of the human knee during locomotion. The ability to store and release energy elastically allows for an increased hopping height compared to the same configuration without a series-elastic element. The concept of functional damping as presented in this study may provide a new theoretical background for the control of such robotic systems.

APPENDIX

A. Upper Falling-Down Barrier Y_{GC}

The spring is only able to store a limited amount of energy. It cannot properly support the point mass if initial energy exceeds this amount. Thus, the spring will not prevent the point mass from ground contact for to large initial energies, i.e. initial apex heights.

For a linear spring with fixed parameters the maximum apex height can be easily calculated. The system is energy-conservative, so system energy fulfills

$$E = Y_0 = Y + \frac{1}{2}Y'^2 + \frac{1}{2}K(1-Y)^2. \tag{9}$$

For the minimum apex height resulting in falling down, Y_{GC}, ground contact, $Y = 0$, is reached with zero velocity, $Y' = 0$. Thus, for successful hopping with a constant linear spring initial values have to satisfy

$$Y_0 < Y_{GC} = \frac{1}{2}K. \tag{10}$$

As the decreasing effect of leg softening exceeds the increasing one of leg lengthening regarding the spring's capacity to store energy, variable-leg-spring hoppers with rest length and stiffness rates lying in the region of stable hopping also observe this limit.

The limit for the maximum apex height can also be calculated for a constant linear spring with additional velocity-dependent damping. However, as the calculation is lengthy, the starting interval for the bisection procedure to identify the maximum apex height was simply chosen as $[Y_s^*; K]$. In this study all simulations resulted in maximum apex heights clearly below $Y_0 = K$.

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