Modeling of a propulsion mechanism for swimming microrobots inspired by ciliate metachronal waves

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Abstract—The envisioned applications of microrobots in bodily fluids have raised the demand for effectively swimming microdevices. Microorganisms have become a source of inspiration because their mechanisms of propulsion are effective at low-Re. We investigated the theoretical performance of swimming microrobots implementing propulsion inspired by metachronal waves. These come from the spontaneous coordination of cilia and are responsible for the high swimming speeds of ciliates. We found that microrobots of typical length below the millimeter could self-propel at speeds of several bodylengths per second. The microrobots were assumed to have a continuous active surface exhibiting traveling-wave deformations that mimic metachronal waves. We developed an FE model for analyzing the performance of propulsion of such bio-inspired microrobots in water. In particular we evaluated how velocity is affected by various parameters, such as the shape and size of the microrobot, and the frequency, wavelength and amplitude of the surface deformations. We believe that the proposed mechanism is advantageous over other methods of propulsion because it does not need external thin and fragile appendages. The results of this analysis could thus guide us towards the design of effective self-propelling microrobots.

I. INTRODUCTION

The term “swimming microrobot” refers to an artificial or hybrid microdevice able to move within a liquid medium [1]. The envisioned applications of these swimming microrobots involve navigation in bodily fluids, such as blood and cerebrospinal fluid [2]. Considering that their typical sizes are within the sub-millimeter range, microrobots swim in a low-Reynolds-number regime. This implies that inertia plays a minor role with respect to viscous damping. Swimming at these scales thus requires mechanisms that are substantially different from those employed at the macroscales [3].

Microorganisms are able to swim at the microscale with different techniques. For this reason they have become a source of inspiration for the design of swimming microrobots. The most studied biological models in microrobotics have been bacteria and eukaryotic flagellates. Tiny microrobots (few tens of microns in length) that swim by means of helical tails have been developed by taking inspiration from bacterial flagella [4-5]. The rotating action exerted on the flagella by the molecular motor in bacteria was replaced by an external magnetic torque. This was applied in a wireless manner by means of a rotating external magnetic field. An elastic tail vaguely resembling eukaryotic flagella was also employed for the magnetic propulsion of a swimming microdevice [6-7]. These two biological mechanisms of propulsion were also investigated for self-propelling microrobots [8]. Another propulsion mechanism relying on magnetically actuated artificial cilia that mimic the natural counterpart was studied [9]. It should also be noticed that cilia propulsion was investigated in order to assess whether such strategy is suitable for controlled fluid transport in microfluidic devices [10-11]. This led to the development of independent artificial cilia, also driven by external magnetic fields.

The swimming microrobots mentioned above involve the employment of thin external propellers. However, external appendages are a hindrance. They could detach from the microrobot body, get stuck or damage the external environment (particularly in medical applications). For this reason, self-propelling microrobots relying on internal mechanisms have been recently proposed [12].

In this work we propose a propulsion mechanism for swimming microrobots inspired by metachronal waves. These consist in waves of deformation propagating on the surface of ciliates and originating from a collective self-coordination of cilia. The proposed propulsion mechanism does not involve external appendages. In fact, it is based on small deformations of the surface of the microrobot.

We thus investigated the expected performance of microrobots relying on the proposed propulsion mechanism. At this scope, we developed a theoretical model of the propulsion mechanism that consists in a simplified version of the classical envelope model for the propulsion of ciliates. The model was employed to predict the performance of Paramecium caudatum, showing substantial agreement between the obtained results and experimental data in literature. We thus employed the model for evaluating the influence of various design parameters on the performance of microrobots. Finally, we found that the proposed mechanism of propulsion could be very effective for microrobots with size within the sub-millimeter range.
II. METACHRONAL-WAVES-INSPIRED PROPULSION

A. Propulsion of Ciliates

Among all microorganisms, ciliates are the larger and faster ones [13]. The larger ciliates such as Paramecia have typical size of few hundred microns and do not have long appendages for propulsion. However, they can self-propel at speeds well above one millimeter per second by means of the coordinated action of the thousands of short and thin cilia covering their surface.

Cilia are short eukaryotic flagella (about 10 µm long), found in densely packed arrays on the external membrane of different kinds of eukaryotic cells [13]. These include a group of microorganisms, which are thus called ciliated. Moreover, cilia can also be found in the human body, e.g. in the airways [13] and in the central nervous system [14], and are involved in many important biological functions. One of their main roles is propelling fluids. In ciliates, this action leads to the propulsion of the whole body of the microorganism through the fluid. Each cilium exhibits a complex asymmetric beating pattern. In particular, this non-reciprocal deformation includes a high-friction power stroke and a low-friction recovery stroke [13]. In order to achieve effective fluid propulsion, many cilia are used simultaneously. Moreover, when they are closely packed on surfaces, cilia display a coordinated collective behavior, termed metachronal waves [15]. These are propagating waves of deformation, in which the cilia beat with a small constant phase difference with respect to their neighbors [13]. Metachronal waves are essential for the effectiveness of ciliary propulsion because of their symmetry-breaking effect. Moreover, metachronal coordination represents an energetic advantage in ciliary beating [16]. Different kinds of metachronal waves were found, depending on the direction of propagation with respect to the direction of the power stroke of the cilium. These can cohabit in a single microorganism, being expressed in different conditions of the environment [17]. The main features of metachronal waves, such as frequency, amplitude and wavelength, can vary widely among the different species that exhibit them. Moreover they can show variation also in a single microorganism because of changes in the surroundings. However, their ranges of variation can be considered as follows:

- amplitudes are typically of a few microns, being limited by the length of the cilium [18];
- frequencies are usually of the order of few tens of Hz [17-19], the same of the beating of each single cilium;
- wavelength ranges from a few tens of microns from few hundred microns[17-18].

B. Concept Definition

Future medical microrobots should be very good swimmers. Both blood and cerebrospinal fluid, for example, can reach peak velocities above 20 mm/s. Achieving such speeds will be an even greater challenge in the case of self-propelled microrobots that do not rely on external actuation.

Having in mind this major constraint, we are aiming at microrobots with high self-propelling performances. We thus chose ciliates as source of inspiration mainly because of their high swimming speeds. Ciliary propulsion has proved to be very effective for the locomotion of bodies of typical size in the hundred-micron range. This is also the target size range for our microrobots.

We thus tried to capture the key features of propulsion of ciliates for implementing them in microrobots. The role of metachronal waves is amazing. The so-called envelope model, introduced by Lighthill in the early ‘50s [20] and further developed by Blake in the ‘70s [21], highlight their fundamental role. This model predicts the propulsion of ciliates by representing metachronal waves as a progressing waving envelope that replaces the individuality of cilia. Therefore we can infer that an object exhibiting propagating waves of deformation of its surface, without any appendages, could self-propel in fluids.

Moving from the above considerations, we defined a concept of microrobot with the following main features:

- typical length ranging from 100 µm to 1 mm;
- propulsion mechanism based on surface deformations inspired by metachronal waves in ciliates;
- no external appendages;
- desired speeds above 10 mm/s.

III. DEFINITION OF THE MODEL

We developed a model of the locomotion of microrobots implementing a mechanism of propulsion inspired by metachronal waves. The modeled microrobots have a continuous deforming surface. Since this is analogous to making the basic assumption of the envelope model for ciliary propulsion, the model is also representative of the propulsion of ciliates. In particular, the model is as a simplification of the envelope model of ciliates propulsion [20-21]. In fact we only considered purely sinusoidal waves of deformation and we neglected deformations tangential to the surface, thus focusing only on the main component of propulsion. Such an approach enables the design of the proposed mechanism of propulsion for swimming microrobots, which is much simpler than that of the living microorganisms. The model is described in detail in the following.

![Fig. 1. The elliptical microrobot in the 3D Cartesian frame and in the 2D axisymmetric cylindrical frame. The v coordinate represents the position on the ellipse and values – γ/4 and + γ/4 respectively at the top and bottom extremities.](image-url)
The swimming microrobot was considered as a prolate spheroid moving along its major axis in unbounded water. The spheroid was a good approximation of the shape of the body of many ciliates, such as those of the genus *Paramecium* [19, 22]. The surface of the spheroid can be expressed in Cartesian coordinates as

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \]  

(1)

where \( a \) and \( b \) are, respectively, the semi-major and the semi-minor axis (Fig. 1).

The metachronal waves were modeled as wave deformations of the surface of the spheroid that travel parallel to its long axis. The model was thus developed exploiting the axial symmetry of both the geometry and the propulsive actions. The problem was represented in a cylindrical frame by means of a 2D axisymmetric model (Fig. 1).

In the 2D axisymmetric cylindrical frame the prolate spheroid is represented by a hemi-ellipse and can be expressed as

\[ \frac{r^2}{b^2} + \frac{z^2}{a^2} = 1 \]  

(2)

In order to model the deformation of the surface we also adopted a coordinate defined as

\[ \nu = \frac{p_{f_e} - p_{f_s}}{D_f}, \gamma = \frac{\gamma(p_{f_e} - p_{f_s})}{8c} \]  

(3)

where \( p_{f_e} \) and \( p_{f_s} \) are the distances of a point on the ellipse from the two foci, \( D_f = 2c \) is the distance between the foci and \( \gamma \) is the length of the ellipse. The \( \nu \) coordinate thus represents the position on the ellipse and varies between \(-\gamma/4\) and \(+\gamma/4\) (Fig. 1).

Assuming a simple sinusoidal waveform, the propagating wave deformation can be expressed as

\[ \delta = \frac{A}{2} \cos \left[ 2\pi \left( \frac{\nu}{\lambda} - \frac{1}{T} \right) \right] \cdot \frac{r}{b} \]  

(4)

where \( A \) is the peak-to-peak amplitude, \( \lambda \) is the wavelength and \( f \) is the frequency. The deformation was assumed to be perpendicular to the surface of the microrobot. The \( r/b \) factor was introduced for obtaining deformation amplitude that is null on the extremities of the spheroid and maximum on the equatorial circumference. This accounts for the lost of coordination between cilia in the proximity of the extremities of the body of the actual microorganisms.

The liquid in which the microrobot swims was considered as an incompressible fluid in the laminar regime. Thus it is governed by the Navier-Stokes equation

\[ \rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \mu \nabla^2 \mathbf{u} + \mathbf{f}_e \]  

(5)

where \( \mathbf{u} \) is the velocity vector field, \( \rho \) is the hydrodynamic pressure scalar field, the \( \mathbf{f}_e \) vector field accounts for external forces, and \( \rho \) and \( \mu \) are, respectively, the mass density and the viscosity of the fluid, which are constant [13, 23].

The surface deformations establish a velocity field in the fluid which, in turn, makes the microrobot advance.

Considering the first part of this process, the deforming surface was assimilated to a moving wall. This was obtained by imposing at the boundary the no-slip condition and the following prescribed velocity of movement of the wall

\[ \mathbf{v}_w = \delta = A \mu \sin \left[ 2\pi \left( \frac{\nu}{\lambda} - \frac{f \cdot t}{T} \right) \right] \cdot \frac{L}{b} \]  

(6)

that is perpendicular to the boundary.

The advancing velocity of the microrobot can now be obtained by considering the coupling between the established fluid movement and the microrobot movement. The fluid acts on the microrobot in two different ways:

- as a pushing force responsible for the propulsion of the microrobot in the fluid (\( F_{prop} \));
- as a resisting drag force that opposes the microrobot movement in the fluid (\( F_{drag} \)).

After a rapid acceleration, this double effect leads to a nearly steady velocity. This is due to the propulsive force being completely counterbalanced by the drag force. Interestingly, both the propulsive and the drag components of the force are due to the action of the fluid on the microrobot body. For this reason, the resulting hydrodynamic force can be obtained by integrating the stress tensor of the fluid on the surface of the microrobot

\[ \mathbf{F} = F_{prop} + F_{drag} = \int \left[ -p \mathbf{I} + \mu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) \right] \mathbf{n} \, dS \]  

(7)

where \( \mathbf{n} \) is the normal vector on the surface \( S \) of the microrobot [13]. This resulting force becomes zero once the velocity has been reached.

Because of the axial symmetry of the model, only the \( z \)-component of the force is not null. Therefore the motion of the microrobot can be described by the following

\[ m_m \ddot{z} = F_z = F \]  

(8)

where \( m_m \) is the microrobot mass. The microrobot thus moves at a velocity \( v_m = \dot{z} \) relative to the fluid.

The model was defined using the reference system of Fig. 1, which is fixed on the swimming microrobot. Consequently, with respect to this moving frame, the microrobot is steady while the fluid goes towards it at a velocity \( v_f = -v_m \) (Fig. 2). Hence, the following volumetric force was applied to the fluid:

\[ \mathbf{f}_v = -\rho \dot{V} \]  

(9)

Fig. 2. Velocity of the fluid with respect to the reference system fixed on the microrobot.

Moreover, the boundary to which the microrobot moves was treated as an inlet with prescribed fluid velocity \( v_f \).

The model was thus implemented in a Finite-Elements simulation software (COMSOL Multiphysics 4.2). The
implementation of the model through the Finite Elements Method will allow us to easily modify it for designing microrobots with different and more complex shapes and features.

IV. PERFORMANCE OF THE SWIMMING MICOROBOTS

A. Comparison with Paramecium caudatum

First of all we tested the model on a biological example. At the best of our knowledge, there are no complete experimental characterizations of the locomotion of a single species of ciliate including all the parameters needed for the model. So we were not able to perform a full validation of the model on a complete dataset. However, we found a quasi-complete set of experimental data about Paramecium caudatum [19]. These data include the dimensions of the body, the frequency of the ciliary beating and the velocity of locomotion in water. The amplitude and wavelength of the metachronal waves were still missing, but we employed typical values in the ranges reported in Section II.A. As a first attempt, we chose 5 µm for the amplitude $A$ and 50 µm for the wavelength $\lambda$, which are very plausible values. The arbitrary choice of these values obviously influences the final results, as described in the following sections. However, this first attempt choice led to results comparable to biological data. A summary of both the experimental data extracted from [19] and the values employed and obtained by the model are reported in Table I.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>COMPARISON WITH BIOLOGICAL DATA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>Experimental [19]</td>
</tr>
<tr>
<td>$a$ (µm)</td>
<td>83</td>
</tr>
<tr>
<td>$b$ (µm)</td>
<td>20</td>
</tr>
<tr>
<td>$A$ (µm)</td>
<td>-</td>
</tr>
<tr>
<td>$\lambda$ (µm)</td>
<td>-</td>
</tr>
<tr>
<td>$f$ (Hz)</td>
<td>40</td>
</tr>
<tr>
<td>Velocity (mm/s)</td>
<td>1.45</td>
</tr>
</tbody>
</table>

Values of parameters and velocity extracted from experimental data in [19] and employed and obtained by the model.

![Fig. 3](image3.png) Fig. 3. A frame from the simulation of the distribution of the fluid velocity with respect to the moving body (red and blue values respectively indicate higher and lower fluid velocities). Parameters from Paramecium caudatum.

![Fig. 4](image4.png) Fig. 4. Velocity of Paramecium caudatum as obtained by the simulation. After a fast acceleration stage, the velocity is nearly steady (average value of 1.343 mm/s). The ripples are due to the intrinsic oscillatory behavior of the propulsion mechanism.

Although the simplifications introduced by the model with respect to the natural propulsion mechanism and the arbitrariness in the selection of $A$ and $\lambda$, the obtained speed is in substantial agreement with that experimentally observed. The result of the simulation is depicted in Fig. 4. In particular, we simulated the first four periods of movement (with respect to the wave frequency – totally 0.1 s), considering a start from the steady state. We obtained a speed of 1.343 mm/s versus a maximum value of 1.45 mm/s reported in [19]. This speed was obtained as an average over the last two periods, in order to discard the initial acceleration stage. This result confirms the substantial validity of the assumption related to the model formulation and to the choice of parameter values.

B. Dependence on Size and Shape

We analyzed whether and how the performance of the microrobots varies with their geometrical characteristics.

![Fig. 5](image5.png) Fig. 5. Dependence of the velocity on the length $L$ of the microrobot. The values of the other parameters are: $a/b = 4$, $A = 5$ µm, $\lambda = 50$ µm and $f = 40$ Hz.

The first parameter we considered was the length $L = 2a$ of the microrobot. The value of $L$ was varied in the range between 100 µm and 1 mm. All the parameters related to the wave deformation were kept at the values of Table 1. The shape ratio $a/b$ was kept fixed, as well. The equatorial diameter of the microrobots thus increased proportionally to the length $L$. 

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The results of the simulations, reported in Fig. 5, show a net increase in the microrobots velocity with an increase of their size. Below the millimeter, the velocity is nearly linearly proportional to size, such that the relative velocity in bodylengths per second remains constant. This suggests, as could be reasonably supposed, that the propelling force is linearly dependent on the surface area of the microrobot. In fact, the surface area of a prolate spheroid is proportional to the square of the length \( L \), so:

\[
F_{\text{prop}} = \alpha \cdot L^2
\]

where \( \alpha \) is a coefficient of proportionality. The drag force on a prolate spheroid is, instead, linearly dependent on \( L \):

\[
F_{\text{drag}} = \beta \cdot L \cdot v_w
\]

where \( \beta \) is another coefficient of proportionality. Therefore the following relation confirms the results showed in Fig. 5:

\[
v_w = \frac{\alpha}{\beta} \cdot L
\]

The role of inertia obviously becomes more and more important when the size increases. This is the reason for an increasing acceleration time that was observed in the transient simulations (settling times below 1 s). Moreover, for larger sizes the terminal velocity shows a sort of saturation behavior, as it can be noticed in Fig. 5.

The shape ratio \( a/b \) is another important parameter that can influence the performance of the microrobot. For this reason we calculated the velocity of the microrobots for shape ratios ranging from 2 to 8. As in the simulations above, the wave deformations features were kept fixed. Moreover, all the microrobots in these simulations had the same volume.

The dependence of the velocity on the shape ratio shows an exponential trend, as can be noticed in Fig. 6. Therefore, an extremely high shape ratio is not much more convenient than a medium one, like, in fact, that of Paramecium caudatum (\( a/b = 4 \)).

C. Dependence on the Wave Deformations

When designing microrobots, size and shape can be strictly constrained by the target application. The performance of the swimming microrobots, however, can be increased by choosing the proper set of values for the parameters that characterize the wave deformations.

Fig. 7 shows the dependence of the swimming speed on the frequency \( f \) of the wave deformation. It can be noticed that the velocity of the microrobots is linearly proportional to \( f \) in the range between 10 and 100 Hz. Interestingly, we did not find any saturation-like behavior in the considered range. Maybe this behavior can arise at even higher frequencies or in fluids that are more viscous than water.

Fig. 6. Dependence of the velocity on the shape factor \( a/b \) of the microrobot. The values of the other parameters are: \( L = 250 \) \( \mu \)m, \( A = 5 \) \( \mu \)m, \( \lambda = 50 \) \( \mu \)m and \( f = 40 \) Hz.

Fig. 7. Dependence of the velocity on the frequency \( f \) of the wave of deformation. The values of the other parameters are: \( L = 250 \) \( \mu \)m, \( a/b = 4, A = 5 \) \( \mu \)m and \( \lambda = 50 \) \( \mu \)m.

Fig. 8. Dependence of the velocity on the wavelength \( \lambda \) of the wave of deformation. The values of the other parameters are: \( L = 250 \) \( \mu \)m, \( a/b = 4, A = 5 \) \( \mu \)m and \( f = 40 \) Hz.

Fig. 9. Dependence of the velocity on the amplitude \( A \) of the wave of deformation. The values of the other parameters are: \( L = 250 \) \( \mu \)m, \( a/b = 4, \lambda = 50 \) \( \mu \)m and \( f = 40 \) Hz.
Analyzing the effect of varying the wavelength of the surface deformation from 50 to 200 µm, we found that the microrobot velocity has a decaying exponential trend over \( \lambda \) (Fig. 8). For wavelengths close to the microrobot length we also observed the rise of an oscillatory and almost non-propulsive behavior.

Finally, the effect of the variation of the amplitude \( A \) in the range between 1.25 and 12.5 µm was analyzed. In particular, we observed a linear trend in the range between 2.5 and 7.5 µm. However, a saturation-like behavior was already noticed with a 10 µm amplitude (Fig. 9).

V. CONCLUSION

In this work we investigated the theoretical performance of swimming microrobots implementing propulsion inspired by metachronal waves in ciliates. In particular, we proposed a mechanism of propulsion based on small deformations of the surface of the microrobots. In this way, no external appendages are needed for propulsion.

An FE model of the bio-inspired propulsion mechanism was developed. Although the simplifications introduced by the model, the predicted velocity was in substantial agreement with experimental data available in literature about Paramecium caudatum (in terms of steady velocity: predicted average value of 1.343 mm/s versus 1.45 mm/s of the natural counterpart).

The model was employed for evaluating the influence of various design parameters on the performance of swimming microrobots. We found that, in the considered range of values, the velocity increases linearly with the microrobot size. Moreover, fixed the volume, a more elongated body shape has a better performance than a more spherical one, although extreme values are not convenient. We also investigated the influence of the features of the wave deformations on the velocity of the microrobot. The performed analysis showed that the velocity is linearly related to both the frequency and the amplitude of the wave. However, the velocity showed a saturation-like behavior already beginning at wave amplitude of 10 µm, while no saturation was found in the considered range for frequency. We also found that the performance of the microrobot decreases with increasing wavelength, until an oscillatory and almost non-propulsive behavior arises for wavelengths close to the microrobot length.

The performed analysis showed that the proposed mechanism of propulsion could be very effective for microrobots with size within the sub-millimeter range. In particular, the model predicted that velocities of several millimeters per seconds could be achieved by means of small surface deformations.

The results of this analysis will thus guide us towards the design of effective self-propelling microrobots. In particular, the model will be employed for investigating aspects related to power demand and hydrodynamic efficiency, in particular as regards their dependence on the main parameters.

VI. REFERENCES