

Modeling kinematic forward model adaptation by modular decomposition

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Abstract—The time needed to adapt to a perturbation depends critically on the amount of the available a-priori information: the more we know about the perturbation, the less experience we need to learn how to compensate for it. The drawback of such a model-based approach is the loss of generality, because rigid assumptions do not allow to rapidly adapt to new perturbations. A possible intermediate solution is represented by a modular strategy, in which the generality is gained through new combinations of pre-learned models. Starting from the assumption that modules might represent a way to store a-priori information in the central nervous system, the present paper explores the consequences of such a modular forward model in human motor learning, in the context of reaching movements. In particular, we tested the prediction that in presence of a modular control, perturbations not compatible with the existing modules should be learned with more difficulty than compatible perturbations. To this aim, we confronted human subjects with two different kinematic perturbations of comparable difficulty: one compatible with the natural kinematic modules (or intra-modular) and one incompatible with them (extra-modular). We observed that human subjects adapt faster to intra-modular perturbations, thus providing evidence in favor of the adoption of a modular strategy by the central nervous system. The obtained results have some interesting consequence within the context of modular learning, hereafter discussed.

I. INTRODUCTION

Reaching, apparently one of the simplest human behaviors, is indeed the result of a complex procedure. In fact, when we reach for an object our central nervous system needs to transform sensory signals into the proper muscle activations to perform the task [18]. This process is thought to rely on internal models, i.e. neural representations of the underlying sensorimotor transformations. The inverse model can transform desired trajectory information into feedforward motor commands, while the forward model can predict sensory consequences from efferent copies of the issued motor commands and thus correct errors. For instance, at a kinematic level and in the context of arm reaching, the inverse model would map the desired hand trajectory to specific joint displacements. The forward model would then help in predicting hand positions given the planned joint trajectories. A question still under debate is how the nervous system builds and adapts these sensorimotor maps. In this study we will essentially focus on kinematic internal models and evaluate the role of modularity in the adaptation of these models to different contextual situations.

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In robotics, within the context of model learning there seems to be a variety of adaptive frameworks, bounded at the two extrema by two very different approaches. On the one hand, model-based approaches rely on rich and accurate mathematical representations of the model to be learned; these representations provide a strong a priori information which is then completed by little information extracted from the supplied data. On the other hand purely data-driven approaches build mathematical representations which rely on minimal a priori information about the model to be acquired (e.g. continuity and smoothness of the function to be learnt); most of the information comes from the supplied data. Within this continuum of possible approaches it is not yet clear which strategy could be adopted by the human central nervous system when learning novel forward models. This scientific question becomes even more interesting if we consider that the more a-priori information is assumed during the learning the smaller the quantity of data necessary to complete this information and therefore the faster the adaptation time. Within this context, it is worth noting that humans might display a significantly long training when exposed to sudden and significant modifications in the environment (e.g. a strong force field - [18]), but similar progressive and slow adaptation is not needed when compensating for the typical kinematic and dynamic changes that we experience everyday. For instance, immediately after grasping a pen, we are perfectly able to move our hand while holding it, naturally compensating for its weight and length in our movements.

In the present paper we investigate the hypothesis that the central nervous system adopts an adaptation strategy that lies in between these two extrema by exploiting a hierarchical and modular organization. Humans might learn through examples and store during their life a certain number of sensorimotor maps, which capture invariants both in the body structure and the environment. Then, they may combine these limited number of elements to rapidly adapt to a wider range of disturbances. In such a modular strategy, therefore, learning is structured in two different processes: the (slow) acquisition of the modules (or maps) and the (fast) adaptation of the combination parameters. Critically testing these predictions could yield indirect evidence for such a modular organization in the brain. D'Avella and Pai's work [6] addressed a similar issue by simulating a motor adaptation experiment with modular and non-modular controllers in the muscle synergies space. In this work we have used another kind of approach, using the properties of

functional analysis metric in the kinematic space and testing these properties in a motor adaptation experiment.

Several are the findings in favor of a modular strategy in the central nervous system, since it has been shown to be able to combine both dynamics and kinematics internal models (e.g. [8], [10], [9]). For instance, when subjects are requested to lift in combination, for the first time, two objects previously lifted separately, they can generate anticipatively the appropriate grip force [8]. Moreover, when subjects are presented with a combined kinematic and dynamic transformation, their performance improves if they had previously learned the two separate transformations [9]. In addition, an entire research field has been dedicated to describe the motor system as a modular structure, characterized by different types of modules or primitives (e.g. spinal force fields [14], kinematic synergies [17] and muscle synergies [7]). In particular, several recent studies have found evidence for muscle synergies by reporting modularity in the context of time-invariant and time-varying muscle activations in different species (e.g. [3], [4], [15], [5]).

Interestingly, modularity could also explain the way we learn to perform actions and why we are so fast at adapting to some changes and not others. For instance holding and manipulating a standard tool (e.g. a pen) is transparent for the human motor system (in contrast to robots). However, more complex changes are much harder to learn (e.g. [13]). We here suggest that the explanation resides in the structure of the human body. For instance, let us describe the arm as a 2 degrees of freedom structure made of two segments connected by rotational joints. The only feature that (slowly) changes during development is the length of the segments, whereas the arm maintains a certain number of kinematic features (e.g. the elbow is always a single rotational joint). Therefore, the mapping from joint angles to end-effector position can be approximated as a combination of simple modules (mathematically, as functions of the joint angles), which is indeed stable during the whole life as the modules depend just on the physical organization of the limb structure. The coefficients of the combination, instead, would depend on the length of the arm and forearm. If this modular interpretation of the arm forward kinematics is correct, learning to reach should be based on a slow development of the modules, for instance learning to associate arm proprioception (e.g. joint angles) to visual position of the hand (i.e. the relevant variable in task space). Indeed, at least 3-4 months seems to be required before infants first reach consistently [12]. Afterwards, the slow changes in arm dimensions due to development and the fast modifications caused by the use of a tool for reaching (e.g. when we point with a stick rather than with our finger) can be rapidly incorporated into the forward model by simply tuning the combination coefficients. Therefore, any variation which is compatible with the learned modular structure does not require a long adaptation process, but rather a fast adjustment of the combination of the learned modules. This would explain the ability to use a great range of pointing tools with no need of prolonged experience. However, the existence of such a

modular structure also implies that learning to cope with any variation not compatible with the existing modules would require much longer training.

Starting from these considerations, we designed an experimental paradigm to test the hypothesis of modularity for the encoding of the forward kinematic internal model. If such modular organization holds, then motor learning should be simplified for tasks that can originate from previous existing modules. At the same time, motor adaptation should become more difficult for tasks which are incompatible with the existing modular structure.

II. MATERIALS AND METHODS

A. Subjects

Six healthy, right-handed subjects (mean age 27 years, ranging from 26 to 29 years; 3 females and 3 males), agreed voluntarily to participate in the experiment. The institutional ethics committee approved the protocol, and all subjects gave informed consent before participating.

B. Experimental protocol

All experiments have been conducted with the use of a manipulandum device (Braccio di Ferro: BdF, see [1] for a complete description of its technical characteristics). Subjects were seated in a chair and grasped the manipulandum handle (see Fig.1). Their chest and wrist were restrained by means of suitable holders. The subjects' right arm was hidden by a towel placed horizontally above the level of their shoulder preventing the use of visual feedback. The learning could have occurred also in presence of continuous visual feedback. However, we chose this protocol based on the work of Mosier et al. [13], to avoid any visually guided movement correction during the training. This choice allowed us to measure for each trial the learning of the feed-forward model. The choice of avoiding continuous visual feedback during the movement was adopted to be able to assess from the blind reaching error the progress of the learning of the remapping with no concurrent visual corrections.

The subjects' right forearm was securely coupled to the robot handle by a custom-molded thermoplastic cuff that also immobilized the wrist joint. The forearm rested on a device linked to the robot arm, that moved over at chest level, constraining movement to the horizontal plane and reducing shoulder and elbow motion to a single degree of freedom each. Prior to the initiation of the experiment we measured the length of the arm and forearm segments. These measures and the knowledge of the kinematic model of the robotic arm allowed to derive subjects' joint angular position on-line, from the instantaneous position of the robotic handle.

Visual feedback of hand and target positions were provided by means of two circles ($\phi = 2\text{cm}$) of different color presented on a large LCD screen with a black background. The screen was positioned in front of the subjects at a distance of about 1 m.

After an acquaintance phase of about one minute in which subjects practiced moving the cursor with the manipulandum, the experiment began.

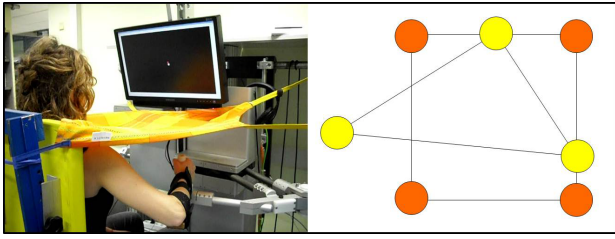


Fig. 1. Experimental setup. On the left, a picture of the experimental setup. On the right, a schema of the two different sets of targets used for the *pre- and post-exposure phases* (yellow dots, in a triangular configuration) and for the *training phase* (orange dots, in a square configuration).

On each trial, participants were requested to position their hand in correspondence of a starting area. Once there, a new target appeared and subjects were instructed to immediately reach it with a one-shot movement. As soon as the hand left the starting zone, the cursor disappeared, reappearing only when the hand stopped (hand velocity lower than 10% of its peak). Subjects could then correct the reaching error under visual guidance. Once inside the target, the procedure was repeated, starting with the appearance of the subsequent target.

Subjects were presented with blocks of different visuomotor remappings (conditions). The first condition was the baseline: the cursor on the screen represented veridically the hand position. After this *no-remapping* condition, subjects were exposed to the *extra-modular* remapping (simulating a non physiological spatial elongation of the links of the subject - see below Sec.II-C for details). At the end of the *extra-modular* block of trials, subjects were presented again with a short block (12 trials) of the baseline condition, to induce a washout of the previous learning. After the washout, subjects were exposed to the *intra-modular* condition (simulating holding a pen).

Each remapping condition was structured in three phases:

- 1) *pre-exposure phase*: 12 reaching movements among three targets arranged in a triangular configuration in pseudorandom order;
- 2) *training phase*: 300 reaching movements among four new targets arranged in a square;
- 3) *post-exposure phase*: 6 reaching movements among the same three targets used in the *pre-exposure phase*.

The *training phase* was aimed at evaluating the learning trend during the exposure to the three different remappings. The *pre- and post-exposure* phases were instead designed to investigate the generalization of the learning of the remappings. In fact, the targets in the triangle set were never experienced during the *training phase* (see Fig.1 right panel). The *pre-exposure* phase was performed to evaluate the starting ability of each subject in performing a reaching under all remapping conditions.

C. Kinematic remapping

Let us consider the arm as a two degrees of freedom kinematic chain made of two segments connected by rotational

joints. The corresponding forward model for a planar motion is defined by:

$$\begin{aligned} x(\theta_1, \theta_2) &= l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2), \\ y(\theta_1, \theta_2) &= l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2), \end{aligned} \quad (1)$$

Where l_1 and l_2 denote the upper arm and forearm length and θ_1 and θ_2 refer to the shoulder and the elbow angular position respectively ¹

From this formulation, the mapping from joint angles to end effector position can be described as a combination of *modules*, represented mathematically by the two functions of joint angles:

$$\begin{aligned} \Phi_1 &: (\theta_1, \theta_2) \rightarrow \begin{pmatrix} \cos \theta_1 \\ \sin \theta_1 \end{pmatrix}, \\ \Phi_2 &: (\theta_1, \theta_2) \rightarrow \begin{pmatrix} \cos(\theta_1 + \theta_2) \\ \sin(\theta_1 + \theta_2) \end{pmatrix}. \end{aligned} \quad (2)$$

This modular description does not change during life, as the arm structure remains unvaried. Only the lengths of the segments change over time (for instance during growth) and this modification can be described by varying the quantities l_1 and l_2 , which correspond to the parameters of the modules combination.

From this mathematical description, two different classes of mappings between joint angle and end effector position can be derived: a class of mappings which are compatible with the existing modules and a class of mappings which are not compatible with this modular structure. In particular a compatible - or *intra-modular* - mapping will belong to $K = \text{span}\{\Phi_1, \Phi_2\}$, a linear vector space spanned by Φ_1, Φ_2 embedded in the Banach space of continuously differentiable functions taking values from the compact set $[-\pi, \pi] \times [0, \pi]$. On the contrary, a incompatible - or *extra-modular* - mapping does not belong to such functional subspace.

In this study we consider two particular mappings, one *intra-* and one *extra-modular*; chosen in order to yield similar spatial distortions in Cartesian space in correspondence of the baseline target positions.

The compatible, *intra-modular* mapping is:

$$M_1 : l_1 \Phi_1 + (l_2 + \delta) \Phi_2, \quad (3)$$

where $\delta = 15\text{cm}$. As observed, this is a natural mapping that still belongs to K .

The *extra-modular* mapping, incompatible with the modules, is instead:

¹Remarkably, we do not claim that the proposed mathematical model is somehow represented in the brain as such. In practice, (θ_1, θ_2) will be represented in the brain by means of suitable combinations of the proprioceptive inputs (e.g. muscle spindles); similarly (x, y) will be represented as combinations of exteroceptive inputs (e.g. visual input). When moving on a plane, the above model is somehow represented in the brain with l_1 and l_2 corresponding to the effective lengths of the upper and forearm. What we are testing here is the robustness of this representation with respect to changes in l_1 and l_2 .

$$M_2 : \begin{pmatrix} (l_1 + \delta) \cos \theta_1 \\ l_1 \sin \theta_1 \end{pmatrix} + \begin{pmatrix} l_2 \cos (\theta_1 + \theta_2) \\ (l_2 + \delta) \sin (\theta_1 + \theta_2) \end{pmatrix}. \quad (4)$$

The *intra-modular* and the *extra-modular* remapping conditions were chosen in order to have two different classes of mappings between joint angle and end effector position: a class of mappings which are compatible with the existing modules and a class of mappings which are not compatible with this modular structure.

D. Data analysis

During the experiments we recorded the time course of the trajectory of the hand, sampled at 100 Hz and transferred off-line for further analysis. All analyses were performed with custom software written in Matlab (Mathworks, Natick, MA, USA). Movement onset and termination were identified by applying a velocity threshold (10% of the peak of velocity) to the cursor speed profile. For each subject we measured and analyzed two main aspects of performance.

- 1) *Reaching error*: the Euclidean distance between the cursor position at movement's end and the target center.
- 2) *Score*: the percentage of successful one-shot reaching movements, identified as reaching movements producing less than 5cm of error.

III. RESULTS

The aim of this work was to test the hypothesis of modularity for the encoding of the forward kinematic internal model. If the central nervous system adopts a modular organization, then motor learning will be simpler for tasks that may originate from the existing modules and more difficult for tasks which are incompatible with them. To address this interrogative we asked subjects to perform reaching movements while they were exposed to visuo-motor remappings which were either compatible (*intra-modular*) or incompatible (*extra-modular*) with the natural kinematic modules (see Sec.II-C).

In Fig.2 we have plotted the average reaching error of all subjects during the exposure to the three remapping conditions. At the beginning of the *pre-exposure phase* the average error for the *intra-modular* and *extra-modular* remapping conditions is similar and significantly higher than the baseline (*no-remapping*) error (Repeated Measures One-Way ANOVA, Tukey post-hoc test, $P=0.02$, see the leftmost data points in Fig.2). However, already during the 12 trials of the *pre-exposure phase*, a learning is apparent for the *intra-modular* remapping condition, as the error at the end of this phase is significantly lower than the one measured during the initial exposure (*pair-sample t-test*, $P=0.02$). The error in the *extra-modular* and *no-remapping* conditions, instead, does not decrease significantly (*pair-sample t-tests*, $P=0.32$ and $P=0.06$ respectively) in this phase.

The *extra-modular* remapping adopted in this study has been chosen to yield the same spatial distortion as the *intra-modular* remapping in Cartesian space in correspondence to

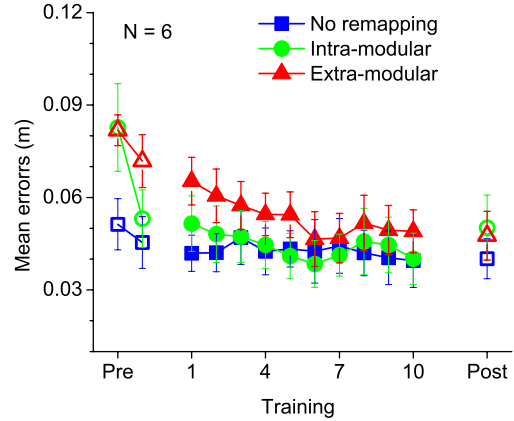


Fig. 2. Average reaching error during all the experimental conditions. Different symbols represent different remappings. The two leftmost sets of data points refer to the average error at the beginning (first 2 trials) and at the end (last 6 trials) of the *pre-exposure phase*, while the right-most set of data points represents the error in the 6 trials of the *post-exposure phase*. The data points in between indicate the average error during the *training phase*, binned in blocks of 30 trials.

the baseline target positions. A proof of the similar initial level of difficulty between the two kinds of remapping is represented by the similarity in the errors in the first bin of the *pre-exposure phase* (see Fig.2, First Bin). As the initial difficulty in dealing with the two remapping is the same, the difference between the conditions should not influence the testing of the hypothesis.

During the *training phase*, subjects show a clear learning trend in both *intra-* and *extra-modular* remapping conditions, with the error in the last block of trials being significantly lower than the error at the beginning of the training (*pair-sample t-tests* between the average error of the first and last set of 30 trials in the *training phase*, $P=0.02$ for the *intra-modular* remapping and $P<0.01$ for the *extra-modular* remapping). As expected, the error in the *no-remapping* condition remains instead almost stable (*pair-sample t-test*, $P=0.59$).

Interestingly, the *extra-modular* remapping also shows a generalization. In fact, the error in the *post-exposure phase*, which required reaching to three targets outside the training area, does not differ significantly from the error made at the end of the training phase, while being significantly lower than the error at the end of the *pre-exposure phase* (Repeated Measures One-Way ANOVAs, Tukey post-hoc test, $P<0.01$). The measure of generalization at the end of the experiment in the case of the *intra-modular* remapping is hampered, as the error reached at the end of the *pre-exposure phase* was already not significantly different from those at the end of the training and in the *post-exposure* (Repeated Measures One-Way ANOVAs, Tukey post-hoc test, $P=0.32$). However, a clear evidence in favor of a generalization process is already present at the very beginning of the training phase, when the learning accomplished in the triangular set of targets of

the *pre-exposure phase* is transferred to the reaching of the new training targets. In fact, the error at the beginning of the training does not differ significantly from the error at the end of the previous phase, while both are significantly lower than the error made during the first exposure to the remapping (Repeated Measures One-Way ANOVAs, Tukey post-hoc test, $P < 0.01$)

These findings shows that learning to perform the reaching in the two remapping conditions generalize also to targets not experienced in the training phase.

Although learning is present in both remapping conditions, the adaptation to the *intra-modular* remapping appears to be easier than that to the *extra-modular* one. Indeed, the reaching error in the mapping compatible with the hypothetical natural modules of arm kinematics (*intra-modular*) reaches its ideal minimum - represented by the performance in the *no-remapping* condition - already during the first 30 trials. From a Repeated Measures One-Way ANOVA on the average error of the first bin in the three conditions, in fact, it emerges that errors in the *no-remapping* and in the *intra-modular* condition are not significantly different, while both differ from the *extra-modular* ($P < 0.01$, Tukey post-hoc test). On the contrary, even after a hundred trials, the adaptation to the *extra-modular* remapping seems not to be completed (i.e. error in the *extra-modular* remapping is significantly higher than the error in the *no-remapping* condition, while *no-remapping* and *intra-modular* errors are similar; Repeated Measures One-Way ANOVA on the average errors of the fifth bin. Tukey post-hoc test, $P < 0.01$). Only during the sixth block of trials (i.e. after about 180 trials) the error becomes similar among all remappings (Repeated Measures One-Way ANOVA on the average errors of the sixth bin. $P = 0.24$).

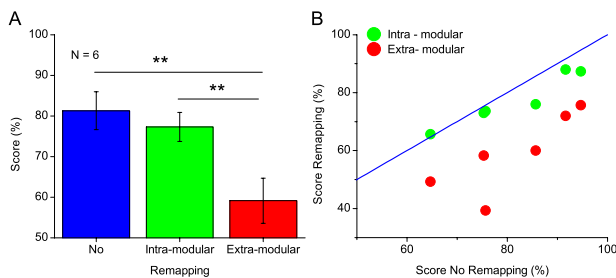


Fig. 3. Reaching score in the training phase. A) Average score in the three remapping conditions, as a percentage of the total number of trials. B) Individual score in each remapping condition plotted against the corresponding score during the baseline (no-remapping). If data points lie under the identity line, it means that the score level reached during the no-remapping condition was higher than that achieved during the exposure to a remapping. Different colors represent different remappings.

The greater difficulty in learning the *extra-modular* remap-

ping emerges also from the analysis of another performance measure, the score (see Fig.3A). Subjects were awarded with one point when they correctly reached the target in a single one-shot movement (i.e. their absolute reaching error was lower than 5cm). Subjects score was significantly lower in the *extra-modular* condition than both the *no-remapping* and the *intra-modular* cases; while the last two did not differ significantly (Repeated Measures One-Way ANOVA, Tukey post-hoc test, $P < 0.01$). Interestingly, this is not just an average effect, as it holds for all the tested subjects (see Fig.3B).

In sum, from these results it emerges clearly that a remapping which is compatible with the natural kinematic modules is learned more easily than one which does not fit the existing modular structure. This is evident despite the two remappings produce a quantitatively similar spatial distortion to the target positions. In fact, although during the first exposure to the remappings subjects showed the same error magnitudes in the two conditions, the learning evolved with a much faster pace in the *intra-modular* remapping.

IV. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

Modularity seems to be a primary feature of the organization of the motor system. More in general, the brain tends to solve a complex problem by developing modules that solve simpler components and by adequately combining them [16]. Such *mixture-of-experts* architecture has been proposed to model a wide variety of abilities, ranging from visuomotor adaptation [10] to object recognition [11] and number representation [2].

One peculiar characteristic of modular systems, is that learning is structured in two phases: learning the modules, which occurs slowly, and learning the parameters of the combination rule, which happens at a much faster pace. A direct consequence of this learning structure is that a modular control should be able to adapt rapidly to changes which are compatible with the pre-learned modules. Indeed, once the modules have been learned, only the combination parameters needs to be changed, which should be fast [6]. On the opposite, learning an apparently similar distortion which however is not compatible with the existing modules should take longer. Modularity would also explain how we may adapt rapidly to a great variety of changes in our limb: e.g. how we are able to use pointing tools of any kind and dimensions without requiring any prolonged experience. This assumption has been recently demonstrated in a simulation study, where a modular controller has been shown to yield to two different learning rates for incompatible and compatible

perturbations, whereas a non-modular controller produced the same learning curves in the two cases [6].

We tested for the first time whether a similar difference in learning rates could be measured in the case of reaching kinematics, to evaluate whether a modular description of reaching could be proven. According to D'Avella and Pai [6], if we introduce subjects in a virtual reality environment where the visual feedback about hand position is remapped in a way which is not describable in terms of the already existent kinematic modules, their adaptation to the visuomotor distortion is much slower than when they are exposed to a remapping compatible with their natural modules. This difference in learning trend is clearly present even though the experiment was controlled in such a way that the two remappings were equivalently difficult (as they induce similar spatial error at the end-point).

These results are compatible with a modular description of the arm kinematics, which is exploited both during reaching in the normal condition and which is potentially adapted to changes (real or virtual) in arm features.

B. Future Works

A possible confound in the current study is that the *intra-modular* remapping is commonly experienced in everyday life (e.g. while holding a tool), while the *extra-modular* remapping is difficult to be found in nature. This is of course expected: a modular control becomes useful only when most of the perturbations it must face are compatible with the known modules. So, it would be less likely for the brain to adopt this strategy in the first place if the use of the modules would not bear any potential advantage. A possible solution to disentangle the relevance of naturalness and compatibility to natural modules will be to test a *intra-modular* remapping which is not usually experienced, i.e. a rigid elongation of the arm between the shoulder and the elbow.

The concept of module is indeed quite flexible and in future works we are planning to test different combinations and different module structures in order to generalize the modular approach. In this study we have shown that subjects were able to adapt to a new remapping generated using the properties of functional analysis and incompatible with the existing modules (the *extra-modular* one), even though the learning was slower. We believe therefore that a form of

adaptation can be achieved even in presence of a task not expressible as the combination of existing modules.

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