Dynamical analysis of human standing model with cyclic motion

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Abstract—Human standing is characterized by large body sway, which cannot be explained by linear control. In past researches, sway has been considered as an uncontrolled biological noise. In contrast, we consider the sway to be a cyclic motion generated by continuous proportional-integral-derivative (PID) control with weak nonlinearity. Through mathematical analysis of nonlinear PID control, cyclic motion is shown to be generated by a stability-gain dependent Hopf bifurcation, and biological noise is shown to help the smooth transition between stationary stable state and cyclic state. The relevance of the proposed sway generation mechanism is verified through human experiment on floors with different stability. As a result, the existence of Hopf bifurcation, i.e., apparent expansion of sway with small decrease of control parameters, realized by the destabilization of floor, is observed.

I. INTRODUCTION

Humans typically maintain a standing posture by swaying backward and forward by about 20 mm at a very slow frequency (<1 Hz). From medical and motor control research, it is known that characteristics of this body sway reflect neural mechanism of posture control in the following ways. 1) Body sway changes are inherent symptoms of certain neurological diseases; for example, the amplitude of sway decreases with Parkinson’s disease [1], [2], and the complexity of sway increases with spinocerebellar ataxia [3]. 2) Biological noise, such as hemodynamic noise, does not by itself adequately explain the amplitude of body sway; to do so requires some nonlinear neural mechanism [4], [5].

One possible explanation for large-amplitude sway is that it is a result of stabilization to a stationary state with a prolonged period of no control. A control procedure in which control is exerted intermittently, called the intermittent control model [6], [7], has been shown to replicate the features of human body sway and to be robust against cognitive delay. This mechanism has been unique explanation of the standing with large body sway.

In contrast, we consider another explanation for posture control with large sway. That is, sway is a cyclic motion generated by the destabilization of continuously controlled stationary state. Human can generate cyclic motion during standing, such as cyclic upper trunk bending (e.g. [8]). The idea of body sway to be a cyclic motion is such a motion involuntary emerges by Hopf bifurcation with destabilization of stationary state. By allowing the initiation of cyclic motion, instead of holding on stationary state, the barrier between standing and motion is lowered, enabling a spontaneous, dynamic reaction to destabilization. That is, it allows a high degree of maneuverability (or variability).

Here we show that this type of posture control for cyclic motion can be realized by a simple proportional-integral-derivative (PID) control with weak (third-order) nonlinearity. First the existence of control parameter-dependent Hopf bifurcation is shown through mathematical analysis and cyclic motion is indicated to be a possible mechanism of sway generation. Then, by numerical evaluation of human standing on floors with different stabilities, the existence of the control parameter-dependent bifurcation is discussed.

II. MODEL

In this research, rotational motion of the center of mass (COM) around the ankle joints is modeled as an inverted pendulum on the sagittal plane.

\[ J \ddot{\theta} = mgh\dot{\theta} + \tau + \sigma \xi \]  \hspace{1cm} (1)

Here, \( \dot{\theta} \) is the elevation angle of the pendulum, \( m \) is body mass, \( g \) is acceleration due to gravity, \( h \) is the length from ankle to the COM, and \( J \) is the inertial moment around an ankle joint. Control torque \( \tau \) is given at the ankle, and biological noise \( \sigma \xi \) is modeled as Gaussian white noise. \( \xi \) is a normalized noise and \( \sigma \) is its magnitude.

Posture control is composed of stiffness control and sensory feedback (Fig. 1). Stiffness control is a real time feedback due to the spring-like nature of muscles, and sensory feedback control is due to cognitive response to sensations with delay for 150 ms. Gains in stiffness and sensory feedback control were experimentally identified, in which sensory feedback was shown to have a gain 10-fold that of stiffness control [9]. Thus we consider sensory
feedback only and treat stiffness control as negligible because of its lower contribution.

As a sensory feedback control, the PID control model

$$\tau = -k_p \Delta - k_d \theta \Delta - k_i \int \theta \Delta dt$$

(2)

has been proposed. This model has been proved to replicate many features of human standing motion [9]. In Eq. (2), $k_p$, $k_d$, and $k_i$ are the proportional, derivative, and integral control gains, respectively; $\theta \Delta$ represents the angle $\theta$ from the COM with a delay.

Peterka [9] identified control parameters of a linear PID model (of the same form as Eq. (2)) by reference to the frequency response for human standing. The identified control parameters were estimated by $k_p = 1.66mgh$, $k_d = 0.54mgh$, and $k_i = 0.20mgh$; this model with the identified parameter values reflected human standing to a significant degree. However, Bottaro et al. [5] calculated that the minimum noise necessary to generate typical body sway amplitude, $0.181^\circ$, by Peterka’s model was $\sigma \xi = 5$ Nm. This value is 10-fold the hemodynamic noise generated by heartbeats (0.4 Nm [4]); hemodynamic noise is considered to be the largest biological noise during standing. Thus, linear PID control cannot explain the magnitude of body sway. For explaining this body sway, intermittent control has been proposed [6], [7], in which proportional control $k_p \theta \Delta$ is intermittently input and body sway is attributed to a no control state. The present research, as a model with weaker nonlinearity than that of the intermittent control model, proposes a nonlinear PID control model with a second-order effect from $\theta$ in $k_p$.

$$k_p = k_{p0} + k_{p1} \theta \Delta + k_{p2} \theta^2 \Delta$$

(3)

We show analytically that this nonlinear PID control model possesses Hopf bifurcation induced by $k_{p0}$ and generates large body sway as a cyclic motion.

III. MATERIALS AND METHODS

A. Model analysis

Analysis of the mechanism of sway generation in the proposed model is conducted as follows. We first analyze stable solutions of the proposed model to remove the effects of noise; body sway is then shown to appear as a cyclic solution determined by Hopf-bifurcation, independent of biological noise. Next, this solution, with noise included, is analyzed; the analysis suggests that biological noise functions to smooth the transition between the stable stationary states and cyclic motion.

1) Fundamental equation and simulation condition: We summarize the equations of the proposed model and the conditions for analysis and dynamical simulation. The proposed equations are Eqs. (1)-(3). They can be written together as Eq. (4):

$$J \ddot{\theta} = mgh \theta - (k_{p0} + k_{p1} \theta \Delta + k_{p2} \theta^2 \Delta) \theta \Delta - k_D \theta \Delta - k_I \int \theta \Delta dt + \sigma \xi.$$  

(4)

The objective motion of the analysis is sufficiently slow, less than 1 Hz, in contrast to the sensory feedback delay, which is on the order of 100 ms. Thus, in the analysis, delay is neglected ($\theta \Delta = \theta$). Moreover, the second-order nonlinear factor $k_{p2}$ contributes only to deviation from the center of motion, so it can be neglected for the sake of simplicity ($k_{p2} = 0$). Thus, the equation becomes

$$J \ddot{\theta} = mgh \theta - (k_{p0} + k_{p1} \theta^2) \theta - k_D \theta - k_I \int \theta dt + \sigma \xi,$$

(5)

and by setting $\psi = \dot{\theta}$ and $\eta = \int \theta dt$, the equation can be written as

$$J \psi = \{(mgh - k_{p0}) - k_{p2} \theta^2\} \theta - k_D \psi - k_I \eta + \sigma \xi.$$  

(6)

Analysis is performed on Eqs. (5) and (6).

The accuracy of the analytical solution is verified by comparison with dynamical simulation. The parameters used for the dynamical simulation are based on the human experiment explained later. The parameter values are the following: $m = 63.7$ kg, $g = 9.8$ m/s$^2$, $h = 0.95$ m, and $J = 57.3$ kg m$^2$ for the inverted pendulum model; $k_{p2} = 115.7 mgh$, $k_{p1} = 0.43 mgh$, and $k_1 = 0.06 mgh$ for control gain; and $\sigma = 0.34$ N for magnitude of biological noise. Moreover, in order to compare the simulated angle $\theta$ from the COM with the angle from the measured value of the center of pressure (COP), $\theta$ is converted using the following relationship [12].

$$COP = h \theta - \frac{h_e h \theta}{\theta},$$

(7)

Here, $h_e$ is the distance from the ankle to the COM modified to consider the deviation of mass, calculated as $h_e = 1.15 h$ [5]. Numerical calculation of stochastic differential equation for the simulation is performed by using the Euler-Maruyama method.

2) Generation of cyclic motion: By simulating the proposed model (Eq. (5)) with various linear proportional gains $k_{p0}$, two steady states were found: a stationary state when $k_{p0} > mgh$ and a limit cycle when $k_{p0} < mgh$.

To understand the characteristics of a solution around the transition point at $k_{p0} \approx mgh$, first the linear response around $k_{p0} \approx mgh$ of Eq. (5) with $k_{p2} = 0$ was investigated. When $k_{p2}$ was set to be $1.1mgh$ and gradually reduced to be $0.9mgh$, the poles of the equation moved along the arrows shown in Fig. 2. Among the three poles of the

![Fig. 2. Root locus. Arrows show the transition of the value of poles with variable $k_{p0}$ from $1.1mgh$ to $0.9mgh$. Gray circles and crosses indicate the values of poles when $k_{p0}$ are $1.1 mgh$ and $0.9mgh$, respectively.](image-url)
system, one pole was located on the real axis with a strictly negative value (the stable pole); the two other poles were complex conjugates of each other. The conjugate poles had a negative real value when \( k_{P0} = 1.1mgh \), but they crossed the imaginary axis at \( k_{P0} \approx mgh \) and became unstable. Thus, the limit cycle can be generated as the effect of these conjugate poles, whose instability at \( k_{P0} < mgh \) is sustained by the effect of the nonlinear control \( k_{P2} \).

Then, to investigate the convergence around \( k_{P0} \approx mgh \) including the effect of the nonlinear control \( k_{P2} \), the conditions necessary for a cyclic steady state and for stability are discussed. To focus on the solution by conjugate poles in Eq. (5), the following general form of solution is considered.

\[
\begin{align*}
\text{Solution:} & \quad \theta = ae^{j\omega t} + a^*e^{-j\omega t}, \\
\text{Restraint condition:} & \quad \dot{a}e^{j\omega t} + \dot{a}^*e^{-j\omega t} = 0
\end{align*}
\]  

(H) Here, for the purpose of simplicity, the integral term is approximated by

\[
\int \theta dt = \frac{1}{j\omega}(ae^{j\omega t} - a^*e^{-j\omega t}).
\]  

By substituting Eqs. (8) and (9) into Eq. (5) and by integrating both members for one cycle, the equation becomes

\[
\dot{a} = -\frac{1}{2j\omega}\left(k_D\omega - k_i\right) a
\]  

\[
-\frac{1}{2j\omega}\left(J\omega^2 + mgh - k_{P0} - 3k_{P2}a^*\right) a.
\]  

(10)

Here, by substituting \( a = re^{j\phi} \), a cyclic solution is extracted. This operation is equivalent to the comparison of the real and imaginary part of Eq. (10) with \( \dot{r} = \dot{\phi} = 0 \), so that the following relationship can be obtained.

\[
\begin{align*}
\dot{r} &= -\frac{1}{2\omega}\left(k_D\omega - k_i\right) r, \\
\dot{\phi} &= -\frac{1}{2\omega}\left(J\omega^2 + mgh - k_{P0} - 3k_{P2}r^2\right)
\end{align*}
\]  

(11)

From the above equations with \( \dot{r} = 0, \dot{\phi} = 0 \), a steady amplitude for the cyclic solution can be obtained.

\[
r = \begin{bmatrix} 0 \\ \sqrt{\frac{k_{P0} + mgh + Jk_i}{k_D}} \end{bmatrix}
\]  

(12)

This equation shows that the fixed solution \( r = 0 \) (i.e., \( \theta = 0 \)) always exists and a cyclic solution simultaneously exists when \( k_{P0} < mgh + Jk_i/k_D \).

To consider the stability of the fixed solution \( \theta = 0 \), Eq. (6) is linearized around \( \theta = 0 \). Then the equation becomes

\[
\begin{bmatrix} \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -\frac{k_D}{J} & \frac{1}{J}\left(mgh - k_{P0} \right) \end{bmatrix} \begin{bmatrix} \theta \\ \psi \end{bmatrix}
\]  

(13)

and the characteristic equation becomes

\[
\lambda^3 + \frac{k_D}{J}\lambda^2 - \frac{1}{J}(mgh - k_{P0})\lambda + \frac{k_i}{J} = 0.
\]  

(14)

From this equation, the stability condition of the solution \( \theta = 0 \) becomes

\[
k_{P0} > mgh + Jk_i/k_D.
\]  

(15)

3) Contribution of biological noise: Next, the features of a system that includes biological noise (i.e., \( \sigma \neq 0 \)) are investigated and the effect of the noise is discussed.

To analyze the transition of COM with variance \( \langle \theta^2 \rangle \), we observe that \( \theta^2 \) and \( \theta^3 \) converge more quickly than \( \theta \) does, so the nonlinear term of \( k_P \) is linearized by using the average of \( \theta \) (mean field approximation [13]). This yields

\[
k_P \approx k_{P0} + k_{P2}\langle \theta^2 \rangle,
\]  

where \( \langle \rangle \) expresses the average and \( \langle \theta^2 \rangle \) becomes the variance because \( \langle \theta \rangle \approx 0 \). Then Eq. (6) can be described by following Langevin equation.

\[
d\theta = AXdt + dW
\]  

(17)

At this time, the correlation matrix about \( X \) and the magnitude of noise \( D \), respectively given by

\[
R = \begin{bmatrix} \langle \theta^2 \rangle & \langle \theta \rangle \\ \langle \theta \rangle & \langle \psi^2 \rangle \end{bmatrix}
\]  

\[
D = \begin{bmatrix} 0 \\ \langle \psi^2 \rangle \end{bmatrix}
\]

have the following relationship by the fluctuation-dissipation theorem:

\[
AR + RA^T = -2D.
\]  

Then, the variance of COM, \( \langle \theta^2 \rangle \), becomes

\[
\langle \theta^2 \rangle = \frac{1}{2k_Dk_{P2}} \left\{ Jk_i + mghk_D - k_Dk_{P0} \right\}^2 + \sqrt{(Jk_i + mghk_D - k_Dk_{P0})^2 + 4k_Dk_{P2}\sigma^2}
\]  

(19)

This equation provides analytical solution of the proposed model. Then the standard deviation of COP is calculated from analytical result (eq. (19)) and simulation with concerning the control gain \( k_{P0} \) is described for discussing bifurcation structure of the system.

B. Experimental method

The experiment is performed under two floor conditions, fixed floor and rotational floor. In the fixed floor condition, participants stand on a force plate with their legs open to shoulder width. Participants are asked to hold their posture for 360 s, and the motion of the COP is measured by time series. In the rotational floor condition, participants stand on a platform that has variable stability in the sagittal plane. The platform possesses a rotational shaft and is connected to the floor by springs. The stiffness of each spring is 10 N/mm; two springs are located in front of the participant and two are located behind the participant (four springs in total). As the effect, the torque transferred from the ankle to the floor is approximately halved. The participants were 9 persons (8 men and 1 woman), 6 participants completed 4 trials and 3 participants completed 3 trials.

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Measurement is performed using a force plate (Tec Gihan TF-4060-A), and the measurement frequency is 1000 Hz. Participants are healthy persons between 21 and 25 years old, with body height from 153 cm to 180 cm and body mass from 42 kg to 76 kg. The participant closed his or her eyes and crossed the arms in front of the chest during measurement. Participants gave written informed consent prior to data collection in accordance with the procedures of the Ethics Committee of Doshisha University or the Ethics Committee of the University of Electro-Communications.

In the proposed model, the stationary state is stable when $k_{P0} > mgh$; cyclic motion appears when $k_{P0}$ falls below $mgh$, so the existence of bifurcation can be analyzed by identification of $k_{P0}$. The parameters are searched so that the RMS error between the power spectrum of the measured data and that of the model obtained by simulation decreases. Searching is performed by simulated annealing method using the MATLAB `Simulannealbnd` function. Search parameters are the control gains $k_{P0}$, $k_{P1}$, $k_{P2}$, $k_{D}$, and $k_{I}$ and the magnitude of noise $\sigma$. Response delay is set constant at 150 ms. Measured values for participants are used for body mass ($m$) and length from ankle to COM ($h$); The moment of inertia ($J$) is calculated as $mh^2$.

IV. RESULTS

A. Sway generation in the proposed control model

In order to focus on the stability of the proposed model (Eqs. (1)-(3)), first the proposed model excepting the effect of biological noise ($\sigma = 0$) was analyzed (see Materials and Methods A2). As a result, system showed Hopf bifurcation induced by $k_{P0}$ as shown in Fig. 3A, i.e., system converged to a stable stationary state when $k_{P0} > mgh + J \frac{k_i}{k_D}$ and this fixed solution destabilized and simultaneously cyclic solution generated when $k_{P0} < mgh + J \frac{k_i}{k_D}$. Here, parameters used for Fig. 3 are set in response to the identified human parameters in the latter discussion and because $mgh \simeq 593$ and $J \frac{k_i}{k_D} \simeq 0.80$ in those parameters, the bifurcation point $k_{P0} = mgh + J \frac{k_i}{k_D} \simeq mgh$. This result indicates that, in the proposed model, the large body sway observed in human motion can be explained as a cyclic solution. Moreover, if $k_{P0}$ is maintained around the bifurcation point, large difference in the amplitude of body sway before and after the bifurcation is potentially observed. This observation of the bifurcation is subsequently discussed through human standing experiment.

On another front, Fig. 3A shows rapid change in the sway amplitude at bifurcation point, which is difficult to believe as a human behavior. We considered that ignoring biological noise caused this rapid change, so we next performed mathematical analysis including the effect of noise (see Materials and Methods A3). Fig. 3B shows the result of analysis including approximately same magnitude of human noise, where solid line represents the result of simulation and dotted line represents the result of analysis. The figure shows that the amplitude of body sway continuously varies around $k_{P0} \simeq mgh$ in both simulation and analysis. Therefore, body fluctuation generated by biological noise is considered to absorb the rapid transition from a stationary state to cyclic motion.

From the above, the proposed model has following characteristics. First, in accordance with the decrease in the linear proportional gain $k_{P0}$, the stationary state destabilizes and cyclic motion appears. Second, the transition from a stable state by a decrease in $k_{P0}$ does not show rapid change of motion, instead it smoothly changes owing to the effect of biological noise. In this way, proposed model can explain large body sway as a cyclic motion. Next, through a human experiment, proposed model is verified to be the mechanism of human control, discussing from the viewpoint of the existence of bifurcation.

B. Bifurcation in human standing

Proposed control mechanism explained human large body sway as a cyclic motion, and predicted the existence of bifurcation in human standing. This bifurcation is supposed to occur in accordance with the decreasing of linear stability gain $k_{P0}$. By decreasing the stability of standing floor, $k_{P0}$ can be equivalently reduced and, if the proposed mechanism is consistent with human control, the transition is possibly observed in human experiment when $k_{P0}$ falls below $mgh$. For discussing this feature, human standing experiment was performed on different stability of floors: fixed floor condition and rotational floor condition. Control gains were identified and the value of $k_{P0}$ and the magnitude of sway represented by standard deviation of COP was compared (see Materials and Methods B).

The identified $k_{P0}$ are 1.06 $mgh$ for fixed floor and 0.97 $mgh$ for rotational floor. This means that the standing state is maintained around the bifurcation point $k_{P0} \simeq mgh$, i.e., stability limit of linear control, in both conditions. Moreover, the value of $k_{P0}$ on a rotational floor was lower than that on a fixed floor, and they were lower than $mgh$ on a rotational floor in average. Based on the identified parameters, the relationship of the magnitude of body sway and control gain $k_{P0}$ was drawn and was compared with analytical result as shown in Fig. 4. In this figure, the standing states for the
fixed floor and rotational floor conditions were separated into a stationary state (white area in Fig. 4) and a cyclic motion state (gray area in Fig. 4). Moreover, increased body sway accompanied by decreasing gain $k_{P0}$ were confirmed to occur as indicated in the analytical solution of the proposed model. Therefore, this experimental result supports the existence of bifurcation structure in human control.

**CONCLUSION**

In this study, we consider instability induced cyclic motion as a possible cause of human body sway during standing and investigated a human posture control mechanism from mathematical analysis of the bifurcation structure and numerical evaluation of the model with human experiment. The following characteristics were found. 1) The proposed control model (a nonlinear PID control model) has a bifurcation from a stationary state to cyclic motion when the value of $k_{P0}$ falls below $mgh + \frac{K_i}{K_P}J$, which is approximately $mgh$. The transition on the bifurcation is smoothly achieved by the effect of biological noise. 2) Linear control gain $k_{P0}$ is maintained at approximately $mgh$ for both fixed and rotational floors; it is higher than $mgh$ on fixed floors and lower than $mgh$ on rotational floors. The standard deviation of body sway on a rotational floor was twice that on a fixed floor. These results indicate the existence of bifurcation from a stationary state to cyclic motion with the destabilization of standing. On the basis of these results, we find that large body sway observed in human standing is a cyclic motion caused by a property of human posture control.

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