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Experimental verification of cusp catastrophe in the gait transition of a quadruped robot driven by nonlinear oscillators with phase resetting

Shinya Aoi[1,2], Takahiro Tanaka[1], Soichiro Fujiki[1], Kei Senda[1], and Kazuo Tsuchiya[1,2]

Abstract— **In this paper, we investigated the dynamic characteristics in the gait of a quadruped robot, which is controlled by an oscillator network constructed based on the physiological concept of central pattern generators and the physiological evidence of phase resetting. From the observation in humans, two-parameter cusp catastrophe is suggested to be embedded in the walk–run transition with the locomotion speed and additional load as control parameters. In our previous work, simulation studies revealed that a quadruped robot model produces the walking and trotting gaits depending on the locomotion speed and it shows the cusp bifurcation due to additional weights similar to humans through dynamic interactions among the robot's mechanical system, the oscillator network system, and the environment. The aim of the present study is to verify such dynamic characteristics in the cusp catastrophe of the gait transition in the locomotion of our quadruped robot model in the real world by using a quadruped robot.**

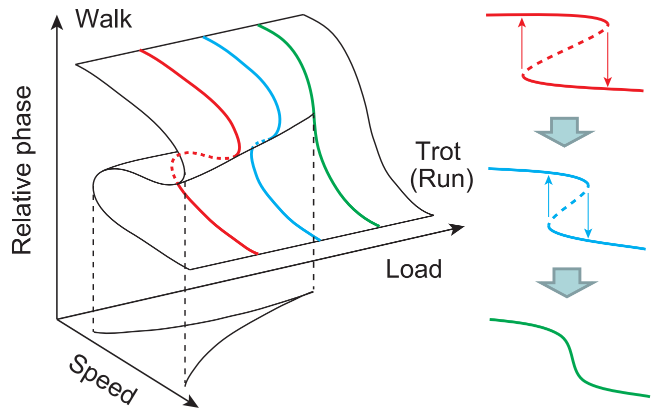

Fig. 1. Cusp catastrophe in gait transitions (modified from [5]). Relative phase determines the gait and shows the folding property for the locomotion speed. The size of the folding reduces as the load increases and the folding disappears at a critical load.

I. INTRODUCTION

Locomotion in humans and animals is a self-organizing phenomenon that emerges through dynamic interactions among the nervous system, the musculoskeletal system, and the environment and has various nonlinear properties. In particular, there are various gaits in their locomotion, such as walking and running gaits for humans and walking, trotting, and galloping gaits for quadruped animals, and hysteresis appears when the gaits change in accordance with the locomotion speed [2], [7], [11], [13], [17], [21], [29], [33], [38]. That is, the gaits vary at different locomotion speeds depending on the speed change direction. Although hysteresis is a typical characteristic of nonlinear dynamic systems, the hysteresis mechanism in the gait transition remains largely unclear.

In our previous works [1], [2], we focused on the walk–trot transition in quadruped animals and developed a neuromechanical model and a quadruped robot based on the physiological concept of central pattern generators (CPGs) [12], [28], [34] and the physiological evidence of phase resetting [6], [8], [30], [31] to elucidate the hysteresis mechanism in the gait transitions from a dynamic viewpoint. The results showed that our models produced the walking and trotting gaits depending on the locomotion speed. In addition, they exhibited the gait transition with hysteresis when the locomotion speed was changed. This was not because we intentionally designed the movements of our models to produce the gait transition and hysteresis; rather, because the stability structure of the gaits changes through the interaction between the nervous control system, the body mechanical system, and the environment. Furthermore, the biological relevance of the gait generation and transition of our models were evaluated by measuring the locomotion in dogs. We conducted a stability analysis using return maps and concluded that the hysteresis in the walk–trot transition of our quadruped models was produced through saddle-node bifurcation.

In addition to the appearance of the hysteresis in the gait transition, Beuter and Lalonde [5] showed that the hysteresis loop regarding the locomotion speed in the walk–run transition of humans reduces as the additional weight put on the subjects increases. That is, the additional load decreases the difference between the walk-to-run and run-to-walk transition speeds. They explained that this result suggests that a two-parameter cusp catastrophe with the locomotion speed and load as control parameters is embedded in the walk–run transition in humans, as shown in Fig. 1. Specifically, the relative phase (between the leg segments for humans [7] and between the leg motions for quadruped animals [2]) determines the gait and shows the folding property for the locomotion speed due to the hysteresis property. In addition, the size of the folding reduces as the additional load increases and the folding disappears at a critical load.

While the walk–run transition of humans suggests the

[1]Shinya Aoi, Takahiro Tanaka, Soichiro Fujiki, Kei Senda, and Kazuo Tsuchiya are with Dept. of Aeronautics and Astronautics, Graduate School of Engineering, Kyoto University, Kyoto daigaku-Katsura, Nishikyo-ku, Kyoto 615-8540, Japan shinya_aoi@kuaero.kyoto-u.ac.jp
[2]Shinya Aoi and Kazuo Tsuchiya are with JST, CREST, 5 Sanbancho, Chiyoda-ku, Tokyo 102-0075, Japan

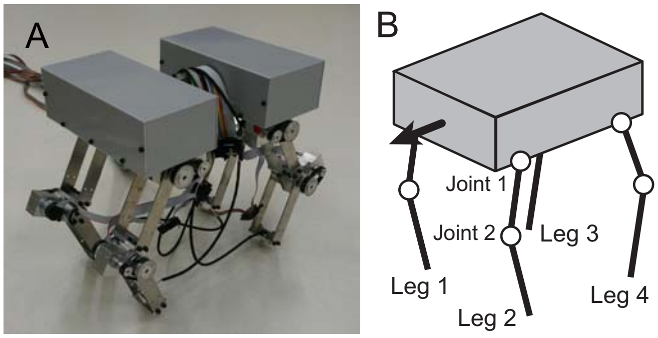

Fig. 2. Quadruped robot (**A**: robot and **B**: schematic model) developed in our previous work [2]. The robot body consists of two sections that are mechanically attached to each other.

existence of cusp bifurcation, it remains unclear if the gait transition of quadruped animals has the cusp catastrophe structure. Therefore, in our previous work [3], we conducted computer simulations using a quadruped robot model and demonstrated that the cusp catastrophe is embedded in the walk–trot transition of the robot model, similarly to the walk–run transition in humans. However, because the simulation study was based on a mathematically ideal model of a robot, discrepancies between simulation results and experimental results are inevitable. In addition, the cusp bifurcation is a characteristic property of nonlinear dynamic systems. It is important to verify our simulation results using an actual robot. The aim of the present study is to verify the obtained dynamic characteristics in the walk–trot transition in the real world using a quadruped robot and additional weights.

This paper is organized as follows: Section II introduces our developed quadruped robot and Section III explains the locomotion control system. Section IV shows the experimental results and Section V presents the discussion and conclusion.

II. QUADRUPED ROBOT

We used a quadruped robot (Fig. 2), which was developed in our previous work [2]. It consists of a body and four legs (Legs 1–4). (The body consists of two sections that are mechanically attached to each other.) Each leg is attached to the body through a pitch joint (Joint 1) and consists of two rigid links connected by a pitch joint (Joint 2). Each joint is manipulated by a motor with encoder. A touch sensor is attached to the tip of each leg. Table I shows the physical parameters of the robot.

The robot walks on a flat floor. Electric power is externally supplied and the robot is controlled by an external host computer (Intel Pentium 4 2.8 GHz, RT-Linux), which calculates the desired joint motions and solves the oscillator phase dynamics in the locomotion control system. The robot receives the command signals at intervals of 1 ms. The robot is connected to the electric power unit and the host computer by cables that are slack and suspended during the experiment so that they do not affect the locomotor behavior.

TABLE I

PHYSICAL PARAMETERS OF THE QUADRUPED ROBOT

Link	Parameter	Value
Body	Mass [kg]	1.50
	Length [cm]	28
	Width [cm]	20
Upper leg	Mass [kg]	0.27
	Length [cm]	11.5
Lower leg	Mass [kg]	0.06
	Length [cm]	11.5

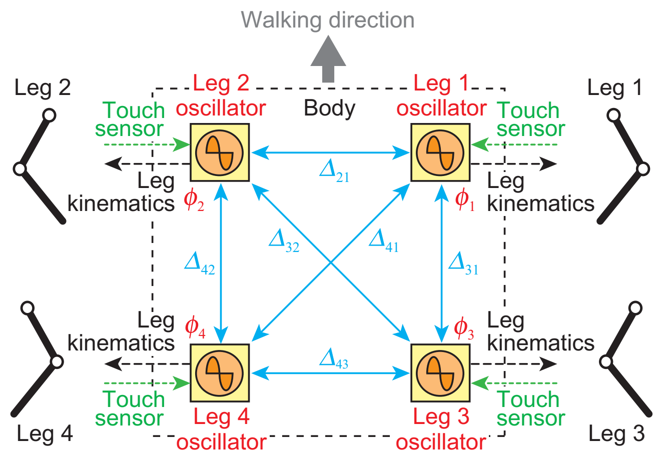

Fig. 3. Locomotion control system composed of an oscillator network using four leg oscillators (Leg 1–4 oscillators). Solid blue arrows indicate interactions between the oscillators based on the phase relationship Δ_{ij}. The oscillator phases are modulated by touch sensor signals (dotted green arrows). The oscillator phases determine the leg joint kinematics (dashed black arrows).

III. LOCOMOTION CONTROL SYSTEM

The physiological concept of CPGs [12], [28], [34] and the physiological evidence of phase resetting [6], [8], [30], [31] have been often used to develop locomotion control systems for legged robots [10], [18–20], [23], [35], [37]. In this paper, we used the locomotion control system developed in our previous work based on the CPGs and phase resetting (Fig. 3) [2]. This consists of a simple oscillator network using four phase oscillators (Leg 1–4 oscillators), which produce the motor torques based on the phase information of the oscillators to create the movements of the corresponding leg and regulate the rhythm and phase information based on the touch sensor signals. We briefly explain this control system below (see details in [2]).

We define the phase of Leg i oscillator as ϕ_i ($i = 1, \dots, 4$, $0 \le \phi_i < 2\pi$) and design the movement of Leg i by the phase ϕ_i. The leg motion is composed of the swing and stance phases (Fig. 4). The swing phase consists of a simple closed curve for the tip of the leg that includes the anterior extreme position (AEP) and the posterior extreme position (PEP). It starts from the PEP and continues until the leg contacts the ground. The line segment between the AEP and the PEP is parallel to the body. The stance phase consists of a straight line that starts from the landing position (LP) and ends at the PEP. We set $\phi_i = 0$ at the PEP and $\phi_i = \phi_{\mathrm{AEP}} (= 2\pi(1-\beta))$

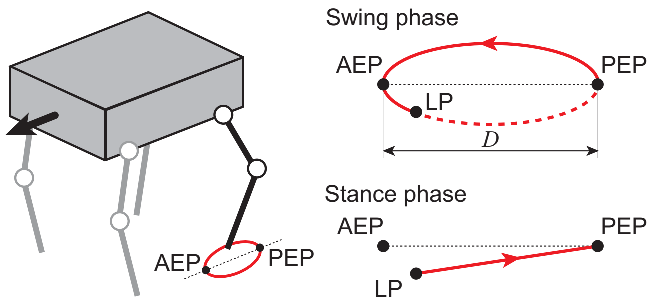

Fig. 4. Leg joint kinematics composed of the swing and stance phases. The swing phase for the tip of the leg is a closed curve that includes the anterior extreme position (AEP) and the posterior extreme position (PEP). The stance phase is a straight line from the landing position (LP) to the PEP. When the leg lands on the ground, it changes from the swing to the stance phase. When the tip of the leg reaches the PEP, it moves into the swing phase.

at the AEP, where β is the duty factor. The desired leg joints are obtained from the inverse kinematics and we controlled the robot joints to follow the designed movements using a PD feedback controller.

We used D for the distance between the AEP and the PEP. We used $T_{\rm sw}$ and $T_{\rm st}$ for the swing and stance phase durations, respectively, for the case when the leg contacts the ground at the AEP (LP = AEP). The duty factor β, the stride length S, and the locomotion speed v are respectively given by

$$\begin{aligned} \beta &= \frac{T_{\rm st}}{T_{\rm sw}+T_{\rm st}} \\ S &= \frac{D}{\beta} \\ v &= \frac{(1-\beta)D}{\beta T_{\rm sw}} \end{aligned} \tag{1}$$

In this paper, we used $D = 10$ mm and $T_{\rm sw} = 145$ ms and varied the locomotion speed v by changing the duty factor β through the stance phase duation $T_{\rm st}$. We used the same values of these parameters for all the legs.

Since the leg movement is determined by the corresponding oscillator phase, the gait is determined by the relative phase between the oscillators. We define the relative phase by

$$\Delta_{ij} = \phi_i - \phi_j, \quad i,j = 1,\ldots,4 \tag{2}$$

where $0 \le \Delta_{ij} < 2\pi$. Because the relationships $\Delta_{ij} = -\Delta_{ji}$, $\Delta_{ij} = \Delta_{ik} + \Delta_{kj}$, and $\Delta_{ii} = 0$ $(i,j,k = 1,\ldots,4)$ are satisfied, the gait is determined by three state variables, such as $[\,\Delta_{21}\ \Delta_{31}\ \Delta_{43}\,]$. For example, $[\,\Delta_{21}\ \Delta_{31}\ \Delta_{43}\,] = [\,\pi\ \pi/2\ \pi\,]$ is satisfied for the walk and $[\,\Delta_{21}\ \Delta_{31}\ \Delta_{43}\,] = [\,\pi\ \pi\ \pi\,]$ is satisfied for the trot.

We used the phase dynamics for the oscillators, which is given by

$$\dot{\phi}_i = \omega + g_{1i} + g_{2i}, \quad i = 1,\ldots,4 \tag{3}$$

where $\omega = 2\pi(1-\beta)/T_{\rm sw}$ and functions g_{1i} and g_{2i} $(i = 1,\ldots,4)$ are the interaction between the oscillators and the sensory regulation by phase resetting based on touch sensor signals, respectively. The function g_{1i} manipulates the relative phases of the oscillators by

$$g_{1i} = -\sum_{j=1}^{4} K_{ij}\sin(\Delta_{ij} - \Delta^*_{ij}), \quad i = 1,\ldots,4 \tag{4}$$

where Δ^*_{ij} $(i,j = 1,\ldots,4)$ is the desired relative phase and K_{ij} $(i,j = 1,\ldots,4)$ is the gain constant $(K_{ij} \ge 0)$. We used

$$\Delta^*_{21} = \Delta^*_{43} = \pi \tag{5}$$

and a large value for the gain constants K_{12}, K_{21}, K_{34}, and K_{43} $(K_{12} = K_{21} = K_{34} = K_{43} = 20)$ so that the right and left legs move in antiphase. We also used the desired value for the relative phase Δ_{31} to produce the antiphase movement between the ipsilateral legs. That is,

$$\Delta^*_{31} = \pi \ (\Delta^*_{42} = \pi) \tag{6}$$

For this interaction, we used the gain constants K_{13}, K_{31}, K_{24}, and K_{42} and set the other K_{ij} to zero. However, we used as small values as possible for the gain constants K_{13}, K_{31}, K_{24}, and K_{42} $(K_{13} = K_{31} = K_{24} = K_{42} = 0.5)$ to minimize this influence and to allow the robot to change its gait from the desired gait through locomotion dynamics due to sensory regulation by phase resetting. The function g_{2i} regulates the locomotor rhythm and phase by touch sensor signals based on the phase resetting mechanism [1–3], [25–27], [40] and is given by

$$g_{2i} = (\phi_{\rm AEP} - \phi_i)\delta(t - t^i_{\rm land}), \quad i = 1,\ldots,4 \tag{7}$$

where $t^i_{\rm land}$ is the time when Leg i contacts the ground $(i = 1,\ldots,4)$ and $\delta(\cdot)$ denotes the Dirac delta function.

Under the constraints $(\Delta_{21} = \Delta_{43} = \pi)$ as assumed in (5) using a large value for the gain constants K_{12}, K_{21}, K_{34}, and K_{43}, the gait of our robot is determined by the dynamics of the relative phase Δ_{31}, which is given from (3), (4), and (7) by

$$\begin{aligned} \dot{\Delta}_{31} &= -(K_{31}+K_{13})\sin(\Delta_{31} - \Delta^*_{31}) \\ &\quad -(\phi_{\rm AEP} - \phi_1)\delta(t - t^1_{\rm land}) \\ &\quad -(\phi_{\rm AEP} - \phi_3)\delta(t - t^3_{\rm land}) \end{aligned} \tag{8}$$

Because we used the desired value for the relative phase Δ_{31} in (6), the trotting gait $(\Delta_{31} = \pi)$ is the only attractor when we do not use phase resetting (7). However, since we used a small value for the gain constants K_{13} and K_{31} and phase resetting modulates the relative phase Δ_{31} by the function g_{2i} in (7), the relative phase Δ_{31} is allowed to move from the desired value Δ^*_{31}. Therefore, the gait is generated through the interactions among the robot's mechanical dynamics, the oscillator dynamics, and the environment, which depends on the physical conditions, such as the locomotion speed and additional weight.

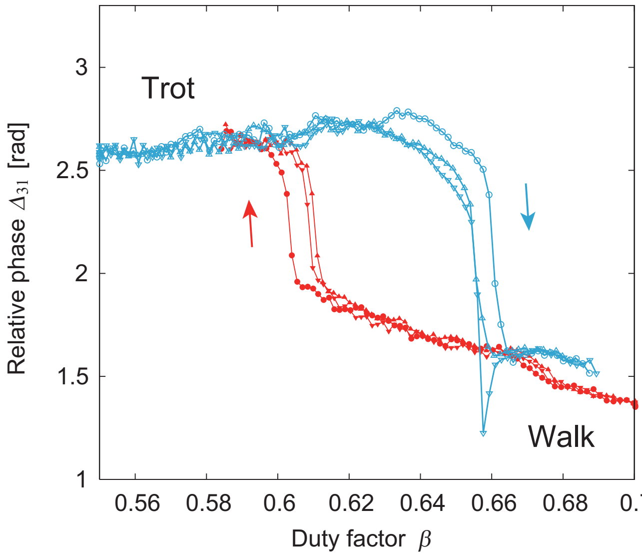

Fig. 5. Gait transition of the quadruped robot without putting additional loads induced by changing the locomotion speed through the duty factor β (three trials for each increase and decrease of the locomotion speed). Relative phase Δ_{31} changed between 1.4 and 2.6 rad, which shows that the gait changed between the walking and trotting gait. The trotting gait changed to the walking gait around $\beta = 0.66$ and the walking gait transitioned to the trotting gait around $\beta = 0.61$. The walk-to-trot and trot-to-walk transition speeds differed and hysteresis appeared.

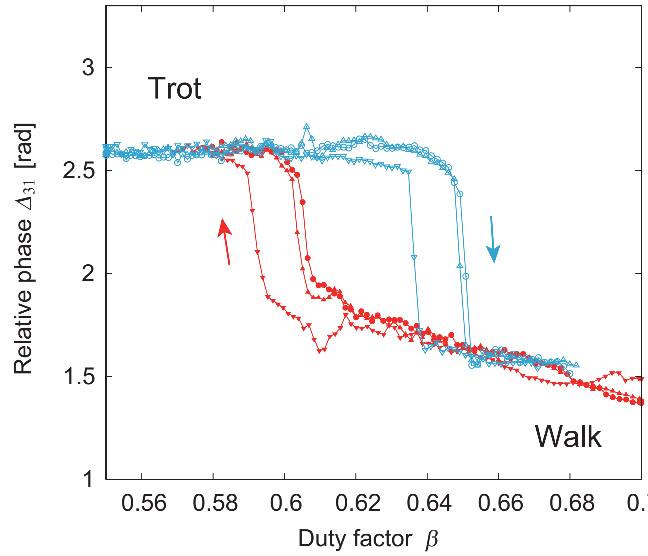

Fig. 6. Gait transition of the quadruped robot induced by changing the locomotion speed through the duty factor β when a weight of 150 g was put on the body. Relative phase Δ_{31} changed between 1.4 and 2.6 rad and hysteresis appeared. Although the walk-to-trot and trot-to-walk transition speeds were fluctuated, the duty factors of the trot-to-walk transition decreased and the hysteresis loop reduced relative to the case without additional weights.

IV. RESULTS

We first examined how the gait of our quadruped robot changes by slowly changing the locomotion speed through the duty factor β without putting additional loads on our quadruped robot, where we alternated accelerating and decelerating trials three times. Figure 5 shows the results of the relative phase Δ_{31} plotted at the foot contact of Leg 1. Around $\beta = 0.55$, the relative phase Δ_{31} is 2.6 rad and the robot established the trotting gait (although 2.6 rad is slightly different from π, we considered this to be the trotting gait to distinguish it from the walking gait described below). On the other hand, around $\beta = 0.7$, the relative phase Δ_{31} is 1.4 rad and the robot achieved the walking gait. This means that the robot produced different gaits depending on the locomotion speed. In addition, when we decreased the locomotion speed by increasing the duty factor β from 0.55 to 0.7, the relative phase Δ_{31} changed from 2.6 to 1.6 rad around $\beta = 0.66$ and the gait changed from the trotting gait to the walking gait. In contrast, when we increased the locomotion speed by decreasing the duty factor β from 0.7 to 0.55, the relative phase Δ_{31} changed from 1.4 to 2.6 rad around $\beta = 0.61$ and the gait transitioned from the walking gait to the trotting gait. The trot-to-walk and walk-to-trot transition speeds clearly differ and hysteresis loop appears in this walk–trot transition. These results are similar to those of the robot experiments without additional load in our previous work [2].

Next, we put a weight of 150 g on the body and investigated how the size of the hysteresis loop changes. Figure 6 shows the results of the relative phase Δ_{31} plotted at the foot contact of Leg 1, showing three trials for each increase and decrease of the locomotion speed. In this case, the relative phase Δ_{31} changes between 1.4 and 2.6 rad and hysteresis appeared, similar to the case without additional weights (Fig. 5). Although the walk-to-trot and trot-to-walk transition speeds were fluctuated, the duty factors of the trot-to-walk transition decreased (the trot-to-walk transition speed increased). This resulted in the reduction of the hysteresis loop.

To further clarify the influence of the additional weight on the gait dynamics of our quadruped robot, we increased the additional load to 250 g and examined how the size of the hysteresis loop changes. Figure 7 shows the results of the relative phase Δ_{31} plotted at the foot contact of Leg 1, illustrating three trials for each increase and decrease of the locomotion speed. In this case, the relative phase Δ_{31} changes between 1.4 and 2.6 rad and hysteresis appeared, which is similar to the cases without additional weights and with a load of 150 g (Figs. 5 and 6). The walk-to-trot and trot-to-walk transition speeds were further fluctuated, but the duty factors of the trot-to-walk transition decreased, which caused further reduction of the hysteresis loop in the walk–trot transition.

To clearly show the changes in the size of the hysteresis loop depending on the additional load, we calculate the qualitative change of the size. For that purpose, we determined the walk-to-trot and trot-to-walk transition speeds by the duty factors when the relative phase Δ_{31} exceeds 2.3 rad for the walk-to-trot transition and falls below 2.1 rad for the trot-to-walk transition. We used the difference between the walk-

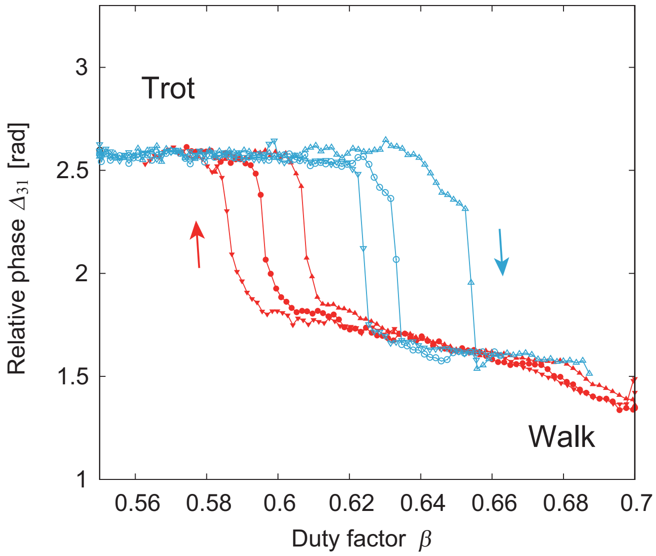

Fig. 7. Gait transition of the quadruped robot induced by changing the locomotion speed through the duty factor β when a weight of 250 g was put on the body. Relative phase Δ_{31} changed between 1.4 and 2.6 rad and hysteresis occurred. Relative to the cases without additional loads and with a load of 150 g, the walk-to-trot and trot-to-walk transition speeds were further fluctuated, but the duty factors of the trot-to-walk transition decreased and the hysteresis loop further reduced.

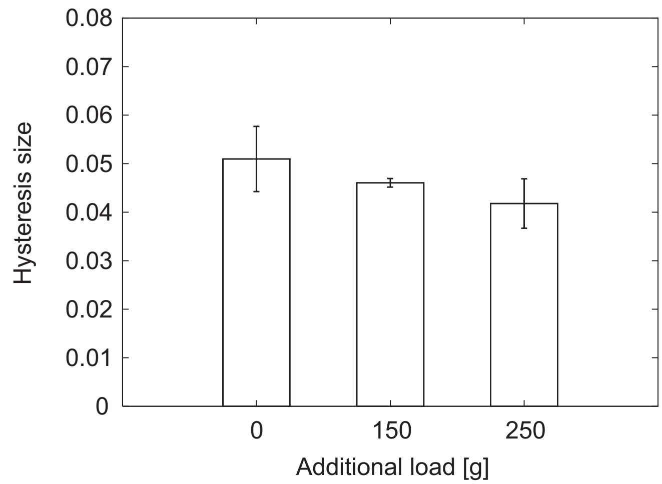

Fig. 8. Dependence of the hysteresis size of the walk–trot transition on the additional load (mean $\pm$ SD), calculated by the duty factors at the walk-to-trot and trot-to-walk transitions. The hysteresis size reduced as the additional weight increased.

to-trot and the trot-to-walk transition speeds for each pair of accelerating and decelerating trials to evaluate the size of the hysteresis loop. Figure 8 shows the result of the hysteresis size (mean $\pm$ SD) for each additional weight. As shown in this figure, the hysteresis size reduced as the additional load increased. This trend is consistent with the locomotion dynamics in which the cusp catastrophe is embedded (Fig. 1).

V. DISCUSSION

In this paper, we investigated the dynamic characteristics in the gait of a quadruped robot whose legs are controlled by nonlinear oscillators with phase resetting. The experimental results showed that the robot produced the walking and trotting gaits depending on the locomotion speed and exhibited the walk–trot transition with hysteresis induced by changing the locomotion speed. In addition, the additional load put on the robot body reduced the hysteresis size, as observed in the locomotion dynamics in which the cusp catastrophe is embedded. However, because of the hardware limitation of our quadruped robot, we could not use more additional weights for the experiment. Therefore, the hysteresis did not yet vanish and we could not completely prove the cusp catastrophe in the gait transition of our quadruped robot. However, our result of the reduction of the hysteresis loop depending on the additional load surely suggests the existence of the cusp bifurcation, as observed in the simulation study of our previous work [3].

So far, to elucidate the determinants of gait transitions in humans and animals, researchers have searched for a potential trigger to change the gait. For example, Margaria [22] and Hoyt and Taylor [14] showed that humans and horses employ gaits that minimize metabolic cost and they suggested that humans and animals switch their gaits to reduce the metabolic cost. On the other hand, Farley and Taylor [9] state that it is difficult to imagine how animals can sense metabolic cost in rapid gait transitions; rather, they consider another criterion, such as biomechanical factors, as the trigger. In addition, Griffin *et al.* [11] suggested that biomechanical and metabolic factors are tightly coupled at the gait transition. However, there are many conflicting reports regarding the roles of metabolic and biomechanical factors in determining gait transitions [15], [16], [24], [29], [39]. An alternative approach is based on dynamic systems analysis [2], [7], [32], as adopted in this paper. In this approach, gaits are viewed as the results of self-organization in complex dynamic system and gait transitions occur when the stability of a gait decreases so much that switching to a new gait improves stability [11]. Simulation studies using neuromechanical models and experimental studies using legged robots by integrating biomechanical and physiological findings must be useful tool to investigate underlying mechanisms of the gait generation and transition in the locomotion dynamics. In particular, elucidating bifurcation structures in the locomotion dynamics must provide meaningful biological insights [4].

Cusp catastrophe is a typical characteristic of nonlinear dynamic systems, as observed in the forced Duffing oscillator [36]. To better understand the underlying mechanism in the gait transition of the locomotion dynamics, in the future we should develop a more sophisticated physical model as well as a more biologically plausible robot.

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