# Heel-contact Toe-off Walking Model Based on the Linear Inverted Pendulum

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Abstract-We propose a new heel-contact toe-off walking model based on the Linear Inverted Pendulum (LIP) model, which due to the linearity and the ease of manipulation of the equations, could be considered to be advantageous for a future online implementation for the generation of walking patterns. This new model is based on the so called functional rockers of the foot (heel, ankle and forefoot rockers), each of which are modeled as an inverted pendulum, changing the ground contact point position of the inverted pendulums for each rocker. We focus on the motion of the Center of Mass (CoM) in the sagittal plane, as it is the plane on which the rockers take place, but also generate the motions on the frontal plane. The model proved to work for constant velocity, accelerating and decelerating gaits, and the effects of the change of pivot point during heel-contact toe-off could be corroborated in the Zero Moment Point (ZMP) graphs. The implementation of this model could improve the human likeness of the motions, as well as the stability of the locomotion.

*Index Terms* - Heel-contact toe-off motion, LIP model, gait pattern generation

## I. INTRODUCTION

Having robust and safe humanoid robots helping us on our daily lives is an idea that is slowly becoming reality. Still, there are many problems that must be solved to fully achieve it. One of the most important is to have stable locomotion in a variety of environments and situations, such as moving on different kinds of terrains, avoiding obstacles, coping with external perturbations like collisions, etc. And if we talk about humanoid robots, the above should be achieved while walking on two feet. For that, it is necessary for the robot to be able to modify its walking to make it stable in different situations, by replanning its steps in real time using the information obtained from inbuilt sensors.

About real time gait pattern generators for humanoid robots, Harada et al. [1] and Morisawa et al. [2] developed a method to analytically calculate the Zero Moment Point (ZMP) and CoM trajectories given the footsteps positions. Nishiwaki et al. [3] developed a real time gait regeneration method that allows changes in the pattern in arbitrary points by changing the ZMP reference, where a simple dynamic

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<sup>5</sup>A. Takanishi is with the Department of Modern Mechanical Engineering and Humanoid Robotics Institute (HRI), Waseda University, Tokyo, Japan. model is used to replan the foot placements and timings, but the effects on the frontal plane motions are not considered. Kryczka et al. [4] developed a two-stage dynamically consistent gait pattern generator, capable of very rapid gait pattern regeneration, which in the first stage uses a simple LIP based dynamic model to define the feet placements and timings and the CoM trajectory, and in the second stage generates a gait pattern using a multibody dynamics model. However, all of them require modifications to generate or regenerate a heelcontact toe-off walking gait pattern.

On the generation of heel-contact toe-off trajectories for the feet, most researches use polynomial interpolations given the foot orientation and position at key points of the motion, namely heel-contact and toe-off angles, swing phase max height, etc. There are researches where the trajectories of all the end-effectors are generated based on ground conditions and stability constraints [5]. Some combine the heel-contact toe-off motion with the stretched knee walking, using genetic algorithms to obtain optimal parameters for the trajectories [6], [7], or the preview control [8]. Others need the step length and time of single and double support phases to generate a walking gait [9]. There are also researches where the foot is modeled as a multi-link mechanism, and the heelcontact toe-off motion is analyzed [10]. But in the aforementioned researches, the trajectories are generated offline and already using a multibody dynamics model. Through the use of a simple model that contains the effects of the change of pivot point during the heel-contact toe-off motion, it could be possible to generate the trajectories online and in real-time, making it possible to take advantage of the heel-contact toeoff motion, for instance, to improve the walking stability.

In this paper we propose a simple model that accounts for the dynamic effects of the heel-contact motion, based on the functional rockers of the foot, which are inverted pendulum like motions that occur during the stance phase of the human gait [11]. Thus, the model is based on the LIP model, and aims to be an extension to the work in [4] to achieve a stable and dynamically consistent real time heel-contact toeoff locomotion. This could improve the human likeness of the motions, and also could be used to improve the locomotion stability, as the heel-contact motion adds a degree of freedom just after the foot contacts the ground.

As for the structure of the present paper, in section II we set the mathematical background from where the calculations for the model proceed. In section III we describe and analyze the details of the proposed model to achieve a heel-contact toe-off motion; in section IV we present results obtained from simulations made with the presented model and finally, in section V we summarize the paper and present limitations of the current model, possible extensions, and applications.

## **II. LIP MODEL EQUATIONS**

To achieve a dynamically consistent heel-contact toe-off walking gait pattern, we propose a simple LIP based model to obtain an initial approximation of the feet placements and CoM trajectory of a humanoid robot. This information can be used afterwards to generate the end-effector trajectories and the reference ZMP trajectory. Then, using a multibody dynamics based gait pattern generator as in [4] or [12] and inverse kinematics, we can obtain a gait pattern for a position controlled humanoid robot, which comprises the angular references for each of its joints.

For the present research, we decided to use the LIP model [13] due to the linearity and the ease of manipulation of its equations. These are derived from the following assumptions:

- The whole system is represented by a single mass inverted pendulum, with the mass as that of the entire system, and placed at the height of the robot's CoM in the free standing position.
- The CoM motion is constrained to a horizontal plane at the height of the mass, i.e., CoM's height is constant.
- There is no torque acting between the system and the ground surface.
- There is no slip between the pendulum and the ground.

With these assumptions, the motions of the pendulum in sagittal and frontal plane can be considered as decoupled, and therefore the trajectories in each plane can be generated separately. The equation which describes these motions is:

$$\ddot{x} = \frac{g}{z}x \quad \left(\ddot{y} = \frac{g}{z}y\right) \tag{1}$$

where x and y are the position of the CoM with respect to the ground contact point of the inverted pendulum in sagittal and frontal plane respectively, g is the gravitational acceleration and z is the height of the CoM. In this model we focus on the motion on the sagittal plane, where the heel-contact toe-off motion occurs.

Integrating (1), it is possible to obtain the equations that describe the position and velocity of the CoM[14]:

$$x(t) = x_0 \cosh\left(\frac{t}{k}\right) + \dot{x}_0 k \sinh\left(\frac{t}{k}\right) \tag{2}$$

$$\dot{x}(t) = \frac{x_0}{k} \sinh\left(\frac{t}{k}\right) + \dot{x}_0 \cosh\left(\frac{t}{k}\right) \tag{3}$$

where  $x_0$  and  $\dot{x}_0$  are the initial position and velocity of the CoM with respect to the inverted pendulum ground contact point, t is the time counted from the moment of the initial conditions and  $k = \sqrt{z/g}$ .

From (2) we can isolate t in order to get the time needed to reach a given final position from a given initial position and velocity. In the same way, we can isolate t from (3) to get the time needed to reach a given final velocity from a given initial position and velocity. These equations are:

$$t_{pos} = k \log\left(\frac{x_1 \pm \sqrt{k^2 \dot{x}_0^2 + x_1^2 - x_0^2}}{x_0 + k \dot{x}_0}\right) \tag{4}$$

$$t_{vel} = k \log \left( \frac{k \dot{x}_1^2 \pm \sqrt{k^2 (\dot{x}_1^2 - \dot{x}_0^2) + x_0^2}}{x_0 + k \dot{x}_0} \right)$$
(5)

where  $t_{pos}$  and  $t_{vel}$  are, the time to reach a final position  $x_1$  or velocity  $\dot{x}_1$ , respectively, given initial conditions,  $x_0$ ,  $\dot{x}_0$ .

If we equate (4) and (5), it means that we want the time to get from some initial conditions to a final position, to be the same as getting to a final velocity given the same initial conditions. Doing so, we can obtain a set of equations which relate initial to final conditions, where we can isolate any of them (position or velocity), to calculate it given that we know the other three conditions. These equations are:

$$x_0 = \pm \sqrt{k^2 (\dot{x}_0^2 - \dot{x}_1^2) + x_1^2} \tag{6}$$

$$\dot{x}_0 = \pm \frac{\sqrt{k^2 \dot{x}_1^2 + x_0^2 - x_1^2}}{k} \tag{7}$$

$$x_{1} = \frac{x_{0}^{2} - k^{2}\dot{x}_{0}^{2} + \left(k\dot{x}_{1} \pm \sqrt{x_{0}^{2} + k^{2}(\dot{x}_{1}^{2} - \dot{x}_{0}^{2})}\right)^{2}}{2\left(k\dot{x}_{1} \pm \sqrt{x_{0}^{2} + k^{2}(\dot{x}_{1}^{2} - \dot{x}_{0}^{2})}\right)}$$
(8)

$$\dot{x}_1 = \frac{x_1^2 - x_0^2 + k^2 \dot{x}_0^2 \pm x_1 \sqrt{k^2 \dot{x}_0^2 + x_1^2 - x_0^2}}{k \left(x_1 \pm \sqrt{k^2 \dot{x}_0^2 + x_1^2 - x_0^2}\right)} \qquad (9)$$

It is worth noting that, even though it is always possible to calculate a condition given the other three from (6), (7), (8) and (9), the solution is not always real, which is the case of final conditions that cannot be achieved given some initial conditions, or initial conditions that cannot produce some desired final conditions. Likewise, when calculating the motion time with (4) and (5), and given that both initial and final conditions are real, there can be different cases:

- $\{t_{pos}, t_{vel} \in \mathbb{R} : t_{pos} = t_{vel} \ge 0\}$ , the motion with the given initial and final conditions is feasible, and it will take time  $t = t_{pos} = t_{vel}$  for the initial conditions to reach the final conditions. (When  $t_{pos} = t_{vel} = 0$ , the initial conditions are the same as the final conditions.)
- $\{t_{vel} \in \mathbb{C}; t_{pos} \in \mathbb{R} : t_{pos} \ge 0\}$ , the motion with the given initial conditions will reach the final position in  $t = t_{pos}$ , but will not reach the desired final velocity.
- $\{t_{pos} \in \mathbb{C}; t_{vel} \in \mathbb{R} : t_{vel} \ge 0\}$ , the motion with the given initial conditions will reach the final velocity in  $t = t_{vel}$ , but will not reach the desired final position.
- {t<sub>vel</sub>, t<sub>pos</sub> ∈ C} or {t<sub>pos</sub>, t<sub>vel</sub> ∈ R : t<sub>pos</sub>, t<sub>vel</sub> < 0}, the motion with the given initial and final conditions is not physically feasible.</li>

Therefore, we will always seek for conditions that fulfill  $\{t_{pos} = t_{vel} > 0\} \in \mathbb{R}.$ 



Fig. 1: Ground contact areas and points during the stance phase for the heel-contact toe-off motion: (a) Human foot (bottom view) [11]. (b) Model foot, side view (up) and bottom view (down).

## III. HEEL-CONTACT TOE-OFF WALKING MODEL

We propose a model for the motion in the sagittal plane of heel-contact toe-off walking, based on the functional rockers of the foot, which are, as the name states, rockerlike motions that happen in the stance phase of a gait[11]. There are three rockers, which are named after the body part that functions as the pivot or fulcrum of the motion: heel, ankle and forefoot. The heel rocker starts with the heel contacting the ground, and finishes when the forefoot strikes the floor. Then the ankle rocker starts when the whole sole of the foot is in contact with the ground, finishing when the Center of Pressure (CoP) on the foot reaches the metatarsal heads (MTH). This leads to the start of the forefoot rocker, concluding when the foot exchange occurs, and the opposite foot's heel strikes the ground. For this model, we assumed the following:

- The CoM velocity at foot exchange is user-defined.
- There is no double support phase, i.e., the foot exchange is instantaneous.

Based on the above, we decided to model each rocker as an inverted pendulum, changing the ground contact point for each. For simplicity, the foot was considered as a rectangular board with a passive toe joint, and the different ground contact points for each phase were placed as in Fig. 1 (b).

For the proposed model, we are using as input the velocity at the moment when the CoM is over the ankle rocker pivot, i.e., when the vector from the ground contact point to the CoM is completely vertical. We will call this point *apex*, as in [4]. With that in mind, we will divide and analyze the motion as follows:

- Phase I will start from the stance foot's apex and will end with the CoM over the same foot's MTH.
- Phase II will begin with the CoM over the same foot's MTH and end with the same foot's toe-off, which will coincide with the opposite foot's heel-contact (foot exchange point).
- Phase III will last from the opposite foot's heel-contact to that same foot's forefoot contacting the ground.
- Phase IV will be from the foot's forefoot contacting the ground to the new apex.



Consistent with the above phases, the time, initial position and velocity, and final position and velocity will be named with each phase's roman number:  $t_I$ ,  $x_{0_I}$ ,  $\dot{x}_{0_I}$ ,  $x_{1_I}$ ,  $\dot{x}_{1_I}$ ,  $t_{II}$ ,  $x_{0_{II}}$ , etc. (Fig. 2)

It should be noted that regardless of the order of the present analysis, as long as the motion is physically feasible and the initial conditions and necessary velocities (apex and/or exchange point velocities) are known, it does not matter from which phase the calculations are started.

### A. Phase I: Motion from Initial Apex to CoM over Metatarsal Heads

In this phase, we know the initial velocity,  $\dot{x}_{0_I} = v_{apex_i}$ , as it is an input, and the initial position,  $x_{0_I} = 0$ , as the CoM is at an apex, and its position is measured with respect to the ground contact point. Also, we know the final position,  $x_{1_I} = d_{a_f}$ , where  $d_{a_f}$  is the distance between the ankle and the forefoot rockers' pivot points. This point was chosen to simplify the calculations, as it will make the initial position of the next phase to be zero. With these three conditions it is possible to calculate the motion time from (4), and the final velocity from (3):

$$t_I = k \log\left(\frac{d_{af} \pm \sqrt{k^2 v_{apex_i}^2 + d_{af}^2}}{k \ v_{apex_i}}\right) \tag{10}$$

$$\dot{x}_{1_I} = v_{apex_i} cosh\left(\frac{t_I}{k}\right) \tag{11}$$

It is worth noting that the final position can be placed before or after the point used in this paper  $(d_{af})$ , as long as the effects that it will have on the motion of this and the next phase are kept in mind.

## B. Phase II: Motion from CoM over Metatarsal Heads to Toe-off (Foot Exchange)

For this phase, we know the initial position and velocity, where the position is calculated from the final position of the previous motion, minus the distance between the ankle and forefoot rocker pivots,  $x_{0_{II}} = x_{1_I} - d_{af}$  (in the present case  $x_{0_{II}} = 0$ ), and the initial velocity is the final velocity of the previous motion,  $\dot{x}_{0_{II}} = \dot{x}_{1_I}$ . For the final conditions, it is necessary to define either the final velocity  $\dot{x}_{1_{II}}$  or position

 $x_{1_{II}}$ . As we are using velocities as inputs, we decided to set the final velocity as the exchange point velocity  $\dot{x}_{1_{II}} = v_{ex}$ , as the foot exchange occurs in the end of this phase. With these three conditions it is possible to calculate the motion time from (5), and the final position from (2):

$$t_{II} = k \log \left( \frac{k \ v_{ex}^2 \pm \sqrt{k^2 (v_{ex}^2 - \dot{x}_{1_I}^2) + (x_{1_I} - d_{af})^2}}{(x_{1_I} - d_{af}) + k \dot{x}_{1_I}} \right)$$
(12)

$$x_{1_{II}} = (x_{1_I} - d_{af})\cosh\left(\frac{t_{II}}{k}\right) + \dot{x}_{1_I}k\sinh\left(\frac{t_{II}}{k}\right) \quad (13)$$

To check for motion feasibility, (8) and (4) can be used, to see if  $t_{pos} = t_{vel}$ . In this case, we found empirically that for a feasible motion,  $v_{ex} > \dot{x}_{1_I}$  is enough.

## C. Phase III: Motion from Opposite Foot Heel-contact to Forefoot Contact

In this phase, the initial velocity is the same as the final velocity of the previous motion,  $\dot{x}_{0_{III}} = v_{ex}$ . As stated in the model assumptions, the motion time is fixed to a constant  $t_{III} = constant$ , which will be later explained in section IV.

For the initial position, we defined  $d_{ex}$  as the distance from the projection of the CoM at foot exchange to the heel contact position, so that  $x_{0_{III}} = -d_{ex}$ , and to define it, as the time of this motion is fixed, it is necessary to know the desired velocity of the next apex, which will occur in the next phase. Because of that we must analyze both Phase III and IV together to completely define all the conditions of both phases. So, with initial position  $x_{0_{III}} = -d_{ex}$  and velocity  $v_{ex}$ , we can use (2) and (3):

$$x_{1_{III}} = -d_{ex} \cosh\left(\frac{t_{III}}{k}\right) + v_{ex} k \sinh\left(\frac{t_{III}}{k}\right) \quad (14)$$

$$\dot{x}_{1_{III}} = \frac{-d_{ex}}{k} \sinh\left(\frac{t_{III}}{k}\right) + v_{ex} \cosh\left(\frac{t_{III}}{k}\right) \qquad (15)$$

where  $-d_{ex}$ ,  $x_{1_{III}}$  and  $\dot{x}_{1_{III}}$  are unknown, and therefore we need either to relate two of those variables or another function, to be able to solve a system of equations for the remaining unknowns. This relation will be obtained from the analysis of the next motion.

#### D. Phase IV: Motion from Forefoot Contact to Next Apex

Finally, for this phase we have the information of the final conditions, where the final position  $x_{1_{IV}} = 0$  as it is an apex, and the final velocity  $\dot{x}_{1_{IV}} = v_{apex_{i+1}}$  is an input. As for the initial conditions, we already know that the velocity corresponds to the final velocity of the previous motion,  $\dot{x}_{0_{IV}} = \dot{x}_{1_{III}}$ , and for the initial position we must take into account the movement of the ground contact point from the heel to the position of the ankle rocker pivot, thus  $x_{0_{IV}} = x_{1_{III}} - d_{ha}$ , where  $d_{ha}$  is the distance between the

heel and the ankle rocker pivot. With this, we can use (7) to relate the initial velocity to the other three conditions:

$$\dot{x}_{0_{IV}} = \dot{x}_{1_{III}} = +\frac{\sqrt{k^2 v_{apex_{i+1}}^2 + (x_{1_{III}} - d_{ha})^2}}{k} \quad (16)$$

where we only take the positive value, as we are expecting a forward motion. Then we can substitute  $\dot{x}_{1_{III}}$  in (15) with the right hand side of (16), and solve the system of the resulting equation and (14) to calculate  $-d_{ex}$  and  $x_{1_{III}}$ :

$$x_{1_{III}} = d_{ha} \cosh^{2}\left(\frac{t_{III}}{k}\right) + k v_{ex} \sinh\left(\frac{t_{III}}{k}\right) \pm \cosh\left(\frac{t_{III}}{k}\right) \sqrt{\left(d_{ha} \sinh\left(\frac{t_{III}}{k}\right) + k v_{ex}\right)^{2} - k^{2} v_{apex_{i+1}}^{2}}$$

$$(17)$$

$$d_{ex} = \sinh\left(\frac{t_{III}}{k}\right)\sqrt{(x_{1_{III}} - d_{ha})^2 + k^2 v_{apex_{i+1}}^2} - x_{1_{III}}\cosh\left(\frac{t_{III}}{k}\right)$$
(18)

Here, as it can be seen from (17), there are two possible final positions for Phase III,  $x_{1_{III}}$ , which in turn will produce two solutions for  $d_{ex}$ . Analyzing the physical meaning of this, if  $d_{ex} > 0$ , it means that the heel-contact will take place in front of the position of the CoM in the foot exchange moment, which will stop the fall of the CoM, redirecting the falling force into a forward motion, which is one of the functions of the heel rocker[11]. On the other hand, if  $d_{ex} \leq 0$ , the heel-contact will take place behind the position of the CoM in the foot exchange moment, allowing the CoM to keep falling and having nothing to stop it, unless  $d_{ex} = 0$  and  $v_{ex} = 0$ , in which case the motion should stop. Therefore, the positive value of  $d_{ex}$  will be chosen, and with it, the value of  $x_{1_{III}}$  that produced it.

Then it only remains to assure that the generated motion is feasible, for which we should once again use (8) and (4) to see if  $t_{pos} = t_{vel}$  using the initial and final conditions for Phase IV. If it is not feasible  $v_{apex_{i+1}}$  should be changed.

#### E. Frontal Plane Motion

For the motion in frontal plane, we used the same method as in [4]. The steps are planned so that the CoM swings do not cross the support foot position, and they have the desired step time, obtained from the sagittal plane motion generation. The CoM motion also depends on (1), (2) and (3).

## IV. SIMULATIONS AND RESULTS

Having the necessary equations for the different phases to obtain the foot positioning and motion of CoM for a heelcontact toe-off motion, we made kinematic simulations to test the behavior of the model. These simulations were done using the software MATLAB<sup>®</sup>. We took three scenarios into account: a constant apex velocity step,  $v_{apex_i} = v_{apex_{i+1}}$ , an



Fig. 3: Position of the CoM and ZMP for the constant velocity simulation.

accelerating step,  $v_{apex_i} < v_{apex_{i+1}}$ , and finally a decelerating step,  $v_{apex_i} > v_{apex_{i+1}}$ . For all the simulations, the value for  $t_{III}$  was defined inside the range 0.1-0.2s, from the assumption that the step time is around 1.0-2.0s, and the heel-contact to forefoot contact motion in humans takes about 10% of the step time. For the presented simulations,  $t_{III} = 0.15s$ . The tests were made under the assumption that the motion would be from apex to apex, having the velocity at those points as inputs. The velocity  $v_{ex}$  at exchange point was selected so that it obeys the condition  $v_{ex} > \dot{x}_{1I}$ , defining it for the simulations as  $v_{ex} = 1.1 \cdot \dot{x}_{1I}$ . To prove the effectiveness of the present model, the reference ZMP which could be used to generate a walking pattern was calculated from the data from the generated motions, to see if the heelcontact toe-off motions had any effects on it:

$$ZMP_{ref} = x - \frac{z}{g}\ddot{x} \tag{20}$$

where x is the position of the CoM in the global coordinate system, and  $\ddot{x}$  is the acceleration on that point.

#### A. Constant Apex Velocity

We simulated a step with constant apex velocity, i.e.  $v_{apex_i} = v_{apex_{i+1}} = 0.1m/s$ . In Fig. 4, the velocity of the CoM in sagittal plane is shown, where the transition from and to each of the modeled inverted pendulums can be clearly seen. In Fig. 3, which shows the ZMP position also in sagittal plane, the effect of the change of placement of the ground contact point of each inverted pendulum can be observed as a stair-like shape, where each "step" shows a ground contact point shift. This proves that the model is successfully coping with the effects of the change of pivot point during the heel-contact toe-off motion.

#### B. Accelerating Apex Velocity

For this test, as the calculations are not possible with a v = 0, we used a very small value for the initial apex,  $v_{apex_i} \approx 0m/s$ , and  $v_{apex_{i+1}} = 0.4m/s$ . Once again, the inverted pendulum transitions can be clearly seen in Fig. 6, as well as the effects of the pivot position shift to the ZMP in Fig. 5. It is also notable that the model does not have problems to get from the desired initial to final apex velocities. A change in the step time and length with respect to the other two



Fig. 4: Velocity of the CoM for the constant apex velocity simulation.



Fig. 5: Position of the CoM and ZMP for the accelerating apex velocity simulation.

simulations is observed, which shows how the model tunes these parameters by itself to achieve the desired velocities.

### C. Decelerating Apex Velocity

For this test, we used  $v_{apex_i} = 0.4m/s$  and  $v_{apex_{i+1}} \approx 0m/s$ . The results are very similar the other simulations, and it is shown that there is no problem to generate a decelerating motion as well, reaching the desired final apex velocity and accounting for the effects of the change of pivot point during each of the pendulums.

#### V. CONCLUSION AND FUTURE WORKS

In the present paper, a model to generate heel-contact toeoff motions based on the LIP model and the foot's functional rockers was presented. It uses apex velocities as inputs, and



Fig. 6: Velocity of the CoM for the constant accelerating apex velocity simulation.



simulation.



Fig. 8: Velocity of the CoM for the constant decelerating apex velocity simulation

gives as outputs foot placements, CoM position, velocity and acceleration during the motion, and step timings and phases. For this, the stance phase of a gait was divided into four phases: from the apex to the CoM over the MTH, from CoM over the MTH to foot exchange time, from the opposite foot's heel-contact to forefoot contact, and from the forefoot contact to the next apex. The methods to calculate initial and final conditions for each phase and to verify the feasibility of the motions were presented, as well as the results of simulations using this model.

In the simulation results, it was possible to see each phase clearly in the velocity graphs, as well as the behavior of the model in each phase, proving that the model successfully emulated each of the functional rockers of the foot. Also in the graphs of the ZMP calculated from the motion data, the effect from each transition between rockers could be clearly seen, regardless of the scenario of the motion, showing that the changes of pivot points during the heel-contact toe-off motion are successfully modeled.

About the current limitations, as one of the goals is to generate more humanlike motions, it is necessary to change some assumptions. The inclusion of a double support phase and the possibility to make the time from heel-contact to forefoot contact variable are being considered. Also, the exchange point velocity is currently being defined given some empirical conditions, which could possibly be based on gait parameters, allowing to have other desired behaviors,

or could be eliminated to make the method more robust.

Moreover, we are planning to implement this model on our humanoid robot to enable online heel-contact toe-off gaits, for which changes must be made to enable the regeneration of the motion at any point.

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