Safety Constrained Motion Control of Mobility Assistive Robots

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Abstract—A systematic approach for the supervision-based safe motion control of mobility assistive robots based on invariance control is presented. It allows the formulation of safety features in the form of a constraint admissible state space region that can be kept invariant by proper switching between a nominal and a corrective controller whenever a predefined safety constraint is about to be in violation. Boundaries on the human-robot distance are considered critical safety features for human forward fall and human-robot collision avoidance. The nominal and corrective controller as well as the switching policy are derived. The approach is validated in simulations and experiments and shows high potential for building a systematic safety framework for mobility assistive robots.

I. INTRODUCTION

Mobility problems can impede independent living and are prevalent in the elderly population. Mobility Assistive Robots (MAR) can help overcoming this situation by incorporating features like posture control, walking and navigation assistance in indoor and outdoor environments, but also come with challenges in terms of safety since they have to operate in close interaction with humans, more specifically elderly or patients with cognitive and/or physical impairments. Safety is thus one of the most important features of mobility assistive robots since any hardware or software failure may bring users at risk.

In [1] robot safety and dependability were discussed with focus on mechanical design, actuation, control architecture as well as fault handling and tolerance. In [2]–[4] safety critical requirements were studied in the contexts of impacts as well as safe planning and control. However, safety in MAR brings new challenges as the user stays in continuous physical contact with the robot. In literature this topic is discussed in the contexts of obstacle avoidance, step avoidance and goal seeking by employing artificial potential fields [5]–[10], as well as fall prevention [11], [12].

In [11] user falls were estimated by evaluating the relative distance between legs and user measured by a laser range finder. In [12] the authors refined their approach by modeling the user with a solid body-link model and online tracking the user configuration with the help of two laser range finders mounted at different heights, determining the user center of gravity and finally checking whether this center of gravity lies within the defined support polygon formed by the area of both feet. The risk of falling was defined to increase if the projection of the center of gravity approaches the border of the support polygon. Human falls were recognized along the horizontal direction caused for example by stumbling and leading to legs that are far apart from the walker, and falls along the vertical direction caused for example by weak legs. Varying admittance control was realized for fall prevention on a passive system by increasing the damping in the desired admittance if the user was found to be in a “falling” state achieved by activating the brakes accordingly, while in the “stopped” state large brake torques were applied to each of the wheels independently of the user applied force to the system.

While this approach works nicely on a passive system, which is intrinsically safe, different safety control approaches have to be derived for active systems. In this paper, we propose invariance control, see [13]–[17], for the supervision of safe control states of an active mobility assistive robot as it allows to define a constraint admissible state space region that can be kept invariant by proper switching between a nominal and a corrective controller whenever a predefined safety constraint is about to be in violation. Specifically we study the case of safety-constrained motion control for human forward fall and human-robot collision avoidance.

This paper is organized as follows: The basic idea of invariance control is reviewed in Sec. II. The example of forward fall prevention and collision avoidance is detailed in Sec. III and simulation and experimental results are reported in Sec. IV.

II. INVARIANCE CONTROL

In the following subsections, the theory of invariance control as reported in [15], [16] is briefly recapitulated before we apply it to the supervision-based safe motion control of mobility assistive robots.

A. Basic Concept

The basic idea of invariance control is to supervise a nominal controller responsible to achieve the main control goal and to correct the control outputs if the system states are about to leave the constrained admissible state space region. This general idea of invariance control is depicted in Fig. 1. When constraints are about to be in violation, the invariance controller switches to the corrective controller to either keep the system on the boundary of the admissible set or to bring the states again inside the boundary.

A general nonlinear affine control system

$$\dot{x} = f(x) + G(x)u \quad \text{and} \quad G = [g_1, \ldots, g_m]$$

(1)

with a state vector $x \in \mathbb{R}^n$, control input variable $u \in \mathbb{R}^m$, and sufficiently smooth functions $f, g_i : \mathbb{R}^n \to \mathbb{R}^n$ and...
an initial state of $x(t = 0) = x_0$ is considered. The state constraints are defined by a series of output functions
\[ y_i = h_i(x) \leq 0 \quad y \in \mathbb{R} \quad \text{for} \ 1 \leq i \leq m, \]
(2)
which when combined result in the maximum admissible set
\[ \chi = \{ x \mid h_i(x) \leq 0, \ \forall i \}, \]
(3)
which separates the constraint and non-constraint admissible states [15], [16]. Each of the output functions needs a well defined relative degree $r_i$ in the maximum admissible set.

**B. Invariant Set Design**

For a single constraint, the set $G_i$ is defined as the zero sublevel set of the invariance function $\Phi(x)$
\[ G_i = \{ x \mid \Phi_i(x) \leq 0 \} \]
(4)
and $\partial G_i$ its boundary
\[ \partial G_i = \{ x \mid \Phi_i(x) = 0 \} . \]
(5)
Following [17] the set $G_i$ is positively invariant for (1) if at least one of the following two conditions is satisfied for each point on its boundary $\partial G_i$:
\[ a) \ y^{(k)}(x) < 0 \ \text{for} \ 1 \leq k \leq r_i - 1 \]
(6)
\[ b) \ y^{(r_i)}(x, u) \leq \gamma_i , \]
(7)
where $y^{(j)}(x) = \frac{d^j y}{dt^j}$, $\gamma_i$ a design parameter and $r_i$ the relative degree.

An extension to multiple constraints foresees that for each constraint $h_i$ a set $G_i$ is designed separately. The combined set $G$ is given by the zero sublevel set of the combined invariance function $\Phi(x)$, the intersection of all sets $G_i$ (see Fig. 2).

**C. Corrective Control**

Condition (7) allows us to keep the set $G$ invariant by switching the control $u$ on the boundary whenever (6) cannot be satisfied. Thus, the corrective controller can be derived by determining the set of active constraints [14]:
\[ \mathcal{I} = \left\{ i \mid x \in G_i^+ \land y_i^{(r_i)}(x, u) > \gamma_i, 1 \leq i \leq m \right\} \]
(8)
with
\[ G_i^+ = \left\{ x \mid \Phi_i(x) \geq 0 \land \exists_{1 \leq j \leq r_i - 1} y_i^{(j)} > 0 \right\} , \]
(9)
and solving the resulting system of equations
\[ A(x)u_{cor} \leq b(x) \]
(10)
for $u_{cor}$ with $A(x) = [a_i^T]$, $b(x) = [\gamma_i - b_i(x)]$ and $i \in \mathcal{I}$, which is formed by the $r_i$-th derivatives of the different output functions $y_i$:
\[ y_i^{(r_i)} = \frac{L_f^r h_i(x) + \left[ L_g L_f^{-1} h_i(x) \ldots L_g L_f^{-1} h_i(x) \right] u}{b_i(x)} , \]
(11)
where $L_f h(x)$ is the directional derivative of the scalar function $h_i(x)$ in the direction of $f$ (Lie-Derivative)
\[ L_f h(x) = \frac{\partial h}{\partial x} f \]
and Lie-Derivatives of higher order $j$ ($L_f^j$) are then defined recursively.

In case of underdetermined problems $u_{cor}$ can be found by solving an optimization problem that reduces the difference of $u_{cor}$ and $u_{nom}$:
\[ \min \ ||u_{cor} - u_{nom}||^2 \]
(12)
\[ s.t. \ A(x)u_{cor} - b(x) \leq 0 \]
with solution
\[ u_{cor} = A_x^\# b_x + (I - A_x^\# A_x)u_{nom} \]
(13)
where $A_x^\#$ denotes the Pseudoinverse of $A_x$. Please note that different corrective controllers can be selected, e.g. by using a weighted Pseudoinverse.

**D. Switching Policy**

The set $G$ is kept invariant by switching between nominal and corrective controllers. The nominal controller is active inside $G$, while on the boundary $\partial G$ the corrective controller is activated. This procedure is illustrated in Fig. 2 for a set of constraints, while the corresponding switching policy can be formulated as follows [16]:
\[ u = \begin{cases} 
    u_{nom}, & \Phi(x) < 0 \lor (6) \lor (7), \\
    u_{cor}, & \text{otherwise.}
\end{cases} \]
(14)
III. EXAMPLE OF CONSTRAINED MOTION CONTROL
In this section, we will demonstrate how invariance control can be applied to the problem of safety-constrained motion control of MAR and more specifically to the example of human forward fall and human-robot collision avoidance by switching between a nominal and corrective controller.

A. System Description
Mobility assistive robots (MAR) in the form of active rollators typically consist of rear and front wheels, chassis, supportive handle bars and a range of sensors to measure environment and human data. We consider a rollator of nonholonomic nature, meaning that the translational motion of the robot along the heading direction as well as rotational motion along its center of rotation are possible while the slippage in the lateral direction is restricted. With reference to Fig. 3, the nonholonomic constraint is given by

\[ \dot{x}_r \sin \theta_r - y_r \cos \theta_r = 0, \]

and the kinematic model can be written as follows,

\[ \dot{q} = \begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\theta}_r \end{bmatrix} = \begin{bmatrix} \cos \theta_r & 0 \\ \sin \theta_r & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ 0 \end{bmatrix} = Ju, \quad (15) \]

where \( v \) and \( \omega \) are two available control inputs for the linear and angular velocity around the vertical axis and \( q = [x_r, y_r, \theta_r]^T \) the states of the robot.

![Fig. 3. Scenario of human and mobility assistive robot in world frame.](image)

B. Nominal Control
To drive the differential drive mobility assistive robot, it is equipped with two force/torque sensors mounted at the handles of the rollator. Force components along and around the heading direction are used for motion control. Position-based admittance control is implemented, which allows to design the desired dynamic behavior of the system with respect to the user’s applied force by selecting proper admittance parameters. A mass-damper system for the linear and angular motion is considered

\[ M_d \ddot{q} + D_d \dot{q} = F_h, \quad (16) \]

where \( M_d \) and \( D_d \) are the desired inertia and damping matrices, respectively, and \( F_h \) the driving forces applied by the user. Thus, the system dynamics can be formulated as follows,

\[ \begin{bmatrix} \dot{q} \\ \dot{\dot{q}} \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & -M_d^{-1}D_d \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ M_d^{-1} \end{bmatrix} F_h \]

\[ y = \begin{bmatrix} 0 \\ M_d^{-1} \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix}, \quad (17) \]

with \( M_d, D_d, F_h \) given by

\[ M_d = \begin{pmatrix} M_{dr,r} & 0 \\ 0 & M_{dz} \end{pmatrix}, \quad D_d = \begin{pmatrix} D_{dr,r} & 0 \\ 0 & D_{d,z} \end{pmatrix}, \quad F_h = \begin{pmatrix} F_r \theta_r \\ M_z \end{pmatrix}. \]

The position of the robot is finally controlled by a low-level position controller.

Considering the situation of a forward fall, the user is loosing stability and thus, unintentionally applying driving forces in the forward direction to the robot handles to keep balance. Given the aforementioned nominal controller, however, this would accelerate the robot and bring the user at risk. This robot behavior can be changed when switching to a corrective controller whenever the distance between human and robot exceeds certain given limits. On the other hand, human-robot collision may occur when the distance is too small. In the following subsection the required safety constraints, switching policy and corrective controller are derived to overcome the aforementioned problems.

C. Safety Constraint Determination
To guarantee human safety during physical interaction with the MAR, specific safety constraints have to be considered. While evaluation of different criteria such as the human Center of Mass (COM) or Zero Moment Point (ZMP) can be used to formulate safety constraints, for the sake of simplicity, we investigated safe human-robot distance as a basic feature for forward fall and human-robot collision avoidance. Forward falls happen when the ZMP of the user leaves the user’s base of support in the heading direction, which results in an increasing distance between the robot and the user’s feet. Considering a distance sensor (e.g. laser, sonar, or Kinect) attached to the mobility assistive robot and pointing towards the user’s feet, this distance \( d \) can be easily determined:

\[ d = \sqrt{(x_r - x_h)^2 + (y_r - y_h)^2} \quad (18) \]

where \( (x_h, y_h) \) and \( (x_r, y_r) \) are the position of the human and robot in the world coordinate frame respectively, see Fig. 3. The midpoint of the user’s feet is determined based on the current position of the left \( (x_{h, left}, y_{h, left}) \) and right foot \( (x_{h, right}, y_{h, right}) \),

\[ x_h = (x_{h, left} + x_{h, right})/2, \quad y_h = (y_{h, left} + y_{h, right})/2. \]

The risk for a human forward fall increases when \( d \) leaves its safe boundary, i.e. \( d > d_{max} \). On the other hand, human-robot collision can happen if the robot comes too close to the human, i.e. \( d < d_{min} \). Thus, we define upper and lower...
bounds on the human-robot distance corresponding to two output functions formulating the safety constraints

\[ h_1 = d - d_{\text{max}} \leq 0, \]
\[ h_2 = d_{\text{min}} - d \leq 0, \]

where \( d_{\text{max}} \) and \( d_{\text{min}} \) represent the maximim and minimum acceptable safe distances between the midpoint of the human feet and the robot. For the sake of computational simplicity we consider the following squared distances as output functions

\[ h_1 = d^2 - d_{\text{max}}^2 \leq 0, \]
\[ h_2 = d_{\text{min}}^2 - d^2 \leq 0. \] (19)

D. Corrective Controller

As can be easily seen, the relative degree of the defined output functions \( y_1 \) and \( y_2 \) (19) is one since their first derivatives depend on the control input \( v \) and \( \omega \):

\[
\frac{dy_1}{dt} = \frac{dy_2}{dt} = 2 \left( x_r - x_h \right) \left( y_r - y_h \right) + 2 \left( x_r - x_h \right) \left( \cos \theta \right) \left( y_r - y_h \right) - 2 \left( x_r - x_h \right) \left( \sin \theta \right) \left( y_r - y_h \right) + \left( \omega \right). 
\]

The desired corrective control input can finally be determined by following the procedure sketched in Sec. II-C, combining the two derivatives and solving the resulting system of equations. The corrective controller of the robot manipulates the user heading velocity during walking while the robot angular velocity can be still used for further safety-related motion corrections such as obstacle avoidance.

E. Modified Switching Policy for Smooth Control Transition

The nominal controller acts until the human-robot distance reaches the boundary of the constraint admissible region. Then, the corrective controller is activated which keeps the constraint on the boundary avoiding constraint violations. Whenever the first derivative of the output function points inward the admissible region, the control switches back from corrective to nominal control.

Following section II-D, the corresponding switching policy for every single constraint is formulated as follows:

\[
u = \begin{cases} 
\nu_{\text{nom}} & y_1(x) < 0, \\
\nu_{\text{nom}} & y_1(x) \geq 0 \land \dot{y}_1(x, \nu_{\text{nom}}) \leq 0, \\
\nu_{\text{corr}} & \text{otherwise}.
\end{cases} \] (20)

However, depending on the invariance function value and its rate of change invariance control may suffer from chattering when moving along the boundary of the admissible region [14], [18]. Considering for example the situation that the system had switched to the corrective controller and the rate of change of the output function decreases with positive value of the output function the system switches back to nominal control. If the nominal controller, however, drives the system again towards the boundary of the admissible set the system will switch back to the corrective controller which results into chattering.

To overcome the aforementioned problem, we modified the switching policy (20) by taking into consideration the rate of change of the nominal control input for every specific constraint as well as similarity of the nominal and corrective controllers

\[
u = \begin{cases} 
u_{\text{nom}} & y_1(x) < 0, \\
u_{\text{nom}} & y_1(x) \geq 0 \land \dot{y}_1(x, \nu_{\text{nom}}) \leq 0 \land \\
u_{\text{nom}} \land \nu_{\text{corr}} \leq \epsilon & \text{otherwise}.
\end{cases} \] (21)

where \( i = 1, 2 \) and \( \epsilon \) is a sufficiently small value.

F. Human Velocity Estimation

The corrective controller requires inputs in terms of human and robot velocity. While the robot velocity can be obtained from odometry, the human velocity can only be determined indirectly by taking into account information about the measured human-robot distance:

\[
\dot{x}_h = \dot{x}_{rh} + \dot{x}_r, \quad \dot{y}_h = \dot{y}_{rh} + \dot{y}_r \] (22)

where \( \dot{x}_{rh} \) and \( \dot{x}_r \) are numerical derivatives of the relative human-robot distances measured in the robot frame. Unfortunately laser and encoder signals are both prone to noise. Thus, third order Butterworth lowpass filters with cut-off frequency of 15 rad/s are used for filtering the human velocity as well as rate of change of the output functions.

IV. RESULTS

In the following subsections we report on simulation and experimental results obtained when implementing the proposed safety-constrained motion controller.

A. Simulation

In a first step the proposed approach was validated in simulations. Simulations were performed in Matlab with the ode45 solver and an accuracy AbsTol = RelTol = 1e-9. Maximum and minimum distance were set to \( d_{\text{max}} = 0.8 \text{ m} \) and \( d_{\text{max}} = 0.1 \text{ m} \), respectively. A user applied forward force with an amplitude of \( 10 \text{ N} \) to the MAR for 3.5 seconds starting from an initial distance of \( 0.4 \text{ m} \) and then changing the direction of motion by applying a backward force with an amplitude of \(-10 \text{ N} \) for 6.5 seconds and finally once more changing the direction of motion by applying \( 10 \text{ N} \) forward for 4 seconds. Simulation results for the two cases of invariance controllers with classical switching policy (20), and with modified switching policy (21) are shown in Fig. 4. As depicted, the human-robot distance is well kept within the safety zone thanks to a timely switching to the corrective controller for the modified switching policy (21), while chattering and thus, violation of the distance boundary occurs for the classical switching policy (20).
Fig. 4. Simulation results for a provoked human forward fall when using switching policies of (21), first row, and (20), second row. First column: human-robot distance using invariance control (solid line) and nominal control (dashed line), second column: constrained output function, \( h_1 \) (blue lines) and \( h_2 \) (red lines) using invariance control (solid lines) and nominal control (dashed lines), third column: robot velocity resulting from nominal controller (blue), corrective controller (red), and invariance controller (black).

### B. Experiment

The MAR shown in Fig. 5 has been used for experimental validation. It is a differential drive mobile robot equipped with two castors as front wheels and actuated rear wheels, two laser range finders, Kinetics, differential encoders, and JR3 force/torque sensors at the handles. The rear wheels are actuated by DC motors. The human applied force is measured at the two handles. The control algorithm of the system is running on Linux/PreemptRT using Matlab/Simulink Real-Time Workshop and a discrete-time Euler solver. The sampling rate of the control unit is 1 kHz while the human-robot distance is measured with update rates of 120 Hz. Velocity is controlled using a low-level PD controller.

Boundary constraints of \( d_{\text{max}} = 0.8 \text{ m} \) and \( d_{\text{min}} = 0.1 \text{ m} \) for safe human-robot distance were considered. In 80 seconds normal walking, the user was asked to first push the robot away two times to increase the risk of a forward fall and then to pull the robot toward him/herself to increase the risk of collision. Figure 6 shows the data captured during this experiment using the two switching policies (21) and (20). As can be clearly seen, in both cases the corrective controller is activated whenever the safety constraints are about to be in violation, although chattering is observed for the classical switching policy (20), which leads to a larger constraint boundary violation compared to the newly proposed switching policy (21).

![Fig. 5. The mobility assistive robot.](image)

### V. Conclusions

A systematic approach for the supervision-based safe motion control of mobility assistive robots based on invariance control was presented, which allows us to formulate safety features in the form of a constraint admissible state space region that can be kept invariant by proper switching of the system control between a nominal controller and a corrective controller whenever a predefined safety constraint is about to be in violation.

The example of human forward fall prevention and col-
Fig. 6. Experimental results for a provoked human forward fall when using switching policies (21), three plots on the left, and (20), three plots on the right. Constrained output functions $h_1$ (blue) and $h_2$ (red) and robot velocities using nominal controller (blue), corrective controller (red), and invariance controller (black).

Lission avoidance was studied by considering human-robot distance as critical safety feature. The corrective controller and a proper switching policy to reduce chattering was proposed. The approach was tested in simulations and experiments and results show high potential for building a systematic framework that can consider safety features in mobility assistive robots.

Future versions will also consider obstacle and step avoidance similar to [19], as well as more context-aware safety features based on the location of the human center of mass and/or zero moment point in order to guarantee human safety during cooperation with MARs, which will require the extension of the constrained motion control approach to higher order dynamics.

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