2014 5th IEEE RAS & EMBS International Conference on
Biomedical Robotics and Biomechatronics (BioRob)
August 12-15, 2014. São Paulo, Brazil

Kinematic Synthesis of the Passive Human Knee Joint by Differential Evolution and Quaternions Algebra: a Preliminary Study

Daniel P. Saldias.[1], Luiz A. Radavelli[1], Carlos R. M. Roesler[2] and Daniel Martins[1]

***Abstract*— The main aim of this preliminar study was to support the orthopedic surgeon with information about the position of the graft placement of the ACL, using the data provided by a spatial model of the knee. A mathematical model available in the literature and based on the theory of mechanism has been here reimplemented. In particular, the numerical approach to the definition of the model has been modified respect to the original by introducing the quaternion algebra and the differential evolution algorithm. This methodology has already shown to be capable to produce mechanisms that match the natural motion of the knee [8]. Any implementation of it may thus be useful in the preoperative planning with information of the position of the graft placement of the ACL.**

I. INTRODUCTION

When surgery of the anterior cruciate ligament (ACL) is required, preoperative planning is a critical step in defining the parameters to be considered prior to the surgery. In this context, orthopedic surgeons need to define scientifically the best insertion position for the graft, which approximates the functionality of an intact ACL. It is a particularly complex case when the graft can not be placed in the natural area of ligament insertion, mainly when this natural area is too small to perform the surgical procedure. In such cases are chosen adjacent places to the natural area of insertion, but there are not knowledge about the repercussions it will have on joint kinematics.

The objective of this research was to propose an implementation of a mathematical model of the human knee, based on the theory of mechanisms and originally proposed in [8], which, respect to the previous versions, will make use of quaternion algebra and the differential evolution algorithm. The mechanical model should simulate the movement that occurs in the knee during the passive flexion. This methodology will be used to implement custom models of the knee, to provide information to assist the medical decision making in the preoperative planning.

The synthesis methodology starts with a preliminary model, obtained by a sistematic approximation of the main kinematic functions of the knee joint. Those main kinematic functions were transformed into kinematic constrains into the model. By using of experimental data from the human knee joint and the optimization by differential evolution algorithm, the preliminary model is refined.

[1]Robotics Laboratory LAR, Federal University of Santa Catarina, University Campus, Trindade, Florianopolis/SC, Brazil. danielpo25@gmail.com

[2]Biomechanics Engineering Laboratory LEBm, Federal University of Santa Catarina, University Hospital, University Campus, Trindade, Florianopolis/SC, Brazil rroesler@hu.ufsc.br

The proposed methodology provides three novel contributions in relation with the existing models: The sistematic approximation used to obtain the preliminary model allows a clear vision about the function of the anatomic elements of the knee for further analysis. The use of quaternion algebra has shown advantages because the processing time has decreased around four times in relation to the time obtained using matrix algebra, where this matrix algebra is traditionally used in the other existing models [4], [8]. The differential evolution algorithm solves global problems and should simplify the algorithm by simply removing of the refinement step. At least two optimization process are found in the literature, the first one is for solve the global problem and the second one is for refinement [4], [8].

This paper begins with the biomechanical analysis of the knee. The background of robotics is then presented, which is used for the analysis and modeling of the knee. The next section presents the proposed methodology for the modeling. Finally, the results and conclusions are presented.

II. BIOMECHANICAL ANALYSIS OF THE HUMAN KNEE

The knee anatomy is divided into four key parts: bone anatomy, muscular anatomy, meniscus and ligament anatomy. The knee bone anatomy: composed by the distal end of the femur, the proximal end of the tibia and patella (Fig. 1a). The muscular anatomy: is composed by the upper

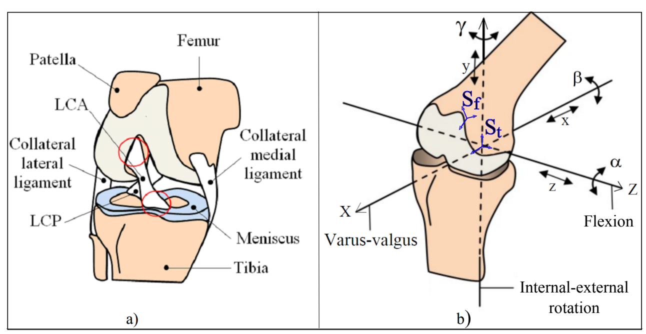

Fig. 1: Structures of the knee and ACL with femoral origin and tibial insertion circled in red.

and the lower muscles and they can classified into flexors, extensors, adductors, abductors and rotators [2]. The meniscus: they are plates of fibrocartilage which dampen impacts between the faces of tibiofemoral contact (Fig. 1a). The ligament anatomy: is composed by the cruciate ligaments (anterior ACL and posterior PCL) and the collateral ligaments (medial CML and lateral CLL) as presented in Fig.

(1a). The natural insertion areas of the ACL are circled in red in Fig. (1a).

The movement of the knee is governed by ligaments and geometric constraints of the articular surfaces. For each knee pose, its spatial position can be described by three coordinates that compose the vector **p**=(x,y,z); and its spatial orientation can be described by three rotations angles that compose the vector **r**=(α, β, γ) (Fig.1b). The vectors **p** and **r** related the movement of the anatomical center of the femur S_f with respect to the anatomical center of the tibia S_t, where S_t is consider the origin of the coordinate system following the biomechanical convention adopted [1]. As shown in (Fig.1b), the direction of the knee position components of **p** are defined as: anterior-posterior x, axial y and lateral collateral z. The directions of the knee rotation components of **r** are defined as: flexion angle α, varus-valgus angle β and internal-external rotation γ.

The knee can develop two ways of movements: active and passive movement. The active movement is considered when the knee is subject to its muscular activation or an external load. The active motion has six degrees of freedom ($6-DOF$) [10], it means that is necessary 6 independent variables to fully described the instantaneous joint pose. The passive movement, or passive flexion, is considered when the knee is not subject to any muscular activation nor external load and it has $1-DOF$ [8], it means that is necessary one independent variables to fully described the instantaneous joint pose. For instance, imposing a flexion angle α it can obtained the position and the orientation of the knee. Also, the knee presents two specific movements in its flexion-extension path: the $screw-home$ and the $rollback$ [2]. The $screw-home$ is a movement developed in the transverse plane, and corresponds to the tibial internal-external rotation in the first $20°$ of flexion, starting from a maximum extension of the joint. $Rollback$ consists on a defined movement in the sagittal plane, where in the first $30°$ of flexion the femur $rolls$ on the tibia without $slipping$. After this point the femur $sliding$ and begins gradually to predominate about $rolling$. Thus, in the end of the flexion, the femur $slides$ without $roll$ over the tibia (pure rotation). The passive motion analysis is very important for the analysis of the articular stability, prostheses design [7] and preoperative planning, for this reason the passive movement is used in this work.

III. Mathematical analysis of parallel platforms to model human knee joint

This section covers the analysis for the parallel platforms which can inspire the modeling of the human knee. Also, a revision and analysis of the quaternion algebra to solve the kinematics of these parallel platforms, is performed.

A. Knee modeling by spatial parallel platforms

The geometric analysis of a parallel manipulator can contribute to modeling the knee, because it has a base and a moving platform which serve as an analogy of the tibia and femur, respectively. The parallel manipulator also has limbs that allow the ligaments and condyles to be modeled. In this regard, is of great interest the the analysis of the Stewart-Gough. This platform is a spatial parallel manipulator of 6-DOF (as the active knee movement), formed by six SPS limbs. The Fig. (2a) shows the Stewart-Gough platform: six identical limbs, with prismatic actuators, connect the moving platform to the fixed base by spherical joints at points B_i and A_i $(i = 1, 2, ..., 6)$, respectively.

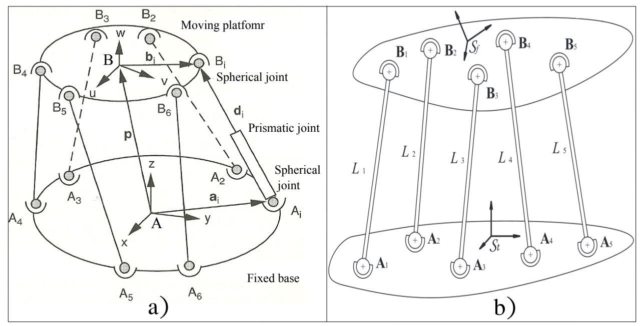

Fig. 2: Parallel platforms: a)Stewart-Gough platform, adapted from [9]. b) $1-DOF$ parallel platform, adapted from [8]

For the analysis, two cartesian coordinate systems, frames $A(x, y, z)$ and $B(u, v, w)$ are attached to the fixed base and moving platform, respectively (Fig. 2a). The transformation from the moving platform to the fixed base can be described by the rotation matrix ${}^{A}R_{B} = R_z\, R_x\, R_y$ and by the position vector **p** which origin is at A and goes up to B [9]. As shown in Fig. 2a, it is consider $\mathbf{a}_i = [a_{ix}, a_{iy}, a_{iz}]^T$ and ${}^{B}\mathbf{b}_i = [b_{iu}, b_{iv}, b_{iw}]^T$ as the position vectors of points A_i and B_i in the coordinate frames A and B, respectively. Considering L_i as the limb length for each limb $\overline{A_i B_i}$ $(i = 1, ...6)$, it can written the $vector-loop$ equation [9] for the ith limb of the manipulator as follows:

$$\|\mathbf{p} +^{A} R_B\ {}^{B}\mathbf{b}_i - \mathbf{A}_i\| = \|\mathbf{L}_i\|, \quad (i = 1, \ldots, 6). \qquad (1)$$

In order to obtain a parallel platform that allows the passive motion of $1-DOF$ to be modeled, several considerations on the geometry of the Stewart-Gough platform has to be performed, as reduce the limb number and eliminate the prismatic joints, turning the six SPS limbs into five SS limbs. The resulting $1-DOF$ parallel platform [8] is shown in Fig. (2b). As the $1-DOF$ parallel platform has 5 limbs, it can be obtain a system of i=5 *vector-loop* equations. In order to solve the position **p** and orientation **r** of the $1-DOF$ moving platform, it imposed the the flexion angle α, resulting in a system of 5 equations and 5 unknowns (β, γ, x, y, z).

B. Quaternions Algebra

Let $\{1, \mathbf{i}, \mathbf{j}, \mathbf{k}\}$ the standard basis of $\mathbb{R}^4$. Quaternions are elements of the form $q = w + x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$, where $(w, x, y, z \in \mathbb{R})$ and $\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{ijk} = -1$. The quaternions space is denoted by $\mathbb{H}$. If $w = 0$, the quaternion q correspond to $3D$ vectors and it is called *pure quaternion*. Therefore it is natural to think of quaternions as the sum of a scalar and a vector, that is,

$$q = w + \mathbf{v} = Sc(q) + Ve(q), \quad w \in \mathbb{R}, \mathbf{v} \in \mathbb{R}^3.$$

Let us consider two quaternions, namely $q_1 = w_1 + x_1\,\mathbf{i} + y_1\,\mathbf{j} + z_1\,\mathbf{k} = w_1 + \mathbf{v}_1$ and $q_2 = w_2 + x_2\,\mathbf{i} + y_2\,\mathbf{j} + z_2\,\mathbf{k} = w_2 + \mathbf{v}_2$. The usual operations are:

$$
\begin{array}{lll}
q^* = w - \mathbf{v} & Sc(q) = \frac{q+q^*}{2} & Ve(q) = \frac{q-q^*}{2} \\
q_1 + q_2 = (w_1 + w_2) + (\mathbf{v}_1 + \mathbf{v}_2) & & \|q\|^2 = qq^* = q^*q \\
\multicolumn{3}{l}{q_1\,q_2 = (w_1 w_2 - \mathbf{v}_1 \cdot \mathbf{v}_2) + (w_1\mathbf{v}_2 + w_2\mathbf{v}_1 + \mathbf{v}_1 \times \mathbf{v}_2)}
\end{array}
$$

For pure quaternions we have $q_1 q_2 = \mathbf{v}_1\mathbf{v}_2 = -\mathbf{v}_1\cdot\mathbf{v}_2 + \mathbf{v}_1\times\mathbf{v}_2$ which establishes the quaternions and $\mathbb{R}^3$ inner and outer product relation. Also for pure quaternions, $\mathbf{v}_1\mathbf{v}_2 + \mathbf{v}_2\mathbf{v}_1 = -2(\mathbf{v}_1 \cdot \mathbf{v}_2)$ and $\mathbf{v}_1\mathbf{v}_2 - \mathbf{v}_2\mathbf{v}_1 = 2(\mathbf{v}_1 \times \mathbf{v}_2)$.

The rotation of a vector $\mathbf{a}$ (pure quaternion) is given by $\mathbf{a}' = q\,\mathbf{a}\,q^*$, where $q = \cos\frac{\theta}{2} + \mathbf{s}\left(\sin\frac{\theta}{2}\right) := q(\mathbf{s},\theta)$ is the quaternion operator which encodes the rotation around the axis represented by unit vector $\mathbf{s}$ and its angle magnitude θ. A more general form of rotations from quaternions is given by

$$\mathbf{a}' = q\,(\mathbf{a} - \mathbf{s}_0)\,q^* + \mathbf{s}_0, \tag{2}$$

where $\mathbf{s}_0$ is the rotational axis position vector. To more definitions and properties of quaternions we suggest [5] and [3].

The *vector loop* equations (Eq.1) for the $1-DOF$ parallel platform (Fig 2b) can solved by quaternions algebra. In this way the rotational matrix ${}^{A}R_{B}$ is replaced by the quaternion rotator $q = q_z\,q_x\,q_y$, where $q_x = q(\mathbf{i},\beta)$, $q_y = (\mathbf{j},\gamma)$ and $q_z = (\mathbf{k},\alpha)$ are the quaternion rotators around the $3D$ axes. Also we must codified the vectors $\mathbf{p}$, $\mathbf{A}_i$ and $\mathbf{B}_i$ into quaternions algebra as following: $p = 0 + \mathbf{p}$, $a_i = 0 + \mathbf{A}_i$, $b_i = 0 + \mathbf{B}_i$. Therefore, the *vector loop* system (Eq.1) is reduced to:

$$\|p + q\,b_i\,q^* - a_i\| = \|\mathbf{L}_i\|, \quad (i = 1,\ldots,5). \tag{3}$$

From the equation components,

$$\sqrt{a_i^T a_i + b_i^T b_i + p^T p - 2a_i^T p + 2(p - a_i)^T\left(q\,b_i\,q^*\right)} = \mathbf{L}_i. \tag{4}$$

Imposing the flexion angle α into the five *vector loop* equations for the $1-DOF$ parallel platform (Fig. 2b) we solve the system in two different ways: from quaternions algebra (Eq.3) and from matrix algebra (Eq. 1). An important observation was performed in this work: using quaternion algebra the solutions were obtained in 13.1s, about four times faster than using matrix algebra.

There are three facts that explain our faster results. First: the quaternions algebra offers an alternatively algebra to model problems, and in our modelling we get a somewhat less nonlinear system. Second: the rotational operator storage requirements are reduced from nine (matrix) to four (quaternions) [3] and the computational cost is better in quaternions algebra [6]. Composition of rotations requires 16 multiplications and 12 additions in quaternion representation, but 27 multiplications and 18 additions in matrix representation [6]. Third: the rotational parameters identification are easily obtained from the quaternion operator.

IV. Proposed Methodology

The proposed methodology for spatial modeling of the human knee, consisting of three steps: (1) experimental session, (2) Preliminary modeling and (3) kinematic modeling. These steps are described below.

A. *Experimental session*

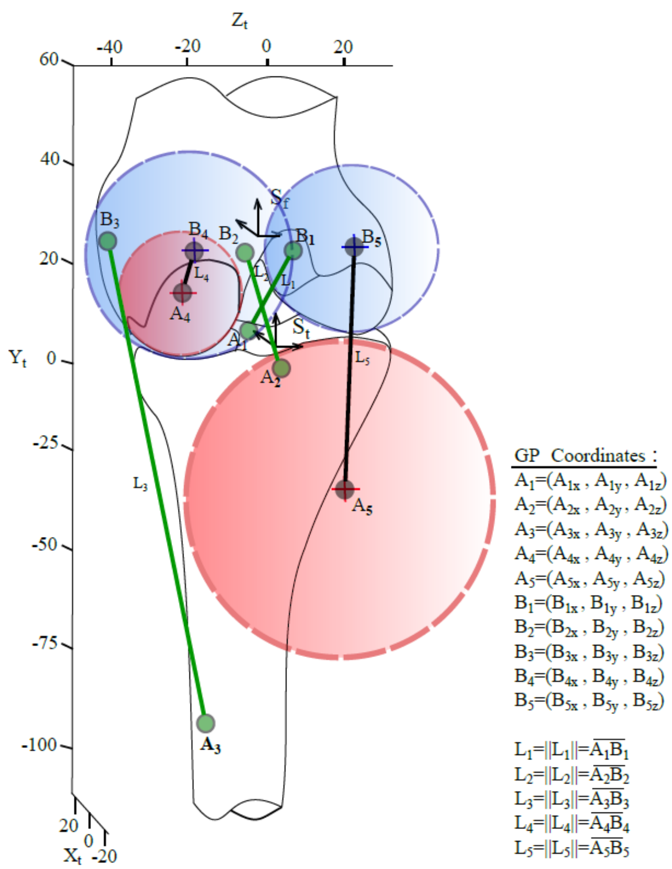

Fig. 3: Preliminary Geometrical parameter GP: A_i, B_i and L_i, $(i = 1...5)$, based on [4]

Consist in the obtaining of experimental data based on two analysis of the human knee: the geometrical analysis and the kinematics analysis. These analysis was performed by [4], who processed the data obtained by an optoelectronic device and knee specimens.

In this paper, is used the geometrical analysis performed by [4] to obtain the $search\ domain$. The $search\ domain$ is used in the optimization process (for syntheses and modeling) and it enclosed anatomical components of interest, as ligament insertion areas and central condylar regions that contains the centrodes. The $search\ domain$ is consider as a spherical volume with center and radius defined by the searcher and it allows the preliminar geometrical parameter GP to be found. The GP are the ligament insertion points, the ligament lengths and the condylar center points. The condylar center point is the center of the sphere that best fit at each condyle. The GP are presented in the Fig. (3) and they are A_i, B_i and L_i $(i = 1, ..., 5)$, where A_i is measured with respect to S_t and B_i is measured with respect to S_f. The GP are explained in detail below. The insertion points of the most isometric ligament fibers of the ACL, PCL and MCL on the tibia are $A1$, $A2$ and $A3$, respectively and on the femur are B_1, B_2 and B_3, respectively. The length of the most isometric fibers of ACL, PCL and MCL are (in green) L_1, L_2 and L_3, respectively. The center points of the medial and lateral condyle on the tibia are $A4$, $A5$, respectively and on the femur are B_4 and B_5, respectively. The length of the links that joint the center of the medial and lateral condyles are (in black) L_4 and L_5, respectively.

As shown in Fig. (3), there are 35 GP (10 points of three dimensional coordinates corresponding to the positions Ai and Bi and five L_i lengths $(i = 1, ...5)$.

The kinematic analysis allows the knee passive flexion to be measure, obtaining experimentally the position and orientation of the femur (S_f) with respect to the tibia (S_t). With this information is obtained the experimental kinematics parameters KP^*: the position vector **p**=(x,y,z) and the orientation vector **r**=(α, β, γ), for each flexion angle α imposed, in the course of the whole passive flexion. The KP^* was obtained by [4] who recorded the passive flexion for each instantaneous flexion angle α by an optoelectronic device and markers with active emitting dioes fixed in to the tibia and femur, as shown in Fig. (4), where m is the number of captures on the whole passive flexion. The KP^* are used later to be compared with the movement of the model to verify its accuracy.

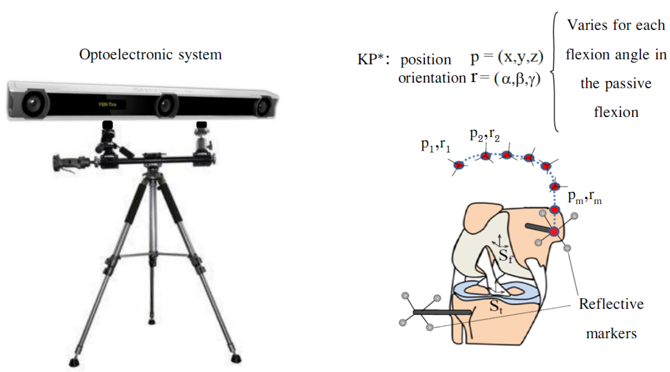

Fig. 4: Experimental kinematic parameters KP^*: **p**=(x,y,z) and **r**=(α, β, γ)

B. Preliminar modeling

This step consists in the synthesis of a $1-DOF$ mechanism composed by elements that represent the fundamental anatomical structures that allow the passive motion of the knee to be performed. This is accomplished through a sistematic approximation (Fig. 5), where each basic movement of the knee is associated with an equivalent kinematic constraint. At the end of this process is obtained, in a preliminary form, a representative functional mechanism of the human knee. This sistematic approximation begins with a mechanism of a simple hinge joint with congruent cylindrical contact surface (Fig. 5a), that represents the

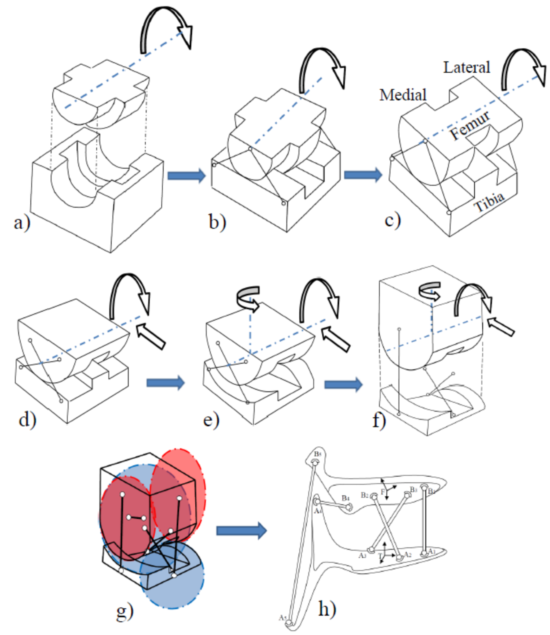

Fig. 5: Sistematic approximation to perform the preliminary model of the human knee joint.

kinematic constrain that only allows the rotation movement to be performed, modeling the main function of the knee: the flexion. The next step follows the first modification (Fig. 5b): introducing incongruent contact surfaces and positioning two set of limbs with spherical joints (SS limbs) in both sides of the mechanism, where the motion guidance has become entirely dependent on limb connections allowing the rotation movement only. Second modification (Fig. 5c): maintaining the rotational movement, the arrangement of the contact surface has been inverted performing two set of condyles (medial and lateral) where the upper part of the mechanism represent the femur and the lower part, the tibia. Third modification (Fig. 5d): one pair of cross arrangement of SS limbs is positioned in each side, allowing the *rollback* movement (rolling and sliding). Fourth modification (Fig. 5e): the medial condyle of the tibia has been changed to a concave shape, and the lateral condyle of the tibia has changed into a convex shape (as the anatomy shapes). The medial condyle is bigger than the lateral condyle, allowing the $screw-home$ movement (axial rotation). Fifth modification (Fig. 5f): one pair of the cross arrangement of SS limbs were excluded. The remaining cross limbs were set in the anatomical cruciate ligaments position. A SS limb was added in the medial side of the mechanism for lateral stability, representing the medial collateral ligament. Sixth modification (Fig. 5g): The circle that best fits on each condyle has been approximated, then the center of each circle was located. The centers of each circle has designated as the spherical joint of a link, resulting in the preliminar mechanical model of the knee (Fig. 5h). This preliminar model is topologically similar to the previously proposed in [8]. This mechanism is a parallel platform composed by five SS limbs, a fixed platform and a moving platform, with a spatial $1-DOF$, according to *Grübler criterion* [9], where λ are the degrees of freedom of the work space, n the number of links of the mechanism, j the number of spherical joints and f_i the degrees of relative motion permitted by joint i:

$$F = \lambda(n-j-1)+\sum_i f_i = 6(7-10-1)+(3\cdot 10) = 6 \quad (5)$$

however, there are 5 passive degrees of freedom associated with the five SS limbs. Therefore, the parallel platform posses $1-DOF$, as the passive flexion of the knee joint.

C. Kinematic modeling

This step is based on an existing proposed method [4], [8], adding novel contributions: the use of the quaternions algebra (instead matrix algebra) to solve the equations that describe the model movement and the use of differential evolution algorithm, that should simplify the optimization by removing the refinement process found the literature [8], [4]. The differential evolution is here implemented (instead genetic algorithms) because has only three control parameters and the influence of these parameters is well known. Also, allows easy implementation and different possibilities of recombination to produce new test populations. In the differential evolution algorithm are optimized the GP in order to obtain a model with KP closer to the experimental KP^*.

As shown in Fig. 6 the process of kinematic modeling start with the data obtained in the experimental session, where is obtained the *search domains* and the experimental kinematic parameters KP^*. The *search domain* data enter in the optimization process (differential evolution algorithm), where the preliminar geometrical parameters GP are determined. These GP (A_i, B_i and L_i) determining a first approximation of the knee model. Seven equally spaced flexion angles α was chosen in the whole passive flexion. At each optimization iteration, the $vector\ loop$ equations where solved at this seven α angles, obtaining the kinematic parameters of the model KP at these seven poses. The KP were iteratively compared with the experimental KP^* for each pose. The sum of the weighted squares of errors between the experimental knee poses and the model poses had to be minimized, constituting the objective function (Eq. 6), where $m = 1, ..., 7$ are the number of poses due to the equally spaced α angle, KP_m are the five i-unknowns, KP^*_m are the desired (experimental) values of the unknowns, and W_m are the weights necessary in order to account for the different dimensions of the unknowns. If the $vector\ loop$ equations are not satisfied at some pose, a very high value is assigned to the objective function as a penalty.

$$O.F. = \sum_{i=1}^{5}\sum_{m=1}^{7} = (KP_m - KP^*_m)^2/W_m^2 \qquad (6)$$

The optimization process is repeated until a small value to reach VTR has found, it mean that the generated model, with the final GP (Fig. 6), allows a movement (KP) very similar to the experimental one (KP^*) obtained from an human knee.

V. Results

The relative orientation and position of the femur with respect the tibia are shown in Fig. 7 and 8 as a function of the flexion angle α. In these figures are presented the experimental data in red [4], [8], the data from other models found in the literature in green [4], [8] and the data obtained from the proposed model (blue). The results of the model found in the literature are sightly better then the proposed

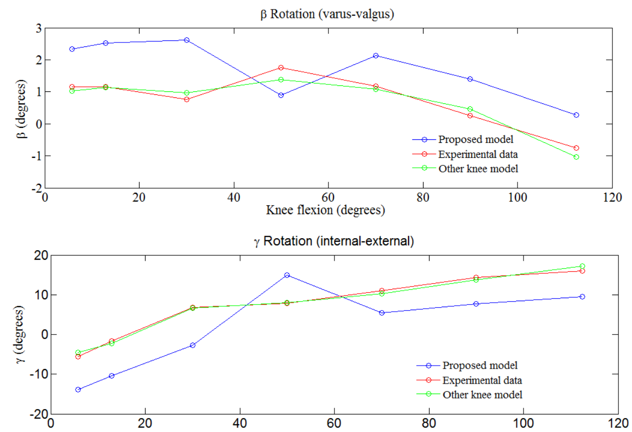

Fig. 7: Orientation of the femur with respect to the tibia: ○ proposed model; ○ other knee model [4]; ○ experimental data [4].

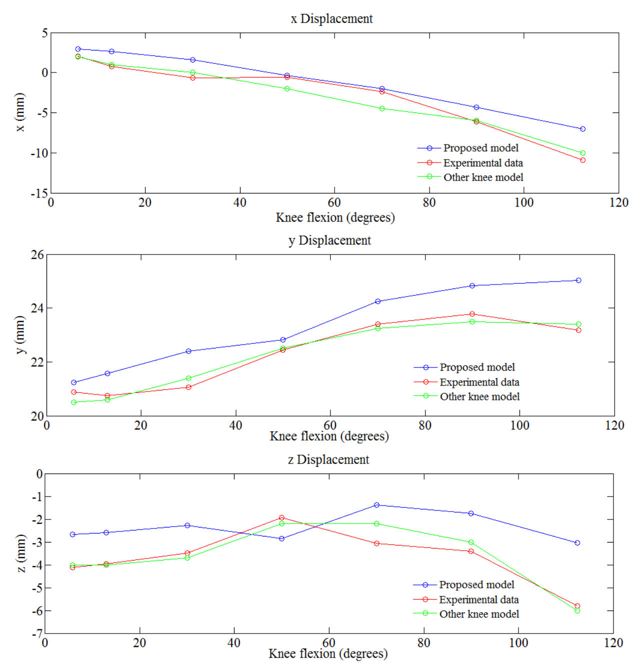

Fig. 8: Position of the femur with respect to the tibia: ○ proposed model; ○ other knee model [4]; ○ experimental data [4].

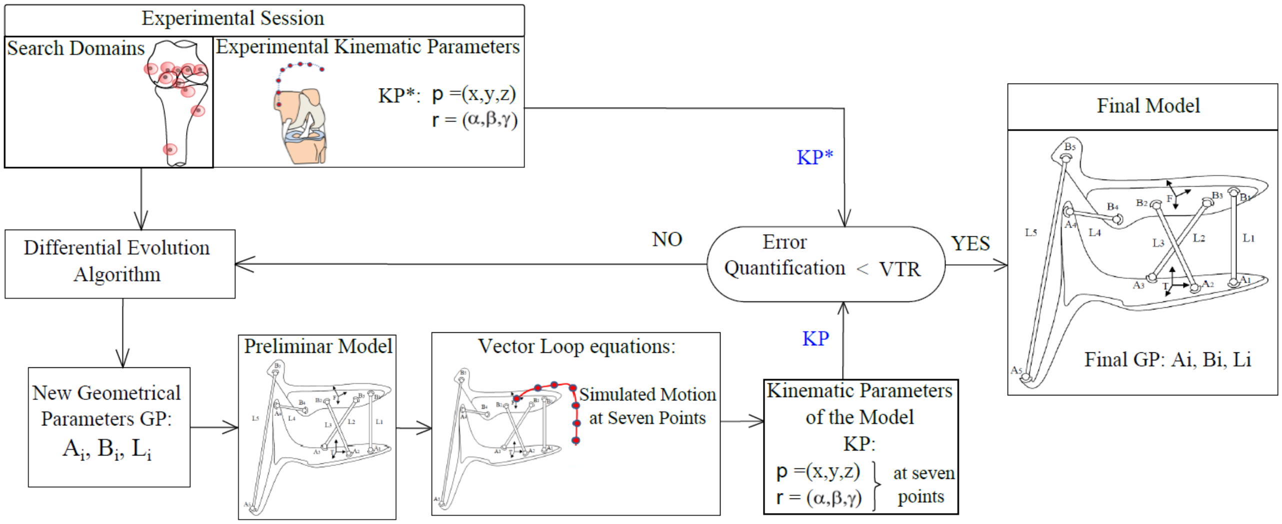

Fig. 6: Kinematic modeling process.

model, but is reasonable because these authors used two nested optimization process [4], [8]. The proposed model can replicate the passive knee motion, therefore, the proposed methodology proved its effectiveness. The final GP of the proposed model are inside of the *search domains* imposed. The final GP are (in mm): $A_1 = (12.86, 0.01, -4.49)$, $A_2 = (-19.32, -10.12, -3.95)$, $A_3 = (14.62, -98.06, -9.78)$, $A_4 = (8.45, -47.31, 20.14)$, $A_5 = (-4.25, 28.04, -31.87)$, $B_1 = (-7.43, 0.08, 11.07)$, $B_2 = (-2.82, -0.39, -3.99)$, $B_3 = (0.71, 4.25, -46.41)$, $B_4 = (-4.34, 1.33, 23.29)$, $B_5 = (1.87, 5.58, -17.77)$, $L_1 = 29.25$, $L_2 = 35.80$, $L_3 = 129.51$, $L_4 = 470.52$ and $L_5 = 12.28$.

VI. Conclusions

This methodology has already shown to be capable to produce mechanisms that match the natural motion of the knee [8]. Any implementation of it may thus be useful in the preoperative planning. Specifically, the proposed methodology could help to found an insertion point for the ACL graft, different to the natural insertion area, without compromising the joint kinematics. It could be accomplish by the use of $search\ domains$ in a different place of these natural insertion areas.

The proposed methodology provides three novel contributions by reimplementation of the existing models [4], [8]: the sistematic approximation to obtain a preliminary model, the use of the quaternion algebra to solve the equations that describe the kinematics of the model and the use of an optimization process by the differential evolution algorithm. The sistematic approximation used to obtain the preliminary model provides a clear vision about the function of the anatomic elements of the knee for further analysis. The use of quaternion algebra has shown advantages in the processing: around four times in relation to the time obtained using matrix algebra, where this matrix algebra is traditionally used in the other existing models. The differential evolution algorithm as an alternative to the genetic algorithms. Whereas in the literature, at least two optimization process are found: one to solve the global problem and another to refinement the solution, and they have shown more accurate results. Finally, the model would be improved by modifying the weight function, and increasing the number of optimization points (more than seven) in order to reduce the difference with the experimental orientation data.

VII. Acknowledgment

The authors would like to thank the CAPES - Coordenação de Aperfeiçoamento de Pessoal de Nível Superior and CNPq - Conselho Nacional de Desenvolvimento Científico e Tecnológico, Brazil, that provided financial support for this research.

References

[1] Edward S Grood and Wilfredo J Suntay. A joint coordinate system for the clinical description of three-dimensional motions: application to the knee. *Journal of biomechanical engineering*, 105(2):136, 1983.
[2] AI KAPANDJI. *Fisiologia articular: Membro inferior. Editorial Médica Panamericana*. Guanabara Koogan, 5ª edição, 2000.
[3] Jack B. Kuipers. *Quaternions and Rotation Sequences*. Princeton University Press, 1999.
[4] V. Parenti-Castelli and N. Sancisi. Synthesis of spatial mechanism to model human joints. In *21th Century kinematics, Ed. M. Mcarthy*, pages 49–84. Springer, 2013.
[5] L.A. Radavelli. Análise cinemática direta de robôs manipuladores via álgebra de clifford e quatérnios. Master thesis, UFSC, 2013.
[6] Eugene Salamin. Application of quaternions to computation with rotations, internal working paper, stanford ai lab., 1979.
[7] N. Sancisi, D. Zannoli, V. Parenti-Castelli, C. Belvedere, and A. Leardini. A one-degree-of-freedom spherical mechanism for human knee joint modelling. *Proceedings of the Institution of Mechanical Engineers, Part H: Journal of Engineering in Medicine*, 225(8):725–735, 2011.
[8] Nicola Sancisi and Vincenzo Parenti-Castelli. A 1-dof parallel spherical wrist for the modelling of the knee passive motion. *Mechanism and Machine Theory*, 45(4):658–665, 2010.
[9] Lung-Wen Tsai. *Robot analysis: The mechanics of serial and parallel manipulators*. Jonh Wiley e Sons, New York, 1999.
[10] Savio LY Woo, Richard E Debski, John D Withrow, and Marsie A Janaushek. Biomechanics of knee ligaments. *The American journal of sports medicine*, pages 533–543, 1999.