Supplementary Material

Anonymous ECCV submission

Paper ID 598

Curve Fitting for Rotationally Symmetric Mirrors

Since the CCP may lie out of the image plane, we complete the image of 3D lines on the mirror in rotationally symmetric mirrors. Recall that the line images in rotationally symmetric mirrors are high-order curves. Therefore, it is infeasible to directly fit the curves using a parametric model. Instead, we adopt a point-aggregate strategy to find all possible pixels on the image plane that can be the image of the 3D line. The complete set is then the line image.

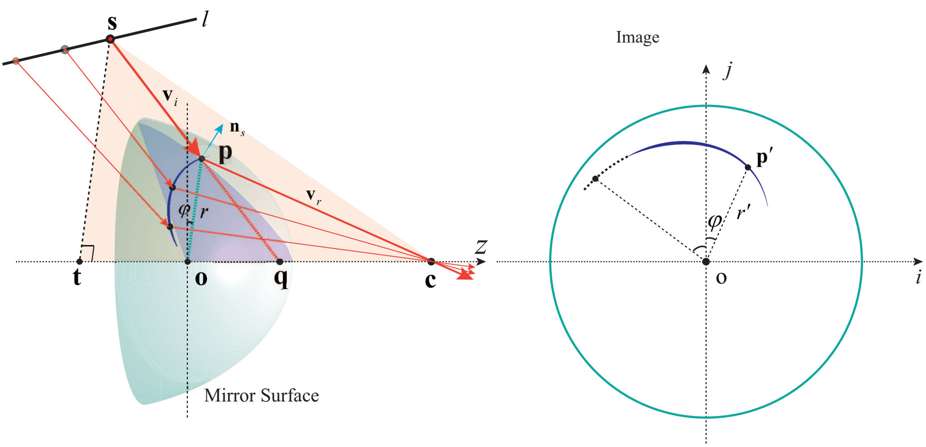


Fig. 1: Line projection in a symmetric catadioptric mirror. Left: the point $\mathbf{s}$ on line must lay on a cone determined by $\mathbf{p}, \mathbf{q}$. Right: The resulting catadioptric image. We show the line image and the mirror profile.

Give a 3D point $\mathbf{s}$: $[x_0, y_0, z_0] + \lambda[dx, dy, dz]$ on line l and its reflection point $\mathbf{p}$: $[x, y, z]$ on mirror surface $\mathbf{p}$. We first establish a constraint on $\mathbf{s}$ and $\mathbf{p}$. Notice the $\mathbf{s}$,$\mathbf{p}$ and $\mathbf{c}$ are coplaner. So we have the following equation:

$$\frac{x_0 + \lambda dx}{y_0 + \lambda dy} = \tan \varphi \tag{1}$$

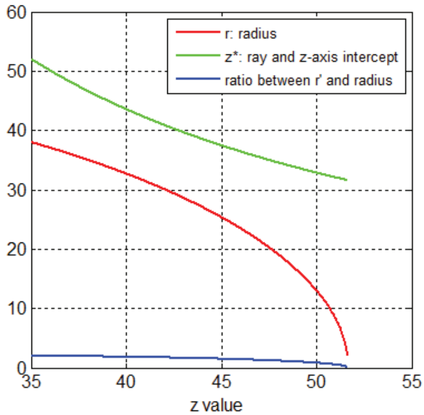
φ is the angle between $\mathbf{op}$ and y-axis. The incident ray $\mathbf{v}_i$ intersects z-axis at $\mathbf{q} = [0, 0, z^*]$. Line $\mathbf{st}$ is perpendicular to the z-axis, we have its length as $\frac{z_0 + \lambda dz - z^*}{z - z^*} r$.

Since $\mathbf{s}$ is on the cone that defined by $\mathbf{p}, \mathbf{q}$, we obtain a constraint on the line equation and its possible reflection point position on the mirror:

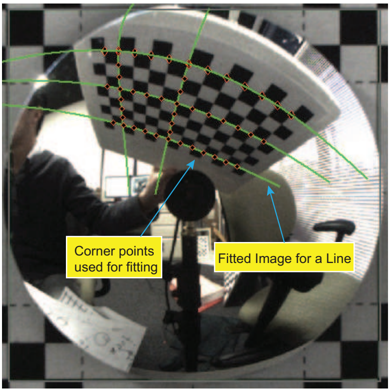
$$(x_0 + \lambda d_x)^2 + (y_0 + \lambda d_y)^2 = \frac{r^2}{(z - z^*)^2}(z_0 + \lambda d_z - z^*)^2$$

Substituting λ with $\tan\varphi$ into Eqn.1, we have:

$$a \cdot \tan\varphi z^* + b \cdot \tan\varphi + c \cdot z^* + d = \sqrt{1 + \tan^2\varphi}\frac{(z - z^*)}{r} \quad (2)$$



(a)



(b)

Fig. 2: The real result using our curve fitting algorithm. (a) The lookup table T we used to estimate the parameters a, b, c and d. Ratio between the r' and radius is used to map the world coordinate to image coordinate.(b) The fitted lines use the estimated a, b, c and d.

where $a = \frac{d_y}{y_0 d_x - x_0 d_y}, b = \frac{y_0 d_z - z_0 d_y}{y_0 d_x - x_0 d_y}, c = \frac{-d_x}{y_0 d_x - x_0 d_y}, d = \frac{z_0 d_x - x_0 d_z}{y_0 d_x - x_0 d_y}$. Since a,b,c and d are uniquely determined by the line parameters, they can be used to identify the line image. In our point-aggregate fitting algorithm, we first estimate the a, b, c and d using the observed line image. We then in turn use a, b, c and d to locate possible pixels on the line image. Given a catadioptric mirrors, we first establish a lookup table T that contains surface point $\mathbf{p}[r, z]$ and its corresponding $\mathbf{q}[0, 0, z^*]$. For each pixel $\mathbf{p}'$ on the observed line image, we compute r and φ corresponding to $\mathbf{p}$. We then consult the lookup table T to find the corresponding z^* of $\mathbf{p}$. By Eqn. 2, we can use φ, r and z^* to solve for a, b, c and d. Recall that we can use a, b, c and d to trace out all the points on the line image. Fig. 2 shows an example of our curve fitting result.