Supplemental Material: A Novel Topic-level Random Walk Framework for Scene Image Co-segmentation

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The reduction of topic-level random walk

We introduce a ranking score vector $\mathbf{r}$ that concatenates the topic-level vectors of all the nodes $v_i, i = 1, \cdots, N$ in the VRN, in which we set $N = |V|$, namely

$$\mathbf{r} = (r[v_1, 1], r[v_1, 2], \cdots, r[v_1, T], r[v_2, 1], \cdots, r[v_2, T], \cdots \cdots, r[v_N, 1], r[v_N, 2], \cdots, r[v_N, T])^T.$$

Thus the equation (3) in the submitted manuscript can be rewritten by the following form:

$$\mathbf{r} = \varepsilon \mathbf{v} + (1 - \varepsilon)\mathbf{U}\mathbf{r} \tag{1}$$

where $\mathbf{v}$ is also a concatenated vector

$$\mathbf{v} = \frac{1}{N}(p(1|v_1), p(2|v_1), \cdots, p(T|v_1), p(1|v_2) \cdots, p(T|v_2), \cdots \cdots, p(1|v_N), p(2|v_N), \cdots, p(T|v_N))^T$$

$\mathbf{U}$ is a $NT \times NT$ sparse matrix, and thus we represent it as a block matrix by

$$\mathbf{U} = \begin{pmatrix} \mathbf{U}_{11} & \cdots & \mathbf{U}_{N1} \\ \vdots & \ddots & \vdots \\ \mathbf{U}_{1N} & \cdots & \mathbf{U}_{NN} \end{pmatrix}$$

Each entry $\mathbf{U}_{ij}$ is a $T \times T$ matrix

- If (v_j, v_i) is an *across-images* edge, $\mathbf{U}_{ij} = \mathrm{diag}(\kappa w_{v_j v_i}, \kappa w_{v_j v_i}, \ldots, \kappa w_{v_j v_i})$, namely a diagonal matrix with diagonal entries set by $\kappa w_{v_j v_i}$.
- If (v_j, v_i) is an *inside-the-same-image* edge which encodes the *part-of* relation, $\mathbf{U}_{ij}$ will be a $T \times T$ matrix with all entries $(1 - \kappa)\tau$ as follows:

$$\mathbf{U}_{ij} = (1 - \kappa)\begin{bmatrix} \tau & \cdots & \tau \\ \vdots & \ddots & \vdots \\ \tau & \cdots & \tau \end{bmatrix}$$

- If (v_j, v_i) is an *inside-the-same-image* edge which encodes the inter-topic spatial relation, then

$$\mathbf{U}_{ij} = \begin{bmatrix} g_{11} & \cdots & g_{T1} \\ \vdots & \ddots & \vdots \\ g_{1T} & \cdots & g_{TT} \end{bmatrix}$$

  where $g_{kl} = (1 - \kappa)c_{v_j v_i}(l, k)$

The formula (1) is a personalized pagerank problem. It can be resolved by any linear system and we use the power iteration to get the final convergent solution.