Supplemental Material for A New Variational Framework for Multiview Surface Reconstruction

Ben Semerjian

Urban Robotics, Inc., Portland, Oregon, USA

This short document lists and works out some of the supplemental material to accompany the ECCV 2014 paper titled "A New Variational Framework for Multiview Surface Reconstruction".

1 Gallery of reconstructions

The subdirectories of this folder contain image sets, with rendered depth images for one of each set, showcasing the performance of this surface reconstruction method for a variety of situations. Also included in each directory is a texture mapped Maya obj file; these are meshes viewable in many tools, *e.g.* Meshlab. The table below lists these directories, with some extra information.

Name	α	Time, s	Lens	Peculiarities
MtHood	1	764	Narrow	Large baseline, obliqueness, contrast
Downtown	0.05	1072	Normal	Small baseline, glare, depth range
Bridge	0.3	494	Narrow	Large baseline, clutter, contrast
Statue	0.4	223	Narrow	Small baseline, focal blur, contrast
Entrance	0.3	684	Narrow	Focal blur, contrast, depth range

The source images are all property of Urban Robotics, Inc.[1] (not for reuse). They have been compressed at 80% JPEG quality to save space, 90% quality was used for all reconstructions; furthermore, all are undistorted.

2 Surface resolving power derivation

In the paper, a quantity termed "surface resolving power" was introduced, an infinitesimal ratio of projected area (pixels^2) to physical area. One way to derive this will be presented here.

Consider a 3D point $\mathbf{X}$ lying on a piecewise differentiable surface with projection $\mathbf{u}$. The goal is to find a way to measure surface resolving power, in other words, a "pixels per area" measure. This may be understood as the reciprocal of the ratio of a small area element ΔA around $\mathbf{X}$ to its projected pixel area ΔQ. To simplify and make "small" precise, the limiting case will be used:

$$\text{Surface resolving power} = \lim_{\Delta A \to 0} \frac{\Delta Q}{\Delta A} = \frac{\mathrm{d}Q}{\mathrm{d}A} \,. \tag{1}$$

[1] www.urbanrobotics.net

To derive this quantity in terms of that which is measurable, consider the outward unit surface normal $\hat{\mathbf{n}}$ at $\mathbf{X}$, which may be expressed in various ways, the best of which depends on surface representation. Two orthogonal tangent vectors $\hat{\mathbf{t}}_1$ and $\hat{\mathbf{t}}_2$ can also be built, arbitrary due to a rotation around the normal.

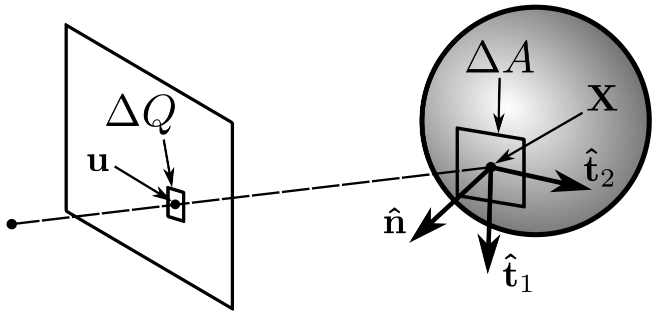

Fig. 1. The projection of a small surface element around $\mathbf{X}$ onto a view.

With this information, the area element may be expressed in terms of a side length Δs and the tangent vectors:

$$\Delta A = \Delta s^2 = |(\Delta s \hat{\mathbf{t}}_1) \times (\Delta s \hat{\mathbf{t}}_2)| \tag{2}$$

which reveals a corresponding pixel area element:

$$\Delta Q = (\Delta s \mathbf{q}_1) \times (\Delta s \mathbf{q}_2) \tag{3}$$

where the cross product of 2D vectors is understood as same but 3D with zero z components, yielding a single z value. $\mathbf{q}_i$ is the projection of a tangent vector:

$$\mathbf{q}_i = \mathbf{u}(\mathbf{X} + \Delta s \hat{\mathbf{t}}_i) - \mathbf{u}(\mathbf{X}) \tag{4}$$

where $\mathbf{u}(\mathbf{X})$ is the projection of $\mathbf{X}$, a function mapping $\mathbb{R}^3 \to \mathbb{R}^2$.

Now, if $\mathbf{u}(\mathbf{X})$ is differentiable, Taylor's theorem may be used to re-express a projected tangent:

$$\mathbf{q}_i = \Delta s \nabla \mathbf{u}(\mathbf{X}) \hat{\mathbf{t}}_i + \mathcal{O}(\Delta s^2)\ . \tag{5}$$

The ratio of the area elements is now fully defined. Taking the limit yields:

$$\frac{\mathrm{d}Q}{\mathrm{d}A} = (\nabla \mathbf{u} \hat{\mathbf{t}}_1) \times (\nabla \mathbf{u} \hat{\mathbf{t}}_2)\ . \tag{6}$$

This form is undesirable in that the tangent vectors appear, which take a little more work to evaluate than the normal due to their ambiguity. Expanding the cross product and rearranging yields the more useful form:

$$\frac{\mathrm{d}Q}{\mathrm{d}A} = -\hat{\mathbf{n}} \cdot \left(\nabla u \times \nabla v\right)\ . \tag{7}$$

Note that if this quantity is negative, the surface element is not viewable.