A Non-local Method for Robust Noisy Image Completion

Wei Li, Lei Zhao, Duanqing Xu, and Dongming Lu

Zhejiang University, Hangzhou, China

Abstract. The problem of noisy image completion refers to recovering an image from a random subset of its noisy intensities. In this paper, we propose a non-local patch-based algorithm to settle the noisy image completion problem following the methodology "grouping and collaboratively filtering". The target of "grouping" is to form patch matrices by matching and stacking similar image patches. And the "collaboratively filtering" is achieved by transforming the tasks of simultaneously estimating missing values and removing noises for the stacked patch matrices into low-rank matrix completion problems, which can be efficiently solved by minimizing the nuclear norm of the matrix with linear constraints. The final output is produced by synthesizing all the restored patches. To improve the robustness of our algorithm, we employ an efficient and accurate patch matching method with adaptations including pre-completion and outliers removal, etc. Experiments demonstrate that our approach achieves state-of-the-art performance for the noisy image completion problem in terms of both PSNR and subjective visual quality.

1 Introduction

In this paper, we aim at recovering an image from a subset of its noisy entries, which means that the image not only suffers from information loss, but also is corrupted by an amount of additive noise. Actually, this problem arises from various practical applications. For example, the task of simultaneously removing impulsive noise and mixed additive noise as described in [29] can be regarded as a practical case of the proposed problem. Since the pixels damaged by impulsive noise contain no information about the true image, it's quite natural to treat them as empty values and thus the task is indeed transformed into a noisy image completion problem. As another case, the problem is involved in the restoration of archived photographs and films [17][16][25][26]. As detailed in [16], archived materials may be degraded due to physical problems or simply chemical decomposition, which typically lead to noise, Dirt and Sparkle, etc. So there is also a necessity to develop techniques to deal with noise and missing data jointly. Besides, in certain difficult imaging situations, e.g., capturing images with a faulted camera or a low-end webcam under low light conditions, it is also a possibility that the captured data contains both missing and noisy intensities.

Compared with traditional image completion and denoising problems, noisy image completion problem is obviously more challenging since both completion

D. Fleet et al. (Eds.): ECCV 2014, Part IV, LNCS 8692, pp. 61–74, 2014.

and denoising techniques cannot handle the problem trivially. Numerical methods are the simplest tools to solve the proposed problem, e.g., total variation [23] and wavelet [6]. However, those methods generally produce coarse recovered results with poor visual quality and noticeable artifacts. Another way to solve the noisy completion problem is to combine image completion and denoising techniques in a straightforward way: one first estimates the missing entries in a corrupted image without considering the noise and then applys a denoising algorithm to the intermediate completed result. This "completion + denoising" scheme is able to produce visually pleasant results in some situations, but the disadvantage is also obvious: estimated values for empty pixels by no matter straightforward interpolation methods or state-of-the-art example-based inpainting approaches are not sufficiently reliable due to the existence of noise, then the generated error will not be further corrected by directly applying denoising techniques. The scheme works even worse when the ratio of the missing pixels increases, which will be demonstrated in experiments.

Recent studies about non-local and patch-based image processing techniques following the idea of "grouping and collaboratively filtering" [4,8,14] provide us a novel way to think about our problem. In general, this kind of methods builds upon a patch-wise restoration for the corrupted image, where three steps are commonly involved. Initially, for a given reference patch in the image, an amount of similar patches are matched and grouped by testing the similarity between the reference patch and the candidate patch located at different spatial position. Then, redundant information among the stacked patches is utilized to perform a restoration, e.g., estimating missing values and removing noises simultaneously. At last, the final restored image is synthesized from all the restored patches. This is also the sketch of our approach proposed in this paper. However, considering the specific noisy image completion problem here, we make two major adaptations within the framework. Firstly, an efficient and robust patch matching algorithm is developed to collect similar patches for each reference patch in the image. Secondly, for each patch group, a patch matrix is formed by stacking the patches and the repair of the matrix is transformed into a problem of low-rank matrix completion from noisy entries, which can be efficiently solved by minimizing the nuclear norm of the matrix with linear constraints [5,15].

Contributions. We propose a robust and accurate noisy image completion algorithm which fills the missing values and removes the noise in an damaged image simultaneously. Our algorithm follows the methodology of "grouping and collaboratively filtering" and combines several recent powerful tools including total variation (TV) [23], adapted PatchMatch [2], and low rank matrix completion with ADMM solver [19] in an effective manner. Experiments demonstrate that our algorithm produces results with better visual quality than existing techniques.

2 Related Works

There are many available methods that can handle the noisy image completion problem. As a straightforward method, the total variation (TV) based image processing method was first proposed in [23] as an efficient tool for denoising. And TV was further extended to deal with more applications like debluring, inpainting and super-resolution [27,30,9]. Actually, a common TV inpainting method can be easily modified to handle an incomplete image with noisy entries by relaxing the constraints of the observed values. In a similar way, wavelet filtering methods [6,10] can also be adapted to solve the noisy completion problem. In [17], a MCMC sampling-based approach was proposed for joint noise reduction and missing data treatment. In [1], a PDE-based method was proposed to settle the problem: after filling the missing pixels by solving a PDE, it then employed the Mean Curvature Flow and a selective diffusion equation to smooth out the noise for the pixels inside and outside the empty regions respectively. However, the above mentioned methods generally produce coarse recovered results with poor visual quality and noticeable artifacts.

The proposed approach in this paper is inspired by the recent progress of the non-local patch-based techniques, which demonstrated impressive capability in solving inpainting [28,7,18,24,22] and denoising [4,8,14,11,20,21] problems. To list a few, in patch-based inpainting algorithms such as [7,24], one first filled the regions by searching and copying proper content from the observed part of the image and then synthesized the result in a visually acceptable way. In [22], the authors extended the traditional quadratic regularization used for inverse problems to non-smooth energies by defining graph-based TV on images. The regularized formalization was utilized to solve inverse problems including image completion. As a pioneer of the non-local based denoising algorithm, the NL-Means algorithm [4] estimated the value of each pixel in an image as an average of the values of all the pixels whose Gaussian neighborhood looked like the neighborhood of the current pixel. The K-SVD algorithm [11,21] utilized highly overcomplete dictionaries obtained via a preliminary training procedure to exploit the 2-D transform sparsity in image patches for the purpose of patch-wise denoising. BM3D proposed in [8] stacked similar image patches in a 3D array based on the similarity between patches and then applied a shrinkage operator in 3D transform domain on the 3D array. The denoised image is synthesized from denoised patches after inverse 3D transform.

3 The Proposed Algorithm

3.1 Overview

Let $I \in \mathbb{R}^{m\times n}$ be a damaged image which is represented as:

$$I = (G + N)_{\Omega}, \tag{1}$$

where $G \in \mathbb{R}^{m\times n}$ is the original image, N is an additive noise and Ω indexes a random subset of pixels which are observed. How to accurately recover original image G from its noisy and incomplete observation I is the main target.

Following the methodology of "grouping and collaboratively filtering", we propose a non-local and patch-based approach for the noisy image completion problem, where "grouping" and "collaboratively filtering" are the two major components. The purpose of "grouping" is to exploit the spatial redundancy in the corrupted image by matching and staking similar patches. Initially, let $p_{i,j}$ be an image patch in I of size $s \times s$ centered at pixel (i, j), and every pixel in the image corresponds to such a patch except for those whose $s \times s$ surrounding regions exceed the image bound. Then given a patch $p_{i,j}$, we search for several similar patches from all the candidates in terms of ℓ_2 distance and put all the matched patches into a patch group $\{p^0_{i,j}, p^1_{i,j}, \ldots, p^k_{i,j}\}$. Here $p^0_{i,j}$ refers to the given patch, or reference patch. By concatenating all columns of each patch in the group into a long vector and stacking all the vectors, a patch matrix $P_{i,j} \in \mathbb{R}^{s^2 \times (k+1)}$ is formed as:

$$P_{i,j} = (\mathbf{p}^0_{i,j}, \mathbf{p}^1_{i,j}, \ldots, \mathbf{p}^k_{i,j}). \tag{2}$$

Corresponding to Equ.(1), $P_{i,j}$ can be also represented as the following equation:

$$P_{i,j} = (Q_{i,j} + E_{i,j})_{\Omega_{i,j}}, \tag{3}$$

where $Q_{i,j}$ denote the patch matrix from the underlying groundtruth G, $E_{i,j}$ is the noise and $\Omega_{i,j}$ indexes the given entries in the patch matrix. Thus recovering G from its incomplete and noisy observation I equals to firstly decomposing Q from P for all of the patch matrices and then combining the results. This is what the "collaboratively filtering" does.

Assuming that I is complete and free of noise ($I = G$, $P_{i,j} = Q_{i,j}$) and the patch matching is still perfect, the grouped patches should exhibit high mutual similarity. Thus $Q_{i,j}$ should be low-rank and $E_{i,j} = 0$. Then in the existence of noise and missing values, if the patch matching is still perfect, $Q_{i,j}$ will also be low-rank and E will contain the noise. Therefore, calculating a low-rank matrix $Q_{i,j}$ from the noisy incomplete patch matrix $P_{i,j}$ is actually a problem of low-rank matrix completion from noisy entries [5,15].

However, as we only have the noisy and incomplete pixels, how to perfectly perform patch matching is a difficult problem. In our approach, several measures are taken to ensure the robustness of patch matching. First, we complete the empty pixels without considering the noise using TV [23]. Then an improved PatchMatch [2] algorithm is applied to the pre-completion result to efficiently search for similar patches for each reference patch. The patch matrix $P_{i,j}$ is formed by utilizing the patch matching result but taking values from the corrupted image I. A visualized description of the procedure is shown in Fig.1. Furthermore, in the formed patch matrix $P_{i,j}$, elements far away from the average of its corresponding row vector are considered as highly unreliable elements to be discarded. Details will be stated in the following sections.

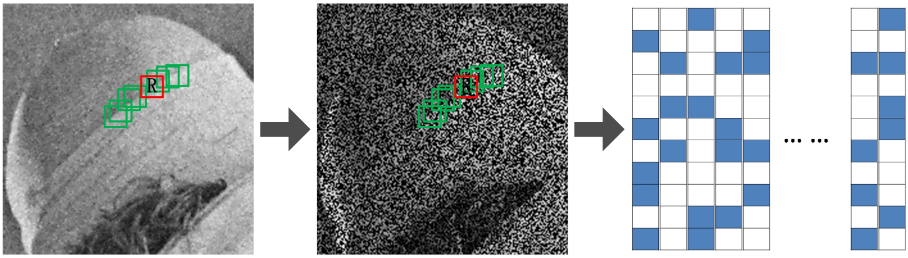

Fig. 1. The forming of a patch matrix. The patch matching is executed on the pre-completion result. Then a patch matrix is formed by taking values from the corrupted image according to the matching result. Reference patch (red box with a 'R') and matched patches (green boxes) are drawn in the two images. For all the patches including the reference patch, by concatenating all of its columns into a long vector and staking all the vectors, we get the right patch matrix. Each column in the patch matrix corresponds to a columns-concatenated patch. And in each column, we represent the missing pixels as the blue grids.

3.2 Robust Patch Matching and Grouping

In this section, we describe how we perform the efficient and robust patch matching and grouping process to search and stack similar patches for each reference patch, where three stages are involved.

Pre-completion. Before we really execute patch matching, we should firstly give proper values to the empty pixels. This is even more necessary in working over highly incomplete data because the captured entries may be insufficient to provide a reliable computation of the similarity between patches. Here we adopt a TV-based image completion algorithm for the pre-completion process which is described as the following equation:

$$\begin{aligned} &\min_{X} \ \left\{TV(X) = \sum_{i,j} \left(|D^h X_{i,j}| + |D^v X_{i,j}|\right)\right\} \\ &s.t. \ \ X_\Omega = I_\Omega . \end{aligned} \tag{4}$$

where $D^h X_{i,j}$ and $D^v X_{i,j}$ are the horizontal and vertical components of the gradient of element $X_{i,j}$ respectively. For colored images, vectorial total variation (VTV) [12] is a better choice than applying TV in a channel-by-channel manner, which is defined as:

$$VTV(X) = \sum_{i,j,k} \left(|D^h X_{i,j,k}| + |D^v X_{i,j,k}|\right), \tag{5}$$

where k represent channels. We refer interesting readers to [27,30,9,13,3] for details and efficient solvers for TV and VTV.

Actually, as we will further employ an improved PatchMatch method and an outlier remover in this robust patch matching stage, excessive computation involved in estimating the missing entries is unnecessary. Therefore, we adopt TV

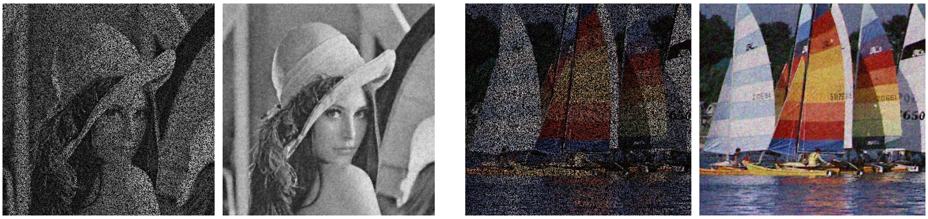

Fig. 2. Two examples of the pre-completion process. The left gray-scale image is completed using TV formulation while the right colored image is processed using VTV.

in the pre-completion mainly for its computational efficiency and that the primary completed result is also accurate enough for the following patch matching process. Fig.2 gives two examples of the pre-completion, where the gray-scale image and the colored image are processed with TV and VTV respectively.

Improved PatchMatch. PatchMatch proposed in [2] is an efficient nearest neighbour searching algorithm, which is employed in our approach for the patch matching and grouping purpose. Applying the PatchMatch algorithm to the pre-completion result will help us find out a given number of similar patches in terms of ℓ_2 distance for all the reference patches. Then for each reference patch, all the matched patches are grouped by considering the reference patch as some sort of "centroid" for the group. Due to the space limitation, the PatchMatch algorithm is not detailed here, and we mainly discuss the two improvements we make for our specific application. At first, as there is not a built-in distance restriction in the PatchMatch algorithm, "diameter" of each group (the largest ℓ_2 distance between any two patches in the group) may be too large that may degrade the following collaborative filtering process. Therefore, the matched patches in each group should be pruned by setting a threshold for the distance between any two patches. Secondly, despite the existence of noise, original captured values are obviously more reliable than the estimated ones in the pre-completion result, thus higher importance should be attached to the observed entries in calculating similarities between two patches. Therefore, the ℓ_2 distance used in PatchMatch is indeed a weighted ℓ_2 distance. Specifically, let p_A and p_B be the two patches from a pre-completion result, and $\omega(\cdot) : \mathbb{R}^{s \times s} \to \mathbb{R}^{s \times s}$ to be a function that reads the weights of every elements in the patch, then the weighted ℓ_2 distance we use in our implementation is:

$$\mathrm{Dist}(p_A, p_B) = \|(p_A - p_B) \odot \omega(p_A) \odot \omega(p_B)\|_2,$$

where the symbol $\odot$ denotes the element-wise multiplication of two equal-sized matrices. The weight for the observed pixels must be no smaller than that of the missing ones, while the specific values should be adapted to the specific noise level and missing rate.

Outliers Removal. Since a low-rank matrix can be recovered from only a small fraction of its entries as proved in [5,15], we only keep those elements of high reliability and discard all the other elements. So detecting and discarding the outliers before solving the low-rank matrix completion problem is necessary. For each row in the formed patch matrix as described in Equ.(2), elements which deviate away from the mean value of the row vector by an amount larger than a pre-defined threshold will be treated as outliers to be discarded. Therefore, in the following low-rank matrix completion problem, the empty elements consist of two sources, one corresponds to the original missing pixels and the other indexes the abandoned outliers.

3.3 Collaborative Filtering Using Low-Rank Matrix Completion

For each grouping formed in the previous stage, a corresponding patch matrix is constructed by concatenating all columns of each patch into a long vector and staking all the vectors, which is a noisy and incomplete version of the underlying original patch matrix. As is discussed in **Section 3.1**, our goal here is to extract the matrix Q with a low rank from P. Mathematically, the task is generalized as the problem of low-rank matrix completion with noisy entries [5,15] in the following form:

$$\begin{aligned} &\min_{Q} \|Q\|_n \\ &s.t.\ \|(Q-P)_\Omega\|_F \le \delta, \end{aligned} \tag{6}$$

where $\delta \geq 0$ measures the noise level, $\|\cdot\|_n$ is the nuclear norm defined as the summation of all singular values which is shown to be the tightest convex approximation for the rank of matrices, and $\|\cdot\|_F$ denotes the Frobenious norm. Ω indexes all the given elements as discussed in previous sections.

We adopt alternating direction method of multipliers (ADMM) [19] in our approach for its implementation simplicity and computational efficiency. Initially, we reformulate Equ.(6) into the following nuclear-norm-regularized least squares problem which is the ADMM applicable form:

$$\begin{aligned} \min_{Q}\ &\|Q\|_n \\ s.t.\ &Q-T=0, \\ &T \in \mathbb{U} = \{\|(U-P)_\Omega\|_F \le \delta\}. \end{aligned} \tag{7}$$

Then the ADMM works by minimizing the augmented Lagrangian function of the above problem:

$$\mathcal{L}(Q,T,Z,\beta) = \|Q\|_n - \langle Z, Q-T\rangle + \frac{\beta}{2}\|Q-T\|_F^2, \tag{8}$$

with respect to the unknown variables Q, T one at a time. Here Z is the Lagrange multiplier of the linear constraint, $\beta > 0$ is the penalty parameter for the violation of the linear constraint and $\langle\cdot,\cdot\rangle$ denotes the standard trace inner produce. The ADMM iteratively updates all the variables as follows:

$$T^{k+1} = arg \min_{T\in\mathbb{U}} \mathcal{L}(Q^k, T, Z^k, \beta), \tag{9}$$

$$Q^{k+1} = arg\min_{X} \mathcal{L}(Q, T^{k+1}, Z^k, \beta), \tag{10}$$

$$Z^{k+1} = Z^k - \gamma\beta(Q^{k+1} - T^{k+1}), \tag{11}$$

where $\gamma \in (0, \frac{\sqrt{5}+1}{2})$ is a constant. In this iterative scheme, the computation of each iteration is dominated by solving the two subproblems Equ.(9) and Equ.(10). At first, the solution for the subproblem Equ.(9) is given by:

$$T^{k+1} = \left(\min\left\{ \frac{\delta}{\|(B^{k+1} - P)_\Omega\|_F}, 1 \right\} - 1 \right)(B^{k+1} - P)_\Omega + B^{k+1}, \tag{12}$$

where $B^{k+1} = Q^k - \frac{1}{\beta} Z^k$.

Secondly, let

$$A^{k+1} = T^{k+1} + \frac{1}{\beta} Z^k = U^{k+1} \Sigma^{k+1} (V^{k+1})^T, \tag{13}$$

be the singular value decomposition (SVD) for A^{k+1}, then we get the closed-form solution for subproblem Equ.(10):

$$Q^{k+1} = U^{k+1} \hat{\Sigma}^{k+1} (V^{k+1})^T, \tag{14}$$

where $\hat{\Sigma}^{k+1} = diag(\max(\sigma_i^{k+1} - \frac{1}{\beta}, 0))$ with $\sigma_i(X)$ denoting the i-th largest singular value.

Synthesizing the Final Output. With all the restored patch matrices, the generation of the final output is straightforward. Firstly, the recovered result of the reference patch in each patch matrix is attached to the corresponding position in the result image. Thus each pixel is covered by several patches with overlapping regions. Then, the final value for the pixel is computed as the average of all the covered patches at this position.

4 Implementation Details and Experiments

In this section, we first give implementation details and then show the advantage of our algorithm over three existing techniques for the noisy image completion problem.

4.1 Implementation Details

Firstly, the famous SplitBregman algorithm proposed in [13] is employed for solving the TV and VTV pre-completion problems. Secondly, in the improved PatchMatch, the size of each patch is set as 7×7 and reference patches are sampled with 3×3 interval. For each reference patch, 20 nearest neighbours are

Table 1. The PSNR values of the recovered results with respect to different missing rates (MR) and noise levels (σ). A simple observation is made that our approach is sensitive to the Gaussian noise but robust to the missing values.

PSNR / σ \ MR	20%	30%	40%	50%
15	35.32	34.66	34.10	33.15
25	34.24	33.80	33.17	32.22
40	32.13	31.59	31.06	30.65
50	30.67	29.84	29.64	28.79

collected in terms of weighted ℓ_2 distance as described in **Section 3.2**. Then, the threshold used in detecting outlier pixels from the patch matrix is chosen to be $2\bar{\sigma}$, where $\bar{\sigma}$ is defined as the mean of the standard deviation of the given entries in each row vector. Besides, the noise level δ in the problem of noisy low-rank matrix completion described in Equ.(6) is evaluated as $\delta = \bar{\sigma}\|\Omega\|_0$, where $\|\cdot\|_0$ counts the amount of the observed entries indexed by Ω. And the ADMM solver for the low-rank matrix completion problem is terminated if the tolerance $RSE < 1 \times 10^{-5}$ is met.

4.2 Results and Comparisons

As stated before, the problem of noisy image completion arises from several applications, so images to be restored in the experiments are artificially trimmed-degraded images rather than real damaged data for the universality. For the same consideration, without loss of generality, the Gaussian noise is mainly tested here. Two set of experiments are conducted in this section: one is to demonstrate how our approach performs with respect to different noise levels and missing rates, the other is to show the advantage of our algorithm compared with other methods for the application. Accuracy of the recovered results is evaluated by PSNR.

Robustness Tests. Table 1 exhibits the performance of our method in dealing with input data of different noise levels and missing rates: the standard deviation σ of the Gaussian noise varies from 15 to 50 while the missing rate (MR) changes between 20% and 50%. It can be easily read from each column in the table that for a certain missing rate, the PSNR drops dramatically when σ increases. On the contrary, when we keep the noise level unchanged, the recovery quality is more stable as is shown in each row of Table 1. This merit is easily comprehended: the estimated value in the pre-completion result is only involved in the patch matching process but not used in collaborative filtering stage, thus the original missing entries can be exactly re-computed by low-rank matrix completion.

Comparisons. In the comparison experiments, we compare our algorithm with three methods, which are mainly derived from state-of-the-art approaches in the

Fig. 3. From left to right: corrupted images, restored results of TV-based method, NL-Means, BM3D and our approach. Our approach outperforms the others for all the situations especially for the examples with high missing rates.

field of image denoising. The first method is similar to the TV pre-completion stage but with soft constraints, which equals to solving the following function:

$$\min_{X} \; TV(X) + \frac{\mu}{2}\|X_{\Omega} - I_{\Omega}\|_F^2, \tag{15}$$

where μ is a constant denoting the regularization parameter. The above equation is easily solvable using the gradient decent method. The other two approaches to be compared with are built upon the scheme of "completion + denoising", which means that we first fill the missing pixels using TV or VTV without considering the noise, which is the same to our pre-completion step, and then apply denoising methods to the intermediate result. The denoising methods adopted here are the former mentioned NL-Means [4] and BM3D [8].

Fig.3 shows the performance of our algorithm and three other methods in dealing with inputs with different missing rates and noise levels. In general, the primary TV-based method performs poorest among all the involved algorithms, which can only provide coarse outputs with noticeable visual artifacts. And

(a) Original (b) Corrupted (c) NL-Means (d) BM3D (e) Ours

Fig. 4. Details are better recovered by our approach than NL-Means and BM3D. And the PSNR values of the three results (c), (d) and (e) are, 24.28dB, 25.04dB and 27.25dB respectively.

Corrupted Original NL-Means BM3D Ours

Fig. 5. The close-ups exhibit the extraordinary capability of our approach in recovering regular structures in a corrupted image

compared with the two "completion + denoising" methods using NL-Means and BM3D, our approach generates the best recovered images in terms of both PSNR and visual quality in all of the examples. Specifically, when we keep the missing rate of the input data fixed, all the methods are sensitive to the changes of standard deviation σ of the Gaussian noise, but our approach obviously has better capability of removing the noise. Things are different when we maintain σ but change the missing rate. As stated before, our algorithm is robust to the existence of missing pixels, so it can be seen from the visualized results in Fig.3 that the final synthesized images are not degraded noticeably when the missing rate increases. On the contrary, in the two methods based on NL-Means and BM3D, since denoising operation is directly applied to the completed result which is indeed not sufficiently reliable, errors produced in estimating the missing values won't be further corrected. As a consequence, visual artifacts are clearly observed in the final repaired results as displayed in Fig.3 especially the two examples with missing rate 50%.

Fig. 6. From top to bottom: original images, corrupted images, pre-completion results and the recovered results of our approach. The damaged images here have a higher missing rate of 60% and $\sigma = 25$. The PSNR values of the recovered results in (a), (b), (c) and (d) are 28.37dB, 30.35dB, 28.64dB and 30.62dB respectively.

Another two comparison examples with close-ups are displayed in Fig.4 and Fig.5, where the original images have clear and repetitive structures. Here we leave out the TV-based method due to its poor performance in processing such sources. In both of the inputs, the missing rate is 50% and $\sigma = 25$. It can be clearly seen from the close-ups in the two figures that regular structures in the images are well reconstructed by our approach while the results of NL-Means and BM3D are quite unpleasant. In Fig.6, we further increase the missing rate of the images to 60% and apply our approach to these data. The visually pleasant results exhibit the brilliant applicability of our algorithm in working over such severely corrupted data. Please refer to the supplementary material for more results.

In summary, we argue that our algorithm has excellent ability of repairing noisy incomplete images. And compared with existing techniques, our approach works much better in processing examples with high missing rate and in recovering structures in images.

5 Conclusions and Future Works

In this paper, we propose a novel robust algorithm following the scheme "grouping and collaboratively filtering" for the problem of noisy image completion. There are two key steps in our approach: firstly, an efficient and robust patch matching method is developed for matching and grouping patches; secondly, the problem of estimating missing values and denoising is transformed to the problem of low-rank matrix completion from noisy entries. Experiments exhibit the extraordinary performance of our approach compared with existing techniques and recovered results will benefit much from multiple circulation of the process. There are also limitations. First, noticeable visual artifacts may exist in working over corrupted images with large missing regions, e.g., wrong completed content or overly smoothed regions. Second, like many other methods of this category, our algorithm also imposes high computational burden. Another limitation comes from the averaging in the last step which obviously blurs the result. So settling the remain problems is our future work.

Acknowledgements. This work was supported in part by the National projects: 2012BAH03F02, 2012BAH43F02, and 2012BAH43F05, the National Basic Research Program (973) project: 2012CB725305, and the Zhejiang Provincial projects: 2012C11010, 2010R50040, and 2012BAH03F03.

References

1. Barcelos, C.A.Z., Batista, M.A.: Image restoration using digital inpainting and noise removal. Image and Vision Computing 25(1), 61–69 (2007)
2. Barnes, C., Shechtman, E., Finkelstein, A., Goldman, D.B.: Patchmatch: a randomized correspondence algorithm for structural image editing. In: SIGGRAPH (2009)
3. Beck, A., Teboulle, M.: Fast gradient-based algorithms for constrained total variation image denoising and deblurring problems. IEEE TIP 18(11), 2419–2434 (2009)
4. Buades, A., Coll, B., Morel, J.M.: A review of image denoising algorithms, with a new one. Multiscale Modeling and Simulation 4(2), 490–530 (2005)
5. Candès, E.J., Plan, Y.: Matrix completion with noise. Proceedings of the IEEE 98(6), 925–936 (2010)
6. Coifman, R.R., Donoho, D.L.: Translation-invariant de-noising. Springer (1995)
7. Criminisi, A., Pérez, P., Toyama, K.: Region filling and object removal by exemplar-based image inpainting. IEEE TIP 13(9), 1200–1212 (2004)
8. Dabov, K., Foi, A., Katkovnik, V., Egiazarian, K.: Image denoising by sparse 3-D transform-domain collaborative filtering. IEEE TIP 16(8), 2080–2095 (2007)
9. Dahl, J., Hansen, P.C., Jensen, S.H., Jensen, T.L.: Algorithms and software for total variation image reconstruction via first-order methods. Numerical Algorithms 53(1), 67–92 (2010)
10. Donoho, D.L., Johnstone, J.M.: Ideal spatial adaptation by wavelet shrinkage. Biometrika 81(3), 425–455 (1994)
11. Elad, M., Aharon, M.: Image denoising via sparse and redundant representations over learned dictionaries. IEEE TIP 15(12), 3736–3745 (2006)

12. Goldluecke, B., Strekalovskiy, E., Cremers, D.: The natural vectorial total variation which arises from geometric measure theory. SIAM Journal on Imaging Sciences 5(2), 537–563 (2012)
13. Goldstein, T., Osher, S.: The split bregman method for l1-regularized problems. SIAM Journal on Imaging Sciences 2(2), 323–343 (2009)
14. Ji, H., Liu, C., Shen, Z., Xu, Y.: Robust video denoising using low rank matrix completion. In: CVPR (2010)
15. Keshavan, R.H., Montanari, A., Oh, S.: Matrix completion from noisy entries. Journal of Machine Learning Research 11(2057-2078), 1 (2010)
16. Kokaram, A.C.: On missing data treatment for degraded video and film archives: a survey and a new bayesian approach. IEEE TIP 13(3), 397–415 (2004)
17. Kokaram, A.C., Godsill, S.J.: Mcmc for joint noise reduction and missing data treatment in degraded video. IEEE TSP 50(2), 189–205 (2002)
18. Komodakis, N., Tziritas, G.: Image completion using efficient belief propagation via priority scheduling and dynamic pruning. IEEE TIP 16(11), 2649–2661 (2007)
19. Lin, Z., Chen, M., Ma, Y.: The augmented lagrange multiplier method for exact recovery of corrupted low-rank matrices. UIUC Technical Report UILUENG-09-2215 (2010)
20. Mairal, J., Bach, F., Ponce, J., Sapiro, G., Zisserman, A.: Non-local sparse models for image restoration. In: ICCV (2009)
21. Mairal, J., Elad, M., Sapiro, G.: Sparse representation for color image restoration. IEEE TIP 17(1), 53–69 (2008)
22. Peyré, G., Bougleux, S., Cohen, L.: Non-local regularization of inverse problems. In: Forsyth, D., Torr, P., Zisserman, A. (eds.) ECCV 2008, Part III. LNCS, vol. 5304, pp. 57–68. Springer, Heidelberg (2008)
23. Rudin, L.I., Osher, S., Fatemi, E.: Nonlinear total variation based noise removal algorithms. Physica D: Nonlinear Phenomena 60(1), 259–268 (1992)
24. Simakov, D., Caspi, Y., Shechtman, E., Irani, M.: Summarizing visual data using bidirectional similarity. In: CVPR (2008)
25. Subrahmanyam, G., Aravind, R.A., Recursive, R.: framework for joint inpainting and de-noising of photographic films. JOSA A 27(5), 1091–1099 (2010)
26. Wang, X., Mirmehdi, M.: Archive film restoration based on spatiotemporal random walks. In: Daniilidis, K., Maragos, P., Paragios, N. (eds.) ECCV 2010, Part V. LNCS, vol. 6315, pp. 478–491. Springer, Heidelberg (2010)
27. Wang, Y., Yang, J., Yin, W., Zhang, Y.: A new alternating minimization algorithm for total variation image reconstruction. SIAM Journal on Imaging Sciences 1(3), 248–272 (2008)
28. Wei, L.Y., Levoy, M.: Fast texture synthesis using tree-structured vector quantization. In: SIGGRAPH (2000)
29. Yan, M.: Restoration of images corrupted by impulse noise and mixed gaussian impulse noise using blind inpainting. SIAM Journal on Imaging Sciences 6(3), 1227–1245 (2013)
30. Zhu, M., Wright, S.J., Chan, T.F.: Duality-based algorithms for total-variation-regularized image restoration. Computational Optimization and Applications 47(3), 377–400 (2010)