$\mathcal{ALC}$(F): a new description logic for spatial reasoning in images

Céline Hudelot[1], Jamal Atif[2], and Isabelle Bloch[3]

1. Ecole Centrale de Paris, 2. Laboratoire de Recherche en Informatique, Université Paris-Sud, 3. Télécom ParisTech, CNRS LTCI - France - email:celine.hudelot@ecp.fr, jamal.atif@gmail.com, isabelle.bloch@telecom-paristech.fr

Abstract. In image interpretation and computer vision, spatial relations between objects and spatial reasoning are of prime importance for recognition and interpretation tasks. Quantitative representations of spatial knowledge have been proposed in the literature. In the Artificial Intelligence community, logical formalisms such as ontologies have also been proposed for spatial knowledge representation and reasoning, and a challenging and open problem consists in bridging the gap between these ontological representations and the quantitative ones used in image interpretation. In this paper, we propose a new description logic, named $\mathcal{ALC}(\mathbf{F})$, dedicated to spatial reasoning for image understanding. Our logic relies on the family of description logics equipped with concrete domains, a widely accepted way to integrate quantitative and qualitative qualities of real world objects in the conceptual domain, in which we have integrated mathematical morphological operators as predicates. Merging description logic with mathematical morphology enables us to provide new mechanisms to derive useful concrete representations of spatial concepts and new qualitative and quantitative spatial reasoning tools. It also enables imprecision and uncertainty of spatial knowledge to be taken into account through the fuzzy representation of spatial relations. We illustrate the benefits of our formalism on a model-guided cerebral image interpretation task.

Keywords: Spatial Reasoning; Ontology-based Image Understanding; Description Logics

1 Introduction

In image interpretation and computer vision, spatial relations between objects and spatial reasoning are of prime importance for recognition and interpretation tasks [5, 6], in particular when the objects are embedded in a complex environment. Indeed, spatial relations allow solving ambiguity between objects having a similar appearance, and they are often more stable than characteristics of the objects themselves. This is typically the case of anatomical structures, as illustrated in Figure 1, where some structures, such as the internal grey nuclei (thalamus, putamen, caudate nuclei), may have similar grey levels and similar shapes, and can be therefore easier distinguished for their individual recognition using spatial relations [14, 34]. Spatial relations also allow improving object and scene recognition in images such as photographs [30, 18], or satellite images [2, 22, 27, 36].

Spatial reasoning can be defined as the domain of spatial knowledge representation, in particular spatial relations between spatial entities, and of reasoning on these entities and relations. This field has been largely developed in artificial intelligence, in particular using qualitative representations based on logical formalisms [1, 37]. In image interpretation and computer vision, it is much less developed and is mainly based on quantitative representations [23, 9]. Bridging the gap between the qualitative representations and the quantitative ones is a challenging and open issue to make them operational for image interpretation.

Description logics (DL) equipped with concrete domains [28] are a widely accepted way to integrate *concrete and quantitative qualities* of real world objects with conceptual knowledge and as a consequence to combine qualitative and quantitative reasoning useful for real-world applications. In this paper, we propose a new description logic, named $\mathcal{ALC}(\mathbf{F})$, dedicated to spatial reasoning for image understanding. In this framework, the combination of a description logic with concrete domains and mathematical morphology provides new mechanisms to derive useful concrete representations of concepts and new reasoning tools, as demonstrated in [20, 21]. This paper builds upon these works by studying in depth the formal properties of this framework and revisiting the tableau decision algorithm. This framework also enables us to take into account imprecision to model vagueness, inherent to many spatial relations and to gain in robustness in the representations [9]. The rest of this paper is organized as follows. In Section 2, we review some related work and we recall how mathematical morphology can be used to derive fuzzy representations of spatial relations. In Section 3, we briefly present the main concepts of a spatial relation ontology used to represent spatial knowledge. We describe our new logic and its properties in Section 4. The reasoning and inference components are detailed in Section 5, and we illustrate the benefits of this framework for image interpretation tasks in Section 6, with the example of brain structure recognition in 3D images.

2 Spatial knowledge representations

As mentioned in Section 1, spatial relations between objects of a scene are of prime importance for semantic scene understanding. Several models for representing spatial relations have been proposed in the literature. These models can be classified according to different viewpoints:

- The nature of the model: quantitative or semi-quantitative models versus qualitative ones. In image interpretation and computer vision, many quantitative or semi-quantitative representations have been proposed. Many of them assimilate objects to basic entities such as centroid or bounding box [23] and others are based on the notion of histograms [31, 29]. On the contrary, in the artificial intelligence field, many qualitative and ontological models have been proposed (for instance, see [13] for a review).

– The type of the spatial relations: many authors have stressed the importance of topological relations and have proposed models for them [32, 15] but distances and directional relative position [9, 24] are also important, as well as more complex relations such as “between”, “surround” or “along” for instance.
– Their ability to model some important characteristics of spatial knowledge and in particular its imprecision [9].

The choice of a representation also depends on the type of question raised and the type of reasoning one wants to perform [10]: (1) which is the region of space where a relation with respect to a reference object is satisfied ? (2) to which degree is a relation between two objects satisfied?

In the following, we briefly present some fuzzy models of spatial relations using mathematical morphology on which we build our logic.

We denote by $\mathcal{S}$ the spatial (image) domain, and by $\mathcal{F}$ the set of fuzzy sets defined over $\mathcal{S}$, defined via their membership functions, associating with each point of space a membership value in $[0, 1]$. The usual partial ordering on fuzzy sets is used, denoted by $\leq_{\mathcal{F}}$, and the associated infimum $\wedge$ and supremum $\vee$. The empty set is denoted by $\emptyset_{\mathcal{F}}$ and the fuzzy set with membership value equal to 1 everywhere by $1_{\mathcal{F}}$. For a t-norm t and its residual implication I, $(\mathcal{F}, \leq_{\mathcal{F}}, \wedge, \vee, \emptyset_{\mathcal{F}}, 1_{\mathcal{F}}, t, I)$ is a residuated lattice of fuzzy sets defined over the image space by $\mathcal{S}$.

As shown in [10] and the references therein, mathematical morphology is a powerful tool to model spatial relations in various settings (sets, fuzzy sets, propositional logics, modal logics...). In the fuzzy set setting, the two main morphological operators, dilation δ and erosion ε, are defined from a t-norm t and its residual implication I as [12]:

$$\forall x \in \mathcal{S}, \delta_{\nu}(\mu)(x) = \vee_{y \in \mathcal{S}} t(\nu(x - y), \mu(x)), \tag{1}$$

$$\forall x \in \mathcal{S}, \varepsilon_{\nu}(\mu)(x) = \wedge_{y \in \mathcal{S}} I(\nu(y - x), \mu(x)). \tag{2}$$

The idea for mathematical morphology based spatial reasoning is to define the semantics of a spatial relation by a fuzzy structuring element ν in the spatial domain, and to use morphological operations to compute the region of space where the relation is satisfied with respect to a reference object. For instance, if ν represents the relation “right of”, then $\delta_{\nu}(\mu)(x)$ represents the degree to which x is to the right of the fuzzy set μ (an example is illustrated in Figure 2). This allows answering the first question above. As for the second question, histogram based approaches can be adopted [31], or pattern matching approaches, applied to the previous result and the fuzzy set representing the second object. A review of fuzzy spatial relations can be found in [9].

3 An ontology of spatial relations

The semantic interpretation of images can benefit from representations of useful concepts and the links between them as ontologies. We build on the work of [19] which proposes an ontology of spatial relations with the aim of guiding image interpretation using spatial knowledge. We briefly recall the main concepts of this ontology using description logics (DLs) as a formal language and we rely on the standard notations of DLs (see [3] for an introduction).

One important entity of this ontology, as proposed in [19], is the concept ***SpatialObject*** ($\mathsf{SpatialObject} \sqsubseteq \top$). As mentioned in [26], the nature of spatial relations is twofold: they are concepts with their own properties but they are also links between concepts and thus an important issue is related to the choice of modeling spatial relations as concepts or as roles in DLs. In [19], a spatial relation is not considered as a role (property) between two spatial objects but as a concept on its own (***SpatialRelation***), enabling to address the two spatial reasoning questions mentioned in Section 2.

- A ***SpatialRelation*** is subsumed by the general concept ***Relation***. It is defined according to a ***ReferenceSystem***:
  $\mathsf{SpatialRelation} \sqsubseteq$
  $\mathsf{Relation} \sqcap \ni \mathsf{type}.\{\mathsf{Spatial}\} \sqcap \exists\ \mathsf{hasReferenceSystem}.\mathsf{ReferenceSystem}$
- The concept ***SpatialRelationWith*** refers to the set of spatial relations which are defined according to at least one or more reference spatial objects RO (hasRO):
  $\mathsf{SpatialRelationWith} \equiv$
  $\mathsf{SpatialRelation} \sqcap \exists\ \mathsf{hasRO}.\mathsf{SpatialObject} \sqcap \geq 1\ \mathsf{hasRO}$
- We define the concept ***SpatiallyRelatedObject*** which refers to the set of spatial objects which have at least one spatial relation (hasSR) with another spatial object. This concept is useful to describe spatial configurations:
  $\mathsf{SpatiallyRelatedObject} \equiv$
  $\mathsf{SpatialObject} \sqcap \exists\ \mathsf{hasSR}.\mathsf{SpatialRelationWith} \sqcap \geq 1\ \mathsf{hasSR}$
- At last, the concept ***DefinedSpatialRelation*** represents the set of spatial relations for which target ($\mathsf{hasTargetObject}$) and reference objects (hasRO) are defined:
  $\mathsf{DefinedSpatialRelation} \equiv$
  $\mathsf{SpatialRelation} \sqcap \exists\ \mathsf{hasRO}.\mathsf{SpatialObject} \sqcap \geq 1\ \mathsf{hasRO} \sqcap$
  $\exists\ \mathsf{hasTargetObject}.\mathsf{SpatialObject} \sqcap = 1\ \mathsf{hasTargetObject}$

4 Proposed logic for spatial reasoning: $\mathcal{ALC}(\mathbf{F})$

In this section, we introduce mathematical morphology as a spatial reasoning tool. In particular, mathematical morphology operators are integrated as predicates of a spatial concrete domain. The main objective is to provide a foundation to reason about qualitative and quantitative spatial relations. The proposed logic is built on $\mathcal{ALCRP}(D)$ [16, 17] with the spatial concrete domain $\mathbf{F}$. We name it $\mathcal{ALC}(\mathbf{F})$ in the rest of the paper.

4.1 $\mathcal{ALC}(\mathbf{F})$ - Syntax and semantics

Definition 1 (Spatial concrete domain). *A spatial concrete domain is a pair* $\mathbf{F} = (\Delta_{\mathbf{F}}, \Phi_{\mathbf{F}})$ *where* $\Delta_{\mathbf{F}} = (\mathcal{F}, \leq_{\mathcal{F}}, \wedge, \vee, \emptyset_{\mathcal{F}}, 1_{\mathcal{F}}, t, I)$ *is a residuated lattice* *of fuzzy sets defined over the image space* $\mathcal{S}$, $\mathcal{S}$ *being typically* $\mathbb{Z}^2$ *or* $\mathbb{Z}^3$ *for 2D or 3D images, with* t *a t-norm (fuzzy intersection) and* I *its residuated implication.* $\Phi_{\mathbf{F}}$ *denotes a set of predicate names on* $\Delta_{\mathbf{F}}$ *which contains:*

- *The unary predicates* $\bot_{\mathcal{S}}$ *and* $\top_{\mathcal{S}}$ *defined by* $\bot_{\mathcal{S}}^{\mathbf{F}} = \emptyset_{\mathcal{F}}$ *and* $\top_{\mathcal{S}}^{\mathbf{F}} = 1_{\mathcal{F}}$.
- *The name of the unary predicate* μ_X *defined by* $(\mu_X)^{\mathbf{F}} \in \mathcal{F}$ *(*$(\mu_X)^{\mathbf{F}} : \mathcal{S} \to [0,1]$*). The predicate associates to a spatial concept* X *a unique fuzzy set in the concrete domain* $\mathbf{F}$*. For each point* $x \in \mathcal{S}$, $\mu_X^{\mathbf{F}}(x)$ *represents the degree to which* x *belongs to the spatial representation of the object* X *in the spatial domain (the image in our illustrative example).*
- *The name of the unary predicate* ν_R *defined as* $\nu_R^{\mathbf{F}} \in \mathcal{F}$ *(*$\nu_R^{\mathbf{F}} : \mathcal{S} \to [0,1]$*). The predicate associates to a spatial relation* R*, the fuzzy structuring element* $\nu_R^{\mathbf{F}}$ *defined on* $\mathcal{S}$ *which represents the fuzzy relation* R *in the spatial domain.*
- *The name of the unary predicate* $\delta_{\nu_R}^{\mu_X}$*, defined by* $(\delta_{\nu_R}^{\mu_X})^{\mathbf{F}} = \delta_{\nu_R^{\mathbf{F}}}(\mu_X^{\mathbf{F}}) \in \mathcal{F}$*, with* δ *a fuzzy dilation defined as in Equation 1.*
- *The name of the unary predicate* $\varepsilon_{\nu_R}^{\mu_X}$*, defined as* $(\varepsilon_{\nu_R}^{\mu_X})^{\mathbf{F}} = \varepsilon_{\nu_R^{\mathbf{F}}}(\mu_X^{\mathbf{F}}) \in \mathcal{F}$*, with* ε *a fuzzy erosion defined as in Equation 2.*
- *The names of two binary predicates* $\sqcap_d, \sqcup_d$*:* $(\mu_{X_1} \sqcap_d \mu_{X_2})^{\mathbf{F}} = \mu_{X_1}^{\mathbf{F}} \wedge \mu_{X_2}^{\mathbf{F}}$ *and* $(\mu_{X_1} \sqcup_d \mu_{X_2})^{\mathbf{F}} = \mu_{X_1}^{\mathbf{F}} \vee \mu_{X_2}^{\mathbf{F}}$*, with* $\wedge$ *and* $\vee$ *the infimum and the supremum of* $\mathcal{F}$.
- *The name of a binary predicate* $\setminus_d$*, defined as* $(\mu_{X_1} \setminus_d \mu_{X_2})^{\mathbf{F}} = \mu_{X_1}^{\mathbf{F}} \setminus \mu_{X_2}^{\mathbf{F}}$*, with* $\setminus$ *the difference between fuzzy sets.*
- *The name of a unary predicate* $-$ *which defines the substraction between the membership function of a fuzzy set with a number into* $[0,1]$.
- *Names for composite predicates consisting of composition of elementary predicates.*

We now illustrate how these fuzzy concrete domain predicates are used to represent spatial relations. As in [16, 17], we assume that each abstract spatial relation concept and each abstract spatial object concept is associated with its fuzzy representation in the concrete domain by the concrete feature hasForFuzzyRepresentation, denoted hasFR (it is a concrete feature because each abstract concept has only one fuzzy spatial representation in the image space).

- SpatialObject $\equiv \exists$ hasFR.$\top_{\mathcal{S}}$. It defines a SpatialObject as a concept which has a spatial existence in image represented by a spatial fuzzy set.
- In the same way, we have: SpatialRelation $\equiv$ Relation $\sqcap \exists$ hasFR.$\top_{\mathcal{S}}$.

Then, the following constructors can be used to define the other concepts of the ontology:

- $\exists$ hasFR.μ_X restricts the concrete region associated with the object X to the specific spatial fuzzy set defined by the predicate μ_X,
- $\exists$ hasFR.ν_R restricts the concrete region associated with the relation R to the specific fuzzy structuring element defined by the predicate ν_R,
- $\exists$ hasFR.$\delta_{\nu_R}^{\mu_X}$ restricts the concrete region associated with the spatial relation R to a referent object X, denoted R_X, to the spatial fuzzy set obtained by the dilation of $\mu_X^{\mathbf{F}}$ by $\nu_R^{\mathbf{F}}$,

- each concept $\mathsf{R\_X}$ can then be defined by:
  $\mathsf{R\_X} \equiv \mathsf{SpatialRelation} \sqcap \exists\ \mathsf{hasRO.X} \sqsubseteq \mathsf{SpatialRelationWith}$ et $\mathsf{R\_X} \equiv \mathsf{SpatialRelation}$ $\sqcap\exists$ $(\mathsf{hasFR},\mathsf{hasRO.hasFR}).\lambda$,
  where λ is a binary predicate built with the mathematical fuzzy operators δ and ε. For a relation R which has a referent object X, we write:

$$(\mathsf{hasFR},\mathsf{hasRO.hasFR}).\delta \equiv \mathsf{hasFR}.\ \delta_{\nu_R}^{\mu_X},$$

- $C \equiv \mathsf{SpatialObject} \sqcap \mathsf{hasSR.R\_X}$ denotes the set of spatial objects which have a spatial relation of type R with the referent object X and we have the following axioms:

$$C \sqsubseteq \exists \mathsf{relationTo.X} \text{ and } C \sqsubseteq \mathsf{SpatiallyRelatedObject}.$$

Examples for distance relations. This new formalism can be used to model different types of spatial relations and to derive useful concrete representations of these spatial relations. We illustrate our approach with distance relations. As for other relations, distance relations can be defined using fuzzy structuring elements and fuzzy morphological operators [8]. For instance, the $\mathsf{Close\_to}$ relation can be defined by the structuring element ν_{Close_To}, which provides a representation of the relation in the spatial domain $\mathcal{S}$. This representation can be learned from examples. We can thus define the abstract spatial relation $\mathsf{Close\_to}$ as: $\mathsf{Close\_To} \equiv \mathsf{DistanceRelation} \sqcap\exists \mathsf{hasFR}.\nu_{Close\_To}$. Let $\mathsf{X} \equiv \exists\ \mathsf{hasFR}.\mu_X$, $\mu_X^{\mathbf{F}}$ being the spatial fuzzy set representing the spatial extent of the object X in the concrete domain (image space). Using the concept-forming predicate operator $\exists f.P$ (see [16]), we can define restrictions for the fuzzy representation of the abstract spatial concept $\mathsf{Close\_to\_X}$ using the dilation operator δ. As a consequence, we have: $\mathsf{Close\_To\_X} \equiv \mathsf{DistanceRelation} \sqcap\exists \mathsf{hasFR}.\delta_{\nu_{Close\_To}}^{\mu_X}$. The value $\delta_{\nu_{Close\_To}^{\mathbf{F}}}^{\mu_X^{\mathbf{F}}}(x)$ represents the degree to which a point x of $\mathcal{S}$ belongs to the fuzzy dilation of the fuzzy spatial representation of X by the fuzzy structuring element $\nu_{Close\_To}^{\mathbf{F}}$. This approach naturally extends to any distance relation expressed as a vague interval.

4.2 Properties

Admissibility of $\mathbf{F} = (\Delta_{\mathbf{F}}, \Phi_{\mathbf{F}})$. A concrete domain $\mathcal{D}$ is called admissible if the set of its predicate names is closed under negation and contains a name $\top_{\mathcal{D}}$ for $\Delta_{\mathcal{D}}$, and the satisfiability problem for finite conjunctions of predicates is decidable [28]. Let us prove that the concrete domain $\mathbf{F} = (\Delta_{\mathbf{F}}, \Phi_{\mathbf{F}})$ is admissible thanks to the algebraic setting of mathematical morphology and fuzzy sets. Indeed, using the classical partial order on fuzzy sets $\leq_{\mathcal{F}}$, $(\mathcal{F}, \leq_{\mathcal{F}})$ is a complete lattice.

1. The name for $\Delta_{\mathbf{F}}$ is $\top_{\mathcal{S}}$.
2. $\Phi_{\mathbf{F}}$ is closed under negation:
   - $\neg\top_{\mathcal{S}} = \bot_{\mathcal{S}}$; $\neg\bot_{\mathcal{S}} = \top_{\mathcal{S}}$;

  - $\forall \mu_X^{\mathbf{F}} \in \mathcal{F}, \neg \mu_X^{\mathbf{F}} \in \mathcal{F}$ (the negation is then a fuzzy complementation and $\mathcal{F}$ is closed under complementation); $\forall \nu_R^{\mathbf{F}} \in \mathcal{F}, \neg \nu_R^{\mathbf{F}} \in \mathcal{F}$;
  - $\forall (\mu, \nu) \in \mathcal{F}^2, \neg \delta_\nu(\mu) = \varepsilon_\nu(\neg \mu)$ and $\neg \varepsilon_\nu(\mu) = \delta_\nu(\neg \mu)$ (duality of erosion and dilation), for dual connectives t and I [11].
3. For decidability of the satisfiability of finite conjunctions of predicates, the same reasoning as in [16] can be applied, leading to the following algorithm:
  - negated predicates can be replaced by other predicates (or disjunctions of predicates), so that only non-negated predicates need to be considered;
  - concrete representations of μ_X and ν_R are computed and considered as variables;
  - relations can be computed between the concrete representations of spatial objects, using classical algorithms of mathematical morphology (here we consider a discrete finite space, and these algorithms are tractable);
  - then it can be directly checked whether a conjunction of predicates is satisfied or not (this is performed in the concrete domain, i.e. a digital finite space, and it therefore tractable).

Let us note that tractability is guaranteed by the fact that the computation of dilations has a low computational complexity. If it is computed using a brute force method, its complexity is in $O(Nn_{se})$ where N is the size of the spatial domain (i.e. number of pixels or voxels) and n_{se} is the size of the support of the structuring element (with $n_{se} \ll N$ in general). Moreover, fast propagation algorithms exist for a number of relations (see e.g. [7] for directions). Additionally, most relations can be computed on sub-sampled images to reduce the computational cost while keeping enough accuracy.

Moreover, several interesting properties for spatial reasoning can be derived from properties of mathematical morphology (for properties of mathematical morphology see [35] and [12, 11] for the fuzzy case). We summarize here the most important ones:

1. **$\vee$-commutativity:** $\delta_{\nu_R^{\mathbf{F}}}(\mu_{X_1}^{\mathbf{F}}) \vee \delta_{\nu_R^{\mathbf{F}}}(\mu_{X_2}^{\mathbf{F}}) = \delta_{\nu_R^{\mathbf{F}}}(\mu_{X_1}^{\mathbf{F}} \vee \mu_{X_2}^{\mathbf{F}})$ and $\delta_{\nu_{R1}^{\mathbf{F}}}(\mu_X^{\mathbf{F}}) \vee \delta_{\nu_{R2}^{\mathbf{F}}}(\mu_X^{\mathbf{F}}) = \delta_{\nu_{R1}^{\mathbf{F}} \vee \nu_{R2}^{\mathbf{F}}}(\mu_X^{\mathbf{F}})$ and therefore we have the following rules:
   Rule 1: $\mathsf{R\_X_1} \sqcup \mathsf{R\_X_2} \equiv \mathsf{R\_(X_1} \sqcup \mathsf{X_2)}$.
   Rule 2: $\mathsf{R1\_X} \sqcup \mathsf{R2\_X} \equiv \mathsf{R12\_X}$,
   where R12 has for representation in the concrete domain $\nu_{R1}^{\mathbf{F}} \vee \nu_{R2}^{\mathbf{F}}$.
2. **$\wedge$-monotony:** $\delta_{\nu_R^{\mathbf{F}}}(\mu_{X_1}^{\mathbf{F}} \wedge \mu_{X_2}^{\mathbf{F}}) \leq_{\mathcal{F}} \delta_{\nu_R^{\mathbf{F}}}(\mu_{X_1}^{\mathbf{F}}) \wedge \delta_{\nu_R^{\mathbf{F}}}(\mu_{X_2}^{\mathbf{F}})$, leading to:
   Rule 3: $\mathsf{R\_(X_1} \sqcap \mathsf{X_2)} \sqsubseteq \mathsf{R\_X_1} \sqcap \mathsf{R\_X_2}$.
3. **Increasingness:** $\mu_{X_1}^{\mathbf{F}} \leq_{\mathcal{F}} \mu_{X_2}^{\mathbf{F}} \Rightarrow \forall \nu_R^{\mathbf{F}} \in \mathcal{F}, \delta_{\nu_R^{\mathbf{F}}}(\mu_{X_1}^{\mathbf{F}}) \leq_{\mathcal{F}} \delta_{\nu_R^{\mathbf{F}}}(\mu_{X_2}^{\mathbf{F}})$ and $\nu_{R_1}^{\mathbf{F}} \leq_{\mathcal{F}} \nu_{R_2}^{\mathbf{F}} \Rightarrow \forall \mu_X^{\mathbf{F}} \in \mathcal{F}, \delta_{\nu_{R_1}^{\mathbf{F}}}(\mu_X^{\mathbf{F}}) \leq_{\mathcal{F}} \delta_{\nu_{R_2}^{\mathbf{F}}}(\mu_X^{\mathbf{F}})$ which implies:
   Rule 4: $\mathsf{X_1} \sqsubseteq \mathsf{X_2} \Rightarrow \forall\ \mathsf{R},\ \mathsf{R\_X_1} \sqsubseteq \mathsf{R\_X_2}$.
   Rule 5: $\mathsf{R_1} \sqsubseteq \mathsf{R_2} \Rightarrow \forall\ \mathsf{X},\ \mathsf{R_1\_X} \sqsubseteq \mathsf{R_2\_X}$.
4. **Iterativity property:** $\delta_{\nu_{R_1}^{\mathbf{F}}}(\delta_{\nu_{R_2}^{\mathbf{F}}}(\mu_X^{\mathbf{F}})) = \delta_{\delta_{\nu_{R_1}^{\mathbf{F}}}(\nu_{R_2}^{\mathbf{F}})}(\mu_X^{\mathbf{F}})$ hence:
   Rule 6: $\mathsf{R_1\_(R_2\_X)} \equiv \mathsf{(R_1\_R_2)\_X}$,
   where $\mathsf{R_1\_R_2}$ is the relation having as fuzzy concrete representation $\delta_{\nu_{R_1}^{\mathbf{F}}}(\nu_{R_2}^{\mathbf{F}})$.

5. **Extensivity:** $\nu_R^{\mathbf{F}}(O) = 1 \iff \forall \mu_X^{\mathbf{F}} \in \mathcal{F}, \mu_X^{\mathbf{F}} \leq_{\mathcal{F}} \delta_{\nu_R^{\mathbf{F}}}(\mu_X^{\mathbf{F}})$, where O is the origin of $\mathcal{S}$ hence:
   Rule 7: $\mathsf{X} \sqsubseteq \mathsf{R\_X}$ for any relation defined by a dilation with a structuring element containing the origin of $\mathcal{S}$ (with membership value 1).
6. **Duality:** $\varepsilon_{\nu_R^{\mathbf{F}}}(\mu_X^{\mathbf{F}}) = 1 - \delta_{\nu_R^{\mathbf{F}}}(1 - \mu_X^{\mathbf{F}})$ for dual t and I, which induces relations between some relations. For instance the fuzzy representation of the mereotopological relation $\mathsf{IntB\_X}$ can be written as: $\mu_X^{\mathbf{F}} \setminus (\varepsilon_{\nu_0}^{\mu_X})^{\mathbf{F}} = \mu_X^{\mathbf{F}} \wedge (\delta_{\nu_0}^{1-\mu_X})^{\mathbf{F}} = (\delta_{\nu_0}^{1-\mu_X})^{\mathbf{F}} \setminus (1-\mu_X)^{\mathbf{F}}$, where ν_0 is an elementary structuring element, hence:
   Rule 8: $\mathsf{IntB\_X} \equiv \mathsf{ExtB\_\neg X}$.

These properties provide the basis for inference processes. Other examples use simple operations, such as conjunction and disjunction of relations, in addition to these properties, to derive useful spatial representations of potential areas of target objects, based on knowledge about their relative positions to known reference objects. This will be illustrated in Section 6 on a real example.

5 Reasoning and inference method

A knowledge base $\langle \mathcal{T}, \mathcal{A} \rangle$ built with our description logic framework is composed of two components: the terminology $\mathcal{T}$ (i.e. Tbox) and assertions about individuals $\mathcal{A}$ (i.e. Abox). Different kinds of reasoning can be performed using description logics: basic ones, including concept consistency, subsumption, instance checking, relation checking, knowledge base consistency, and non-standard ones [25]. In [3], it has been shown that basic inference services can be reduced to Abox consistency checking. For instance, concept satisfiability, (i.e. C is satisfiable with respect to $\mathcal{T}$ if there exists a model $\mathcal{I}$ of $\mathcal{T}$ such that $C^{\mathcal{I}}$ is not empty) can be reduced to verifying that the Abox $\mathcal{A} = \{a : C\}$ is consistent.

In description logics, this reasoning is often based on tableau algorithms, also known as completion algorithms. A good overview on these algorithms can be found in [4]. The principle of these algorithms is the following: starting from an initial Abox $\mathcal{A}_0$ whose consistency is to be decided, the algorithm iteratively applies completion rules to transform the given Abox into more descendent Aboxes. The algorithm results in a tree of Aboxes (or a forest in the case of Aboxes involving multiple individuals with arbitrary role relationships between them). The algorithm stops either if the produced Abox is complete, i.e. no more rules are applicable, or all leafs in the tree are contradictory (i.e. with clashes). Tableau algorithms often assume that all the concept terms occurring in the Abox are converted in their negation normal form.

In our framework, to combine terminological with quantitative reasoning in the concrete domain, the tableau calculus proposed in [17] is slightly modified.

First, the properties of description logics derived from properties of mathematical morphology can be directly used to expand the knowledge base and to facilitate the consistency checking. For instance, each disjunction of spatial relations is replaced according to the following equivalences: $\mathsf{R\_X_1} \sqcup \mathsf{R\_X_2} \equiv \mathsf{R\_(X_1} \sqcup \mathsf{X_2)}$ and $\mathsf{R1\_X} \sqcup \mathsf{R2\_X} \equiv \mathsf{R12\_X}$.

Moreover, in our framework, we assume as an ontological commitment that each instance of abstract concept is associated with its fuzzy spatial representation in the image space with the feature hasFR. As a consequence, each step of the tableau calculus algorithm enables us to derive spatial constraints on the fuzzy concrete representation using the properties of mathematical morphology. Thus, we consider that an instance of a concept C describing an object having a spatial relation R with the instance of another concept X (i.e. R_X) is satisfiable if and only if the fuzzy representation in the image domain of the instance of C fits with the fuzzy representation of the instance of the relation R_X with the function $fit : \mathcal{F} \times \mathcal{F} \to \{0, 1\}$ which verifies the strict inclusion between fuzzy sets[1]:

$$fit(\mu^{\mathbf{F}}_{X_1}, \mu^{\mathbf{F}}_{X_2}) = 1 \Leftrightarrow \mu^{\mathbf{F}}_{X_1} \leq_{\mathcal{F}} \mu^{\mathbf{F}}_{X_2}.$$

If it does not fit, a clash occurs in the Abox. More precisely, this clash occurs when we have the following assertions in the Abox:

- $s : X, (s,t) : hasSR, t :$ R_Y, $(s, \mu_X) : hasFR, (t, \lambda^{\mu_Y}_{\nu_R}) : hasFR$ and, in the spatial domain, $fit(\mu^{\mathbf{F}}_X, (\lambda^{\mu_Y}_{\nu_R})^{\mathbf{F}}) = 0$ where λ is the fuzzy predicate enabling the building of the fuzzy representation of the spatial relation R_Y.

Others occuring clashes are:

- $a : C \in \mathcal{A},\ a : \neg C \in \mathcal{A}$
- $(a, x) : f \in \mathcal{A},\ (a, y) : f \in \mathcal{A}$ with $x \neq y$

As an example, let us detail some completion rules introduced in our framework for spatial reasoning:

- **Spatial Object Conjunction Rule** ($\mathcal{R}_{\sqcap}$)
  - **Premise**: $(a : X \sqcap Y) \in \mathcal{A}$, $(a, \mu) : hasFR$, $(a : X) \notin \mathcal{A}$, $(a : Y) \notin \mathcal{A}$.
  - **Consequence**: $\mathcal{A}' = \mathcal{A} \cup \{a : X, a : Y\}$ and we have the spatial constraint $\mu = \mu_X \sqcap_d \mu_Y$.

  This rule means that if a conjunction is included in $\mathcal{A}$, then each part of the conjunction should be included in $\mathcal{A}$ as well (this is what is meant by "completion"). Here the novelty when using concrete domains is that the constraint $\mu = \mu_X \sqcap_d \mu_Y$ is added as well.
- **Spatial Relation 1** ($\mathcal{R}1_{R_X}$)
  - **Premise**: $(a :$ R_X$) \in \mathcal{A}$, $((a, \mu) :$ hasFR$) \in \mathcal{A}$, $\neg\exists r, (r : R) \in \mathcal{A}, \neg\exists x, (x : X) \in \mathcal{A}$.
  - **Consequence**: $\mathcal{A}' = \mathcal{A} \cup \{r : R, (r, \nu_R) :$ hasFR$, x : X, (x, \mu_X) :$ hasFR$, (a, x) :$ hasRO$\}$ and we have the spatial constraint $\mu = \lambda^{\mu_X}_{\nu_R}$.

    This rule means that from $a :$ R_X$(a \in ($R_X$)^{\mathcal{I}})$, we can deduce that there must exist an individual r which is an instance of the relation R having the fuzzy representation ν_R (i.e. the relation is well defined in the abstract and the concrete domains) and an individual x which is an instance of X (having the fuzzy representation μ_X), such that $(a, x) \in ($hasRO$)^{\mathcal{I}}$

[1] Other functions could be used.

and the binary predicate $\lambda_{\nu_R}^{\mu_X}$ holds in the concrete domain. λ is the fuzzy predicate enabling the building of the fuzzy representation of the spatial relation $\mathsf{R\_X}$. Note that this rule is a shortcut of the application of the conjunction rule, the exist restriction rule and the complex role rule of [17] on the assertion $a : \mathsf{R\_X}$ where $\mathsf{R\_X} \equiv \mathsf{SpatialRelation} \sqcap \exists (\mathsf{hasFR},\mathsf{hasRO.hasFR}).\lambda$.

- **Spatial Relation 2** ($\mathcal{R}2_{R_X}$)
  - **Premise**: $(a : \mathsf{R\_X}) \in \mathcal{A}$, $((a,\mu) : hasFR) \in \mathcal{A}$, $\exists r, (r : R) \in \mathcal{A}$ and $((r,\nu_R) : hasFR) \in \mathcal{A}, \exists x, x : X \in \mathcal{A}$ and $((x,\mu_X) : hasFR) \in \mathcal{A}$.
  - **Consequence**: $\mathcal{A}' = \mathcal{A} \cup \{(a,x) : hasRO\}$ and we have the spatial constraint $\mu = \lambda_{\nu_R}^{\mu_X}$.
- **In Spatial Relation** ($\mathcal{R}_{\exists hasSR}$)
  - **Premise**: $(a : \exists \mathsf{hasSR.R\_X}) \in \mathcal{A}$, $((a,\mu) : \mathsf{hasFR}) \in \mathcal{A}$, $\neg\exists b, (b : \mathsf{R\_X}) \in \mathcal{A}$ and $((a,b) : hasSR) \in \mathcal{A}$.
  - **Consequence**: $\mathcal{A}' = \mathcal{A} \cup \{b : \mathsf{R\_X}, (a,b) : \mathsf{hasSR}, (b, \lambda_{\nu_R}^{\mu_X}) : \mathsf{hasFR}\}$ and we have the spatial constraint $fit(\mu^{\mathbf{F}}, (\lambda_{\nu_R}^{\mu_X})^{\mathbf{F}}) = 1$.

6 An illustration in the domain of medical image interpretation

In this section, we illustrate on a simple but real example how our framework can be used to support terminological and spatial reasoning in a cerebral image interpretation application. In particular, our aim is to segment and recognize anatomical structures progressively by using the spatial information between the different structures. The recognition is performed in 3D magnetic resonance images (MRI) obtained in routine clinical acquisitions. A slice of a typical 3D MRI is shown in Figure 1.

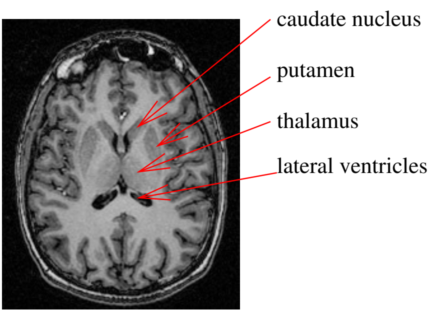


Fig. 1. An example of a slice of a 3D MRI of the brain, with a few anatomical structures indicated.

6.1 Modeling and reasoning

Anatomical knowledge, derived from anatomical textbooks [38] and from existing medical ontologies, such as the FMA [33], is converted in our formalism as follows.We denote respectively LV, RLV and LLV the *Lateral Ventricle*, the *Right Lateral Ventricle* and the *Left Lateral Ventricle*. The other anatomical structures we consider are the *Caudate Nucleus* (denoted CN, RCN, LCN) which are *grey nuclei* (denoted GN) of the brain. We have the following TBox ($\mathcal{T}$) describing anatomical knowledge using our spatial logic $\mathcal{ALC}(\mathbf{F})$:

AnatomicalStructure $\sqsubseteq$ SpatialObject
GN $\sqsubseteq$ AnatomicalStructure
RLV $\equiv$ AnatomicalStructure $\sqcap \exists$ hasFR.μ_{RLV}
LLV $\equiv$ AnatomicalStructure $\sqcap \exists$ hasFR.μ_{LLV}
LV $\equiv$ RLV $\sqcup$ LLV
Right_of $\equiv$ DirectionalRelation $\sqcap \exists$ hasFR.$\nu_{IN\_DIRECTION\_0}$
Close_to $\equiv$ DistanceRelation $\sqcap \exists$ hasFR.ν_{CLOSE_TO}
Right_of_RLV $\equiv$ DirectionalRelation $\sqcap \exists$ hasRO.RLV $\sqcap \exists$ hasFR.$\delta^{\mu_{RLV}}_{\nu_{IN\_DIRECTION\_0}}$
Close_to_RLV $\equiv$ DistanceRelation $\sqcap \exists$ hasRO.RLV $\sqcap \exists$ hasFR.$\delta^{\mu_{RLV}}_{\nu_{CLOSE\_TO}}$
RCN $\equiv$ GN $\sqcap\exists$ hasSR.(Right_of_RLV $\sqcap$ Close_to_RLV)
CN $\equiv$ GN $\sqcap\exists$ hasSR.(Close_to_LV)
CN $\equiv$ RCN $\sqcup$ LCN

The role forming predicate allows defining explicitly the dilation or erosion as a role (for instance the dilation which leads to the definition of the region to the right of the lateral ventricle):

dilate $\equiv$ (hasFR,hasRO.hasFR).δ
Right_of_RLV $\equiv$ Right_of $\sqcap\exists$ dilate.RLV

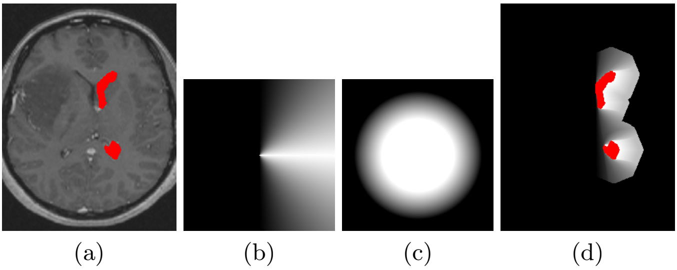


Fig. 2. (a) The right ventricle corresponding to the image region S_1 is superimposed on one slice of the original image (3D MRI). (b) Fuzzy structuring element representing the semantics of Right_of in the image. (c) Fuzzy structuring element representing the semantics of Close_to in the image. (d) $(\delta^{\mu_{S_1}}_{\nu_{IN\_DIRECTION\_0}})^{\mathbf{F}} \wedge (\delta^{\mu_{S_1}}_{\nu_{CLOSE\_TO}})^{\mathbf{F}}$.

The situation in Figure 2(a) corresponds to the following Abox $\mathcal{A}$:

c_1: RLV , (c_1,μ_{S_1}): hasFR
r_1: Right_of, (r_1,$\nu_{IN\_DIRECTION\_0}$): hasFR
r_2: Close_to, (r_2,ν_{CLOSE_TO}): hasFR

It means that we can observe an instance of the Right Lateral Ventricle (RLV) on Figure 2(a) and that we know its spatial extent in the image domain (μ_{S_1}). Moreover, the spatial relations Right_of and Close_to has been defined in the spatial domain by the learning from examples of the structuring elements $\nu_{IN\_DIRECTION\_0}$ and ν_{CLOSE_TO}.

First scenario. In a first example, our aim is to find some spatial constraints in the image domain on an instance c_2 of the Right Caudate Nucleus (RCN) given available knowledge, i.e. $\mathcal{K} = (\mathcal{T}, \mathcal{A})$. Our objective is to infer spatial constraints on concrete domains to ensure the satisfiability of RCN given $\mathcal{K}$. Using the basics of description logics reasoning, it means that the Abox enriched with $\{c_2 : \mathsf{RCN}, (c_2, \mu_{S_2}) : \mathsf{hasFR}\}$ is consistent.

First, we replace the concept RCN by its definition in $\mathcal{T}$:

$$\mathcal{A} \cup \{c_2 : \mathsf{GN} \sqcap \exists \mathsf{hasSR}.(\mathsf{Right\_of\_RLV} \sqcap \mathsf{Close\_to\_RLV}), (c_2, \mu_{S_2}) : \mathsf{hasFR}\}.$$

Then, completion rules are used to transform the given Abox into more descendent Aboxes and to derive constraints on the fuzzy representations of concepts in the concrete domain (in our case, the image domain). For instance, the completion rule adds the assertion:

$$c_2 : \mathsf{GN}, c_2 : \exists \mathsf{hasSR}.(\mathsf{Right\_of\_RLV} \sqcap \mathsf{Close\_to\_RLV})$$

and we have an individual name c_3 such that:

$$c_3 : \mathsf{Right\_of\_RLV} \sqcap \mathsf{Close\_to\_RLV}, (c_2, c_3) : \mathsf{hasSR}, (c_3, \mu_{S_3}) : \mathsf{hasFR}$$

In the spatial domain, it means that $\mu^{\mathbf{F}}_{S_2}$ and $\mu^{\mathbf{F}}_{S_3}$ must fit, i.e. $\mathrm{fit}(\mu^{\mathbf{F}}_{S_2}, \mu^{\mathbf{F}}_{S_3}) = 1$.

As c_3 is an instance of a conjunction of spatial objects, its fuzzy spatial representation in the concrete domain is:

$$((\mu_{\mathrm{Right\_of\_RLV}}) \sqcap_d (\mu_{\mathrm{Close\_to\_RLV}}))^{\mathbf{F}}$$

and we add the following assertions in the ABox:

$$c_3 : \mathsf{Right\_of\_RLV}, c_3 : \mathsf{Close\_to\_RLV}$$

The completion rule $\mathcal{R}2_{R_X}$ is applied and we have:

$$\mu_{S_3} = \delta^{\mu_{S_1}}_{\nu_{IN\_DIRECTION\_0}} \sqcap_d \delta^{\mu_{S_1}}_{\nu_{CLOSE\_TO}}$$

and the following assertion in the ABox : $(c_3, c_1) : hasRO$.

The set of inferred spatial constraints is:

$$fit(\mu^{\mathbf{F}}_{S_2}, \mu^{\mathbf{F}}_{S_3}) = fit(\mu^{\mathbf{F}}_{S_2}, (\delta^{\mu_{S_1}}_{\nu_{IN\_DIRECTION\_0}} \sqcap_d \delta^{\mu_{S_1}}_{\nu_{CLOSE\_TO}})^{\mathbf{F}}) = 1$$

and the following constraint must be verified in the image domain :

$$(\mu_{S_2})^{\mathbf{F}} \leq_{\mathcal{F}} (\delta^{\mu_{S_1}}_{\nu_{IN\_DIRECTION\_0}})^{\mathbf{F}} \wedge (\delta^{\mu_{S_1}}_{\nu_{CLOSE\_TO}})^{\mathbf{F}}.$$

The region corresponding to the the right-hand side of the inequality is illustrated in Figure 2(d). No more completion rules can be applied so the concept RCN is satisfiable given $\mathcal{K}$ if and only if this constraint is satisfied.

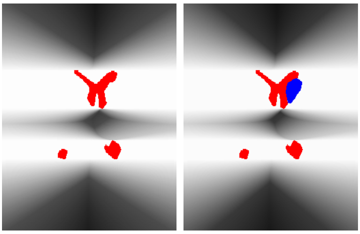
Second scenario. In this second example, illustrating disjunctions of relations, we are interested in all the instances of Caudate Nuclei in the image. A caudate nucleus is a grey nucleus which is either to the right or to the left of the lateral ventricles. This information can be represented by the following axioms:

$$\mathsf{CN} \equiv \mathsf{GN} \sqcap \exists \mathsf{hasSR}.(\mathsf{Right\_of\_LV} \sqcup \mathsf{Left\_of\_LV})$$

Using Rule 2 introduced in Section 4.2, we obtain:

$$\mathsf{Right\_of\_LV} \sqcup \mathsf{Left\_of\_LV} \equiv \mathsf{SpatialRelation} \sqcap \exists \mathsf{hasFR}.\delta^{\mu_{LV}}_{\nu_{RIGHT\_OF} \sqcup_d \nu_{LEFT\_OF}}.$$

As a consequence, the search space for the caudate nuclei is computed by: $\delta_{\nu^{\mathbf{F}}_{RIGHT\_OF} \vee \nu^{\mathbf{F}}_{LEFT\_OF}}(\mu^{\mathbf{F}}_{LV})$, which is equivalent to $\delta_{\nu^{\mathbf{F}}_{RIGHT\_OF}(\mu^{\mathbf{F}}_{LV})} \vee \delta_{\nu^{\mathbf{F}}_{LEFT\_OF}}(\mu^{\mathbf{F}}_{LV})$. The corresponding fuzzy region is represented in Figure 3(a).



(a) (b)

Fig. 3. (a) Fuzzy interpretation of the disjunction of the relations "to the left or to the right of LV". (b) One of the caudate nuclei is displayed.

7 Conclusions

In this paper, we extended the work described in [19] by the proposition of a framework for spatial relationships and spatial reasoning under imprecision based on description logics with fuzzy interpretations in concrete domains and fuzzy mathematical morphology. The resulting framework enables us to integrate qualitative and quantitative information and to derive appropriate representations of concepts and reasoning tools for an operational use in image interpretation. The benefits of our framework for image interpretation has been illustrated in the domain of medical image interpretation for the progressive segmentation and recognition of brain anatomical structures. Future work aims at formalization the spatial reasoning in the concrete domain as a constraint satisfaction problem and at further developing the brain imaging example.

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