

Attractor-Shape for Dynamical Analysis of Human Movement: Applications in Stroke Rehabilitation and Action Recognition

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Abstract

In this paper, we propose a novel shape-theoretic framework for dynamical analysis of human movement from 3D data. The key idea we propose is the use of global descriptors of the shape of the dynamical attractor as a feature for modeling actions. We apply this approach to the novel application scenario of estimation of movement quality from a single-marker for future usage in home-based stroke rehabilitation. Using a dataset collected from 15 stroke survivors performing repetitive task therapy, we demonstrate that the proposed method outperforms traditional methods, such as kinematic analysis and use of chaotic invariants, in estimation of movement quality. In addition, we demonstrate that the proposed framework is sufficiently general for the application of action and gesture recognition as well. Our experimental results reflect improved action recognition results on two publicly available 3D human activity databases.

1. Introduction

Human movement analysis from portable 3D sensing systems has opened the door to several applications in home-based health monitoring and well-being. In this paper, we focus our interest towards movement quality assessment for stroke rehabilitation using a single marker-based 3D motion capture system. A comprehensive study conducted by the World Health Organization reveals that approximately 15 million people suffer a stroke worldwide each year, making it the most common neurological disorder [16]. Stroke leaves millions of people disabled with chronic impairments often left untreated due to insufficient coverage by insurance for long-term treatment. Recent directions in stroke rehabilitation research have been focused on the development of portable and personalized rehabilitation systems [4], which can provide

low-cost home-based rehabilitation for long-term therapy administered at home. With advances in 3D motion capture technology, researchers from various backgrounds, including computer vision, have shown interest in the development of objective measures for improvement in movement quality assessment during and following rehabilitation. Visual monitoring of

movements by experienced and trained physical therapists has been the standard protocol for evaluating movement quality for decades [14]. Widely accepted quantitative scales for rating movement such as the Fugl Meyer Test [10] and the Wolf Motor Function Test (WMFT) [29], have proven to be useful and effective in evaluating movement quality. For

example, the WMFT has been used to quantify the upper extremity motor ability through timed and functional tasks [17]. Since these methods are based on visual monitoring for movement evaluation, they can be subjective, as each evaluator rely on their individual training and impressions for evaluating a subject's movement quality. This laborious, time consuming and expensive task would greatly benefit from the development of a non-subjective computational framework for movement quality assessment. The aim here is to develop standardized methods to describe the level of impairment across subjects. The Wolf Motor Function Test (WMFT) quantifies the upper extremity motor ability through timed and functional tasks. Kinematic Impairment



Figure 1: Home-based adaptive mixed reality rehabilitation systems designed for stroke survivors.

Measure (KIM) by Chen *et al.* [8] employ kinematics using a heavy marker-based system to quantitatively evaluate the quality of movements (e.g., reach and grasp). While a heavy marker-based system provides one with rich data, there is an increasing interest to deploy simple and reduced marker-based systems at home to reduce the cost of rehabilitation therapy. One example of a single marker based system is shown in Figure 1, which shows a home-based adaptive mixed reality rehabilitation system for upper extremity stroke rehabilitation [4]. In this paper, we develop a novel computational framework for movement quality assessment, combining the theoretical concepts of dynamical system analysis and ideas in shape theory. We show the utility of the proposed action modeling framework for quantifying the quality of reaching tasks using a single marker on the wrist, and obtain comparable results to a heavy marker-based setup.

Related Work: Human activity analysis has attracted the attention of many researchers providing extensive literature on the subject. A detailed review of approaches in literature for modeling and recognition of human activities is discussed in [2, 11]. Since our present work is related to dynamical system analysis for action modeling, we restrict our discussion to related methods.

Human actions have been modeled using dynamical system theory in computer vision [3, 6] and biomechanics [9, 19, 25]. Differential equations can be used to model such a system, which requires access to all independent variables of the system. This approach would facilitate an understanding of the system behavior and also allow for the prediction of future states using present and past state information. However, this is not realizable in practice, as it is extremely hard to determine the independent variables and the interactions governing the dynamics of human actions. Further, the task of modeling human actions for action recognition is non-trivial due to several factors, including inter-class similarities between actions (e.g., running and walking), intra-class variations due to multiple strategies for an action (e.g., dance) and inter-subject variations. In previous works, human actions have been modeled with the assumption that the underlying dynamical system is linear [6] or nonlinear [3]. Chaotic invariants, like largest Lyapunov exponent have been extensively used to model human actions [3, 9, 19, 25]. However, [20] and [27] have shown that these nonlinear dynamical measures need large amounts of data to produce stable results. We explore the use of proposed action modeling framework for action recognition with our experiments demonstrating the strength and flexibility of our framework. It should be noted that we are *not* trying to solve the tracking problem and therefore we assume that the trajectories of skeletal joints are available.

The focus of existing approaches for human movement quality assessment has been towards finding typical patterns

in kinematics which differ between healthy and impaired subjects. While these approaches are successful in giving an insight into understanding human movement, they fail to utilize the inherent dynamical nature of the movement. Rehabilitation therapies are composed of repetitive movements (e.g., reach to a target) that are strongly periodic (see Figure 2) with some variability. Traditional methods have assumed that this variability arises from noise in the system. However, it is evident that variability is an integral part of repetitive movements due to the availability of multiple strategies for the movement. Also, it is believed that variability produced in human movement is a result of nonlinear interactions and have deterministic origin [25]. Extensive research has been carried out to model this variability using nonlinear dynamical system theory [9, 19, 25].

In the broader vision community, Bissacco *et al.* [6] used linear dynamical systems to approximate the dynamics of human gait and learn parametric models. Ali *et al.* [3] used chaotic invariants like largest Lyapunov exponents, correlation dimension and correlation integral to analyze the nonlinear dynamics of human actions. Junejo *et al.* [12] used self-similarity matrix, a graphical representation of distinct recurrent behavior of nonlinear dynamical systems, to learn an action descriptor. Recently Bregonzio *et al.* [7] proposed the use of global spatio-temporal distribution of interest points for action recognition from 2D videos.

Contributions: We treat the reconstructed phase space of a dynamical system as a 3D point cloud and extract discriminative point cloud shape features. We show how the proposed framework is useful for movement quality assessment for application in home-based stroke rehabilitation, action recognition and gesture recognition.

Outline: In section 2, we discuss theoretical concepts of phase space reconstruction and methods to estimate the parameters for the same. We propose our framework for modeling human actions in section 3. In section 4, we present results on extensive experiments carried out on stroke rehabilitation dataset [8], motion capture dataset [3] and MSR Action3D dataset [15].

2. Phase Space Reconstruction

The phase space is defined as the space with all possible states of a system. In a deterministic dynamical system that can be mathematically modeled, future states of the system can be determined using present and past state information. However, for human actions, the system equations are complex. Furthermore, the home-based setting for stroke rehabilitation (single marker-based system) does not allow us to observe all variables of the system. To address these problems, we have to employ methods for reconstructing the attractor to obtain a phase space which preserves the important topological properties of the original dynamical

system. This process is required to find the mapping function between the one-dimensional observed time series and the m -dimensional attractor, with the assumption that all variables of the system influence one another. The concept of phase space reconstruction was explained in the embedding theorem proposed by Takens, called Takens' embedding theorem [26]. For a discrete dynamical system with a multidimensional phase space, the time-delay vectors can be written as

$$X_i(n) = \{x_i(n), x_i(n + \tau), \dots, x_i(n + (m - 1)\tau)\} \quad (1)$$

where ' m ' is the embedding dimension and ' τ ' is the embedding delay. These parameters should be carefully selected in order to facilitate a good phase space reconstruction. For a sufficiently large m , the important topological properties of the unknown multidimensional system are reproduced in the reconstructed phase space. The embedding method has proven to be useful, particularly for time series generated from low-dimensional deterministic dynamical systems, by providing a way to apply theoretical concepts of nonlinear dynamical systems onto observed time series. The embedding theorem does not suggest methods to estimate the optimal values for m and τ . We use false nearest neighbors [13] approach to estimate m and the first zero crossing of the autocorrelation function [23] to estimate τ . Figure 2 shows an example of phase space reconstruction from a one-dimensional observed time series.

Embedding Dimension: The aim here is to estimate an integer embedding dimension which can *unfold* the attractor thereby removing any self-overlaps due to projection of the attractor onto lower dimensional space. Hence, the embedding dimension can be defined as the minimum dimension required to unfold the attractor completely. The false nearest neighbor approach finds this minimum embedding dimension to remove any *false* nearest neighbors (neighbors due to projection onto lower dimension). Consider a vector in reconstructed phase space in dimension m given by

$$X(k) = \{x(k), x(k + \tau), \dots, x(k + (m - 1)\tau)\} \quad (2a)$$

Consider a nearest neighbor in the phase space given by

$$X^{NN}(k) = \{x^{NN}(k), x^{NN}(k + \tau), \dots, x^{NN}(k + (m - 1)\tau)\} \quad (2b)$$

If the vector $X^{NN}(k)$ is a true neighbor of $X(k)$, then it should be because of the underlying dynamics. The vector $X^{NN}(k)$ can be a false neighbor of $X(k)$ when dimension m is unable to unfold the attractor. Hence, moving to the next dimension $m + 1$ may move this false neighbor out of the neighborhood of $X(k)$. This process of finding false neighbors to every vector $X_i(k)$ sequentially removes self-overlaps and identifies m where the attractor is completely unfolded. The embedding dimension m suggested by the false nearest neighbor algorithm for exemplar trajectories

was either 3 or 4 on stroke rehabilitation database. We select a constant embedding dimension $m = 3$ to reconstruct all relevant phase space. Even with this fixed value of m , we obtain excellent results as shown in our experiments.

Embedding Delay: Theoretically, the embedding process allows any value of τ if one has access to infinitely accurate data ([1], chap. 3). Since this is practically impossible, we try to find a value τ which makes the components of the vector $\{x(k), x(k + \tau), x(k + 2\tau)\}$ in the embedding sufficiently independent. A low value of τ makes adjacent components to be correlated and hence they cannot be considered as independent variables. On the other hand, a high value of τ may make the adjacent components uncorrelated (almost independent) and cannot be considered as part of the system that supposedly generated them. The shape of the embedded time series will critically depend on the choice of τ [23]. A good selection of τ should ensure that the data are maximally spread in phase space resulting in smooth phase space reconstruction. We use the first zero-crossing of the autocorrelation function as an estimate of τ as suggested in [23] for strongly periodic data, which is a suitable choice for our experiments (see Figure 2).

3. Shape Features from Attractors

In this section, we present a framework which combines the strong theoretical concepts of nonlinear dynamical analysis and ideas in shape theory to effectively represent human movement. From Figure 2, the '*shape*' of the reconstructed phase space can be seen as a discriminative feature for classification between unimpaired and impaired subjects. Shape analysis of 3D surfaces is a well-studied problem in the computer vision community. Osada *et al.* [18] present a method for finding a similarity measure between 3D shapes by computing shape distributions of the 3D surface sampled from the shape function by measuring their global geometric properties. We use the *shape distribution* of reconstructed phase space as the dynamical feature in our experiments. Similar to the **D2** shape function from [18], we measure the distance between two random vectors of the attractor (phase space) which can be represented as

$$D_{ij} = \|X_i - X_j\|_2 \quad (3)$$

where X_i and X_j are embedding vectors in the reconstructed phase space. A set of these distances for randomly chosen embedding vector pairs are computed. From this set, we construct a histogram by counting the number of samples which fall into each of $B = 50$ fixed sized bins. Several metrics exist in literature to calculate distance between histograms including chi-squared statistic (χ^2 distance), Bhattacharyya distance [5], Riemannian analysis [24] and Earth Mover's Distance (EMD) [21]. In our experiments, we use Euclidean distance as our *similarity measure* (see Figure

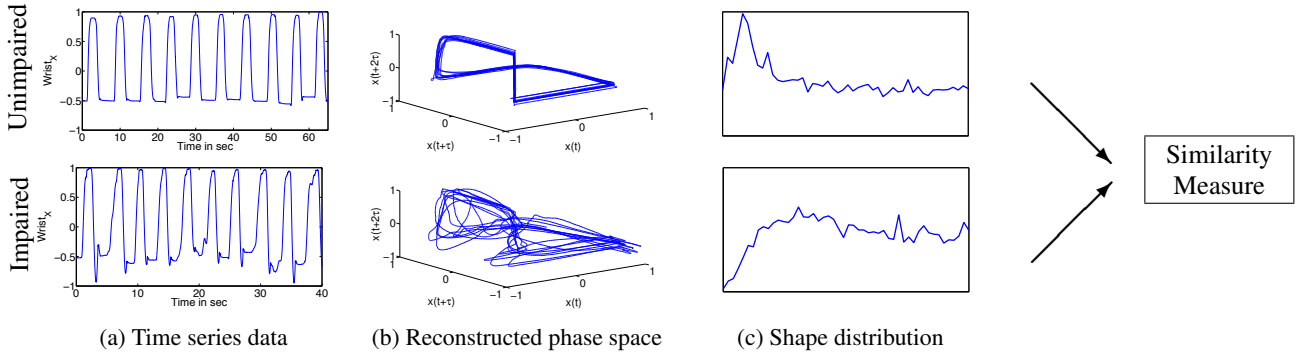


Figure 2: Proposed framework for movement quality assessment and action recognition by extraction of dynamical shape feature from reconstructed phase space. (a) shows the time series of x -location of wrist marker; its respective reconstructed phase space is shown in (b). These two exemplar trajectories are collected from the stroke rehabilitation dataset [8] and belong to unimpaired and impaired subjects respectively. The corresponding dynamical shape feature represented by shape distribution is shown in (c). Similarity measure (e.g., Euclidean distance) can be used to classify these trajectories.

2) to measure the distance between histograms and classify movements.

Test on Models: The framework was tested on Lorenz and Rossler models to find whether the shape feature can be effectively used to classify differences in shape of reconstructed phase space of nonlinear dynamical systems. We compare the performance of the proposed framework with that of largest Lyapunov exponents. Chaos theory has found its application in the analysis of chaotic dynamical systems. In comparison, largest Lyapunov exponent is a widely used measure of chaos in various engineering applications, including computer vision [3, 22]. A practical method for estimating the largest Lyapunov exponent from a time series proposed by Rosenstein [20] quantifies chaos by monitoring the rate of divergence of closely spaced trajectories over time. The algorithm claims to be fast, easy to implement and robust to changes in embedding dimension, size of dataset, embedding delay and noise level. Rosenstein’s algorithm was developed to address the limitations of Wolf’s algorithm [28] and has been shown in [27] that it is more robust to changes in data length than Wolf’s algorithm. However, experimental results on Lorenz and Rossler models for different time series lengths (N) with fixed embedding dimension and embedding delay shows that the estimate approaches the true value only after $N = 5000$ and 2000 , respectively. Furthermore, both Rosenstein and Wolf suggest that the minimum number of data samples required for accurate estimation of largest Lyapunov exponent is 10^m (where m is the embedding dimension) [27]. Therefore, we believe that the use of largest Lyapunov exponent may not be a suitable approach in modeling human actions where the number of data samples is small. Also, from Figure 3, the shape distribution was found to be stable for different time-series lengths. This striking ability of our feature to be robust to changes in data length will be useful in applications related to human activity analysis, where the signal observation time is small/variable.

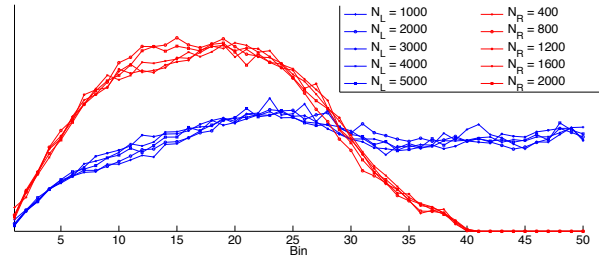


Figure 3: Shape distribution of reconstructed phase space for Lorenz (blue) and Rossler (red) models for different time series length N (N_L and N_R represents time series lengths of Lorenz and Rossler systems respectively). Embedding parameters m and τ were chosen to be same as reported by Rosenstein *et al.* [20]. It is reported in [20] that largest Lyapunov exponent estimation on these models give significant error for the above shown data lengths.

4. Experiments and Results

The proposed framework was tested on stroke rehabilitation dataset [8], motion capture dataset [3] and MSR Action3D dataset [15].

4.1. Stroke Rehabilitation Dataset

Our aim in this experiment is two-fold: a) to classify movements of unimpaired (neurologically normal) and impaired (stroke survivors) subjects, b) to quantitatively assess the quality of movement performed by the impaired subjects during repetitive task therapy. The experimental data was collected using a heavy marker-based system (14 markers on the right hand, arm and torso) in a hospital setting. Seven unimpaired and 15 impaired subjects perform reach and grasp movements, both on-table and elevated (the subject must move against gravity to reach the target). The stroke survivors were also evaluated by the Wolf Motor Function Test (WMFT) [29] on the day of recording, which evaluates a subject’s functional ability on a scale of 1 – 5 (with 5 being least impaired and 1 being most im-

	Proposed Method		KIM [8]		Largest Lyapunov Exponent [20]	
Subject	Impaired	Unimpaired	Impaired	Unimpaired	Impaired	Unimpaired
Impaired	55	5	53	7	43	17
Unimpaired	5	23	6	22	18	10

Table 1: Confusion table for stroke rehabilitation dataset using the proposed dynamical shape feature, KIM and largest Lyapunov exponent from a single wrist marker giving 88.6%, 85.2% and 60.2% classification rate respectively.

Method	Classification Rate
KIM [8]	85.2 %
Largest Lyapunov exponent [20]	60.2 %
Proposed method	88.6 %

Table 2: Comparison of classification rates for different methods using leave-one-reach-out cross-validation and nearest neighbor classifier on the stroke rehabilitation dataset.

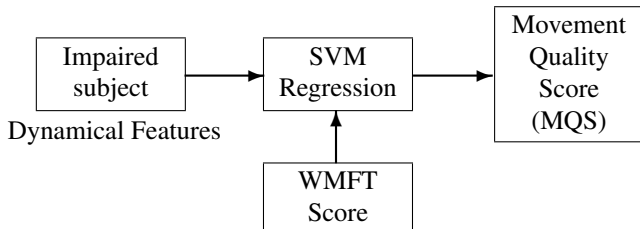


Figure 4: Block diagram representation for learning a regressor for movement quality assessment using Functional Activity Score (FAS) from the Wolf Motor Function Test (WMFT).

paired) based on predefined functional tasks. Since our focus is on development of quantitative measures of movement quality for a home-based rehabilitation system that would use a single marker on the wrist (as shown in Figure 1), we only use the data corresponding to the single marker on the wrist from the heavy marker-based hospital system. Although WMFT scores are based on various functional tasks (e.g., folding a towel, picking up a pencil) and are not based on evaluation of reach and grasp movements, we utilize these WMFT scores as an approximate high-level quantitative measure for movement quality of impaired subjects performing reach and grasp movements because both WMFT evaluation and 3D marker data on the wrist were obtained on the same day.

The focus of traditional methods for quantitative assessment of movement quality has been towards kinematics. Hence, in Table 2, we compare our results with an approach which uses kinematic analysis on the same dataset [8]. We also compare our results with the performance of largest Lyapunov exponents, a widely used measure in human movement analysis in Table 2. Table 1 shows the confusion table for classification of movements using respective measures on stroke rehabilitation dataset. Our method performs better than the two promising quantitative measures for movement analysis.

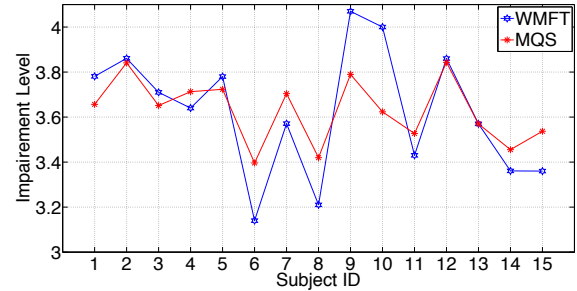


Figure 5: Comparison between impairment level (with 5 being least impaired and 1 being most impaired) given by actual WMFT score and MQS for 15 impaired subjects. The Pearson correlation coefficient was found to be 0.8527 with a two-tail P-value of 5.35×10^{-5} , proving its statistical significance.

We also propose a framework for movement quality assessment (shown in Figure 4) for stroke rehabilitation. Using the WMFT scores of impaired subjects, we learn a regression function using SVM (using radial basis function kernel) to compute a movement quality score from dynamical shape features. The regressor is trained using leave-one-reach-out cross-validation technique. The outputs of the regressor were averaged per subject to get the Movement Quality Score (MQS). Figure 5 shows a comparison between actual WMFT score and the quality assessment score by the proposed method (MQS). The Pearson correlation coefficient between the MQS and the Function Activity Score (FAS) of the WMFT was found to be 0.8527. When we repeat the same experiment with kinematic attributes on a single wrist marker, the correlation coefficient was found to be 0.6481. In comparison, kinematic analysis from data from all 14 markers gave a correlation coefficient of 0.9041. This experiment clearly shows that the proposed framework achieves comparable results obtained by the heavy marker-based system even when using a single wrist marker, which is facilitated by the phase space reconstruction and robust feature extraction from phase space using shape distributions.

While this is a promising step towards development of non-subjective framework for movement quality assessment for stroke rehabilitation in a home-based setting, there are limitations of our framework that require more investigation prior to being used as a rehabilitation therapy tool and are discussed in future work.

Action	Dance	Jump	Run	Sit	Walk
Dance	31	0	0	0	0
Jump	0	13	1	0	0
Run	0	0	28	0	2
Sit	0	0	0	35	0
Walk	0	1	1	0	46

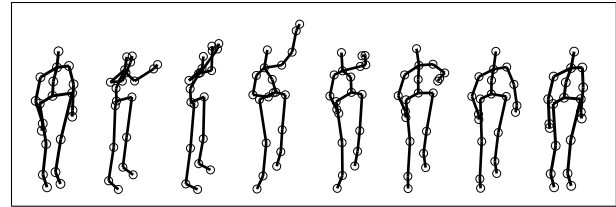
Table 3: Confusion table for motion capture dataset achieving mean classification rate of 96.84% when compared to 89.7% reported by Ali *et al.* in [3].

4.2. Motion Capture Dataset

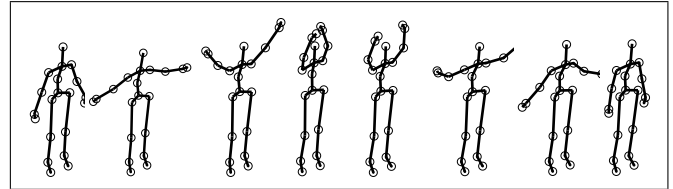
In the next experiment, we show that the proposed framework can be applied to the well-studied problem of action recognition. For this experiment, we use the dataset released by FutureLight, R&D division of Santa Monica Studios which is a collection of five actions: *dance*, *jump*, *run*, *sit* and *walk* with 31, 14, 30, 35 and 48 instances respectively. The classification problem on this dataset is shown to be challenging due to the presence of significant intra-class variations [3]. The data is in the form of trajectories of 18 body joints. We use all body joints except data from hip joint, to remove any effect of translational movement of body. The 3D time series from these 17 joints were divided into scalar time series (x, y & z) resulting in a 51-dimensional vector representation for each action. Phase space reconstruction and dynamical shape feature extraction was performed. We use the leave-one-out cross-validation approach using nearest neighbor for classification. The results are tabulated in Table 3 and we achieve a mean accuracy of 96.84% in comparison with 89.7% reported by Ali *et al.* in [3]. Our results show that there was some error made in classification of *Jump*, *Run* and *Walk* actions, which is reasonable considering the similarity between these actions.

4.3. Kinect Dataset

The framework was also tested on a more comprehensive dataset released by Microsoft Research called *MSR Action3D* dataset [15] having 20 action classes (see Figure 6 for example actions) with 10 subjects performing each action thrice. The dataset provides 3D joint positions (x, y & z) and will be used as our input. These 20 action classes were further divided into 3 Action Sets: *AS1*, *AS2* and *AS3* by Li *et al.* in [15] to account for the large amount of computation involved in classification of these actions. The action sets 1 and 2 were intended to group actions with similar movement and action set 3 to group complex movements. The classification results are tabulated in Table 4 and as seen, the proposed framework performs better than the Bag of 3D points approach proposed by Li *et al.* [15] for two action sets on the cross-subject test setting using a linear SVM. It should be noted that we have used ten subjects as



(a) Tennis serve



(b) Two hand wave

Figure 6: Example actions from action class *Tennis serve* (a) and *Two hand wave* (b) from the MSR Action3D dataset. Skeleton data of 20 joints provided in the dataset will be used in our action recognition experiment.

Action Set	Proposed method	Bag of 3D points
AS1	77.5 %	72.9 %
AS2	63.1 %	71.9 %
AS3	87.0 %	79.2 %
Overall	75.9 %	74.7 %

Table 4: Classification results for cross-subject test setting where 50% subjects were used for training and the remaining 50% subjects for testing in proposed method. While a total of seven subjects were used by Li *et al.* [15], our results presented are for 10 subjects.

opposed to seven subjects by Li *et al.* [15].

5. Conclusion and Discussion

In this paper, we proposed a new shape theory based dynamical analysis framework for movement quality assessment and action recognition. To address the drawbacks of traditional measures from chaos theory for modeling the dynamics of human actions, we proposed a framework combining the concepts of nonlinear time series analysis and shape theory to extract robust and discriminative features from reconstructed phase space. The proposed framework for movement quality assessment was used in assessing a stroke survivor's level of impairment. Furthermore, our goal is to apply the proposed framework to home-based stroke rehabilitation systems using a single marker. While the information contained in a single marker on the wrist is much impoverished compared to a heavy marker-based system, our experiments indicate that with advanced dynamical features and machine learning tools, we are able to achieve comparable performance levels to a heavy marker-based system in movement quality assessment. These results also suggest that it is feasible to reduce the requirement of multiple markers, leading to more portable and cheaper rehabilitation systems for home-based deployment.

Experimental results also suggest that the proposed method can be used for recognition of complex actions (e.g., AS3 in Table 4). Furthermore, kinematic analysis for complex movements (e.g., lift and transport an object) is difficult, as it is impossible to define a “reference” trajectory for such cases. Since the proposed framework does not require a predefined reference trajectory, we believe that our computational framework will be suitable for quality assessment of complex movements and will be explored in our future work. A current limitation is that the proposed framework only specifies a Movement Quality Score (level of impairment) and does not give any information about underlying movement components contributing to the score (e.g., elbow versus torso movement in a reaching task). Hence, it cannot be used as a rehabilitation therapy tool yet and we focus our future work in the same direction.

Our action recognition experiments on motion capture and MSR Action3D datasets showed the strength and flexibility of the proposed framework and can be convincingly used for human action recognition from 3D data as well.

Future Work: In this paper we consider a rehabilitation system in the context of repetitive task therapy (periodic data) in which the stroke survivor repeats an activity for a defined number of times. However, we would like our future work to include non-periodic data from daily life activities performed by stroke survivors. As mentioned earlier, the WMFT and our framework are not rating the same activities. To address this, we are in the process of data collection from six stroke survivors performing simple and complex tasks and have developed a rating scale in collaboration with physical therapists that will be used to rate these activities. Within this scale, physical therapists provide us both an overall rating and a component rating. We are currently collecting both 3D marker position data and physical therapist ratings in order to make comparisons among the kinematics, our proposed measure, and the therapist ratings, across the same action. Utilizing the expert knowledge of the therapist ratings for these rated actions will also help us better contextualize the data to better shape our framework as a therapy tool.

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