Improving the Visual Comprehension of Point Sets
Supplemental: Proofs of lemmas

Sagi Katz
Technion
sagi.katz@gmail.com

Ayellet Tal
Technion
ayellet@ee.technion.ac.il

Lemma 4.1 *$TPO(P) \subseteq P_O$, i.e., every point p marked occluding by the operator is indeed an occluding point.*

Proof: Let $p \in TPO(P)$. Suppose, by way of contradiction, that $p \notin P_O$. Then, the ray from C to p passes through some point $q \in S$ whose distance to C is greater than that of p. After the transformation, q will be still farther away from C compared to p, since the transformation is strictly monotonically increasing in the norm. Thus, p is internal to the convex hull. □

Lemma 4.2 *When $\gamma \to 0$, the set of points marked by the TPO operator is equal to the set of occluding points.*

Proof: One side of the equality was proved in Lemma 4.1. To prove the other side, we will show that if $p \in P_O$, then $p \in \lim_{\gamma \to 0} TPO(P)$. Without loss of generality, let $p = (r, 0)$ (in spherical coordinates), i.e., p lies on the $X-$axis. Let $U = \sup\{\|p - C\| \, |p \in S\}$, be the maximum distance to C for points in P.

Then, applying F to p and another arbitrary point $q \in S$, we get $\hat{p} = (r^\gamma, 0)$ and $\hat{q} = (r_q^\gamma, \theta)$, $(\theta \neq 0)$. To show that $p \in \lim_{\gamma \longrightarrow 0} TPO(P)$, we will find a condition for γ that will ensure that for γ values for which it holds, $\hat{q}$ is on one side of a line through $\hat{p}$, $\forall q \in S$. The line we choose is $x = r^\gamma$, which is parallel to the $y-$axis. Now, the x coordinate of $\hat{q}$ is $q_x = r_q^\gamma \cos\theta$, but since $r_q < U$, then $q_x < U^\gamma \cos\theta$. Therefore, for the following to hold

$$r^\gamma > U^\gamma \cos\theta, \tag{1}$$

we require that

$$0 < \gamma < \frac{\log(\cos\theta)}{(\log r - \log U)} \, \square. \tag{2}$$

Lemma 4.3 $p \in TPO_\gamma(P)$ if p is visible to an observer positioned on the ray p-C, farther away than any point in P, and the curvature κ at p satisfies

$$\kappa > \frac{\gamma(1-\gamma)\sin(\beta)(\cos^2(\beta) - \gamma \sin^2(\beta))}{d(\gamma^2 \sin^2(\beta) + \cos^2(\beta))^{\frac{3}{2}}}.$$

Proof: The proof of this lemma is similar to the proof of Lemma 3.1 in the paper. Note that the local curvature is computed such that the normal is directed towards the point C in both lemmas. However, the viewpoint in our case is in the opposite direction from C relative to p. Therefore, the condition for a point to be an occluding point changes so that $\kappa > \kappa_\Lambda$, where κ_Λ is defined in a similar way as in Lemma 3.1.□

Lemmas 4.3 and 3.1 hold for both the infinite case and the finite case, where the distances between samples of the surface are larger than 0. The reason for this is that the Λ curve (Figure 12) adjusts itself to the shape of the point cloud. Therefore, its curvature is identical to the curvature of the surface in the infinite case and is a good approximation in the finite case.