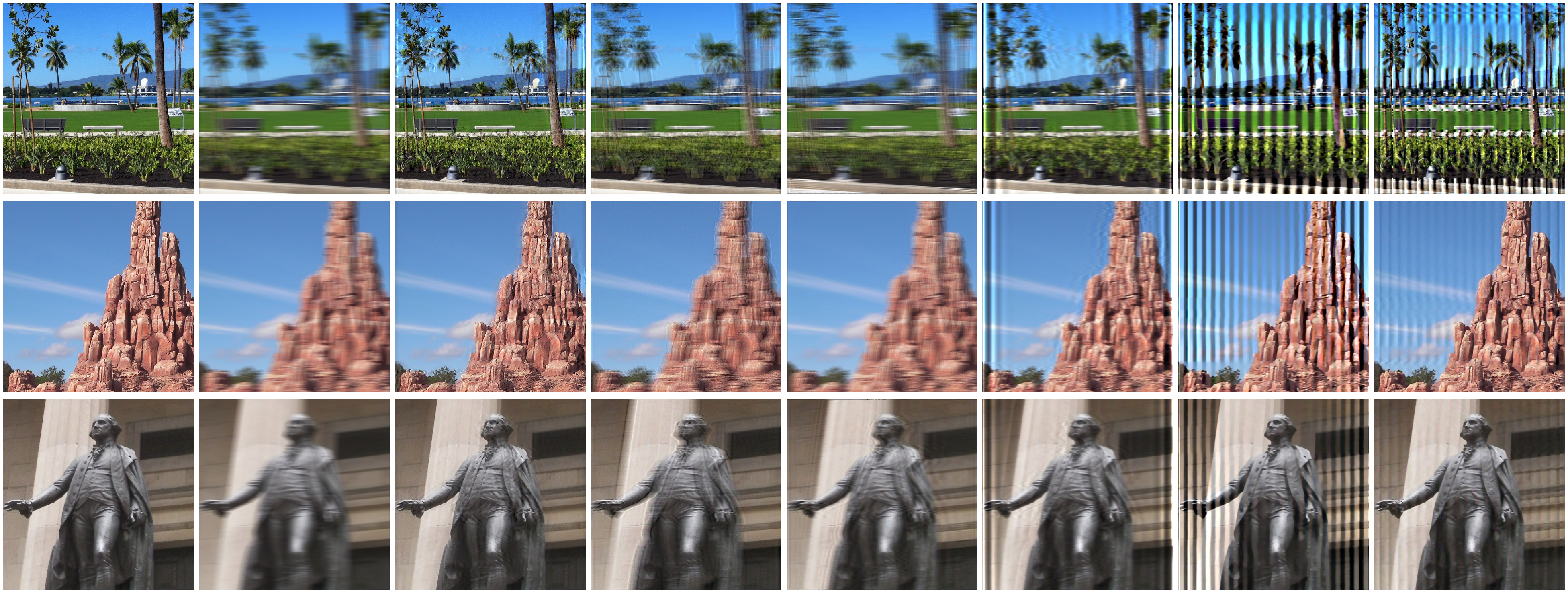
Blur Processing Using Double Discrete Wavelet Transform
Supplementary Material

1 Additional Examples

For object motion and defocus blur, we have two categories: real camera data with spatially varying blur and synthetic experiment with uniform blur. We use Nikon D90 to obtain raw image data for spatially varying blur and point-and-shoot camera for ground truth of synthetic data. The experiment was developed in Matlab using a Intel core i7 cpu at 3.4GHz and 16GB of RAM. The execution time for deblurring is typically from 1 to 5 seconds according to different image size.

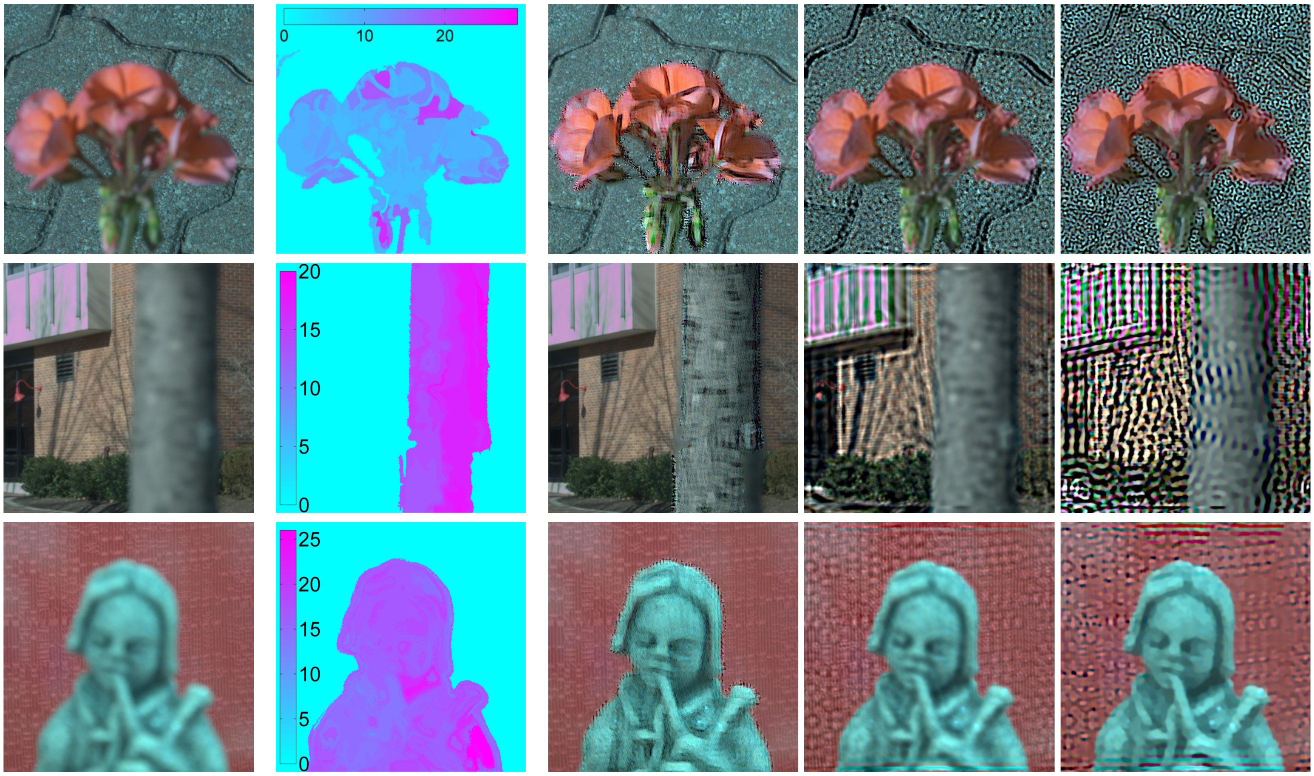
Figure 1 and 2 are examples of non-uniform and uniform object motion blur, respectively. Figure 3 shows examples of non-uniform defocus blur. Figure 4 shows uniform defocus blur and figure 5 are examples of camera shake blur.

Figure 1: Examples with real camera data (spatially varying blur). Output images from the proposed and method in [6] are produced by blind deblurring. The others are non-blind deblurring based on blur kernel estimated by the DDWT technique



Ground truth | Input | Proposed | Cai et al.[1] | Whyte et al.[6] | Lucy[4] | Chan et al.[3] | Shan et al.[5]

Figure 2: Example of non-blind deblurring using synthetic data (global blur).

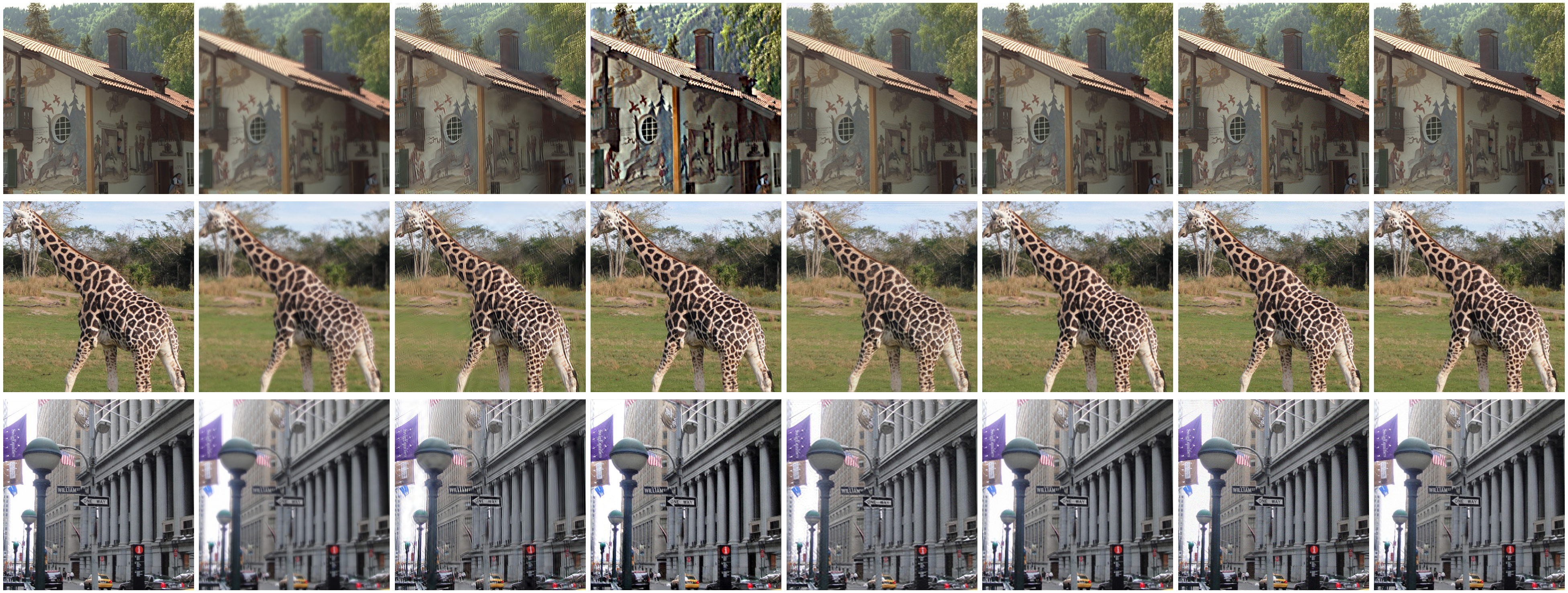


Input | Blur estimate with DDWT | Proposed | Lucy[4] | Chan et al.[3]

Figure 3: Example of using real camera data (spatially varying blur). Output image from the proposed method is produced by blind deblurring, the others are non-blind deblurring based on blur kernel estimated by the DDWT technique.

Ground truth | input | Proposed | Lucy[4] | Chan et al.[3]

Figure 4: Example of non-blind deblurring using synthetic data (global blur).



Ground truth | input | Proposed | Cai et al.[1] | Whyte et al.[6] | Lucy[4] | Chan et al.[3] | Shan et al.[5]

Figure 5: Example of non-blind deblurring using synthetic data.

2 Proof of Claim 1

Claim 1 (Robust Regression)**.** *Let*

$$\hat{v}(\boldsymbol{n}) = u(\boldsymbol{n} + \tbinom{0}{k/2}) - u(\boldsymbol{n} - \tbinom{0}{k/2}).$$

Suppose further that the probability density function of u is symmetric (with zero mean), and $P[u(\boldsymbol{n}) = 0] = \rho$ (u is said to be "ρ-sparse"). Then

$$P\Big[u(\boldsymbol{n} + \tbinom{0}{k}) = 0 \Big| \|\hat{v}(\boldsymbol{n} + \tbinom{0}{k/2})\| < \|\hat{v}(\boldsymbol{n} - \tbinom{0}{k/2})\|\Big] \geq \rho.$$

Proof. We have $P\Big[u(\boldsymbol{n} + \tbinom{0}{k}) = 0\Big] = P[u(\boldsymbol{n}) = 0] = \rho$, and $P\Big[u(\boldsymbol{n} + \tbinom{0}{k}) \neq 0\Big] = 1 - \rho$.

$$P\Big[u(\boldsymbol{n} + \tbinom{0}{k}) = 0 \Big| \|\hat{v}(\boldsymbol{n} + \tbinom{0}{k/2})\| < \|\hat{v}(\boldsymbol{n} - \tbinom{0}{k/2})\|\Big]$$

$$= \frac{P\Big[\|\hat{v}(\boldsymbol{n} + \tbinom{0}{k/2})\| < \|\hat{v}(\boldsymbol{n} - \tbinom{0}{k/2})\| \Big| u(\boldsymbol{n} + \tbinom{0}{k}) = 0\Big]\rho}{P\Big[\|\hat{v}(\boldsymbol{n} + \tbinom{0}{k/2})\| < \|\hat{v}(\boldsymbol{n} - \tbinom{0}{k/2})\| \Big| u(\boldsymbol{n} + \tbinom{0}{k}) = 0\Big]\rho + P\Big[\|\hat{v}(\boldsymbol{n} + \tbinom{0}{k/2})\| < \|\hat{v}(\boldsymbol{n} - \tbinom{0}{k/2})\| \Big| u(\boldsymbol{n} + \tbinom{0}{k}) \neq 0\Big](1 - \rho)}$$

We know that $\hat{v}(\boldsymbol{n} + \tbinom{0}{k/2}) = u(\boldsymbol{n} + \tbinom{0}{k}) - u(\boldsymbol{n})$, and $\hat{v}(\boldsymbol{n} - \tbinom{0}{k/2}) = u(\boldsymbol{n}) - u(\boldsymbol{n} - \tbinom{0}{k})$.

Hence

$$P\Big[\|\hat{v}(\boldsymbol{n} + \tbinom{0}{k/2})\| < \|\hat{v}(\boldsymbol{n} - \tbinom{0}{k/2})\| \Big| u(\boldsymbol{n} + \tbinom{0}{k}) = 0\Big] = P\Big[\|u(\boldsymbol{n})\| < \|u(\boldsymbol{n}) - u(\boldsymbol{n} - \tbinom{0}{k})\|\Big]$$

With the zero mean symmetric probability density function of u, we can have the right side of above equation is equal to or greater than $\frac{1}{2}$, so

$$P\Big[\|\hat{v}(\boldsymbol{n} + \tbinom{0}{k/2})\| < \|\hat{v}(\boldsymbol{n} - \tbinom{0}{k/2})\| \Big| u(\boldsymbol{n} + \tbinom{0}{k}) = 0\Big] \geq \frac{1}{2} \tag{1}$$

Similarly,

$$P\Big[\|\hat{v}(\boldsymbol{n} + \tbinom{0}{k/2})\| < \|\hat{v}(\boldsymbol{n} - \tbinom{0}{k/2})\| \Big| u(\boldsymbol{n} + \tbinom{0}{k}) \neq 0\Big] \leq \frac{1}{2} \tag{2}$$

Thus

$$\frac{P\Big[\|\hat{v}(\boldsymbol{n} + \tbinom{0}{k/2})\| < \|\hat{v}(\boldsymbol{n} - \tbinom{0}{k/2})\| \Big| u(\boldsymbol{n} + \tbinom{0}{k}) = 0\Big]}{P\Big[\|\hat{v}(\boldsymbol{n} + \tbinom{0}{k/2})\| < \|\hat{v}(\boldsymbol{n} - \tbinom{0}{k/2})\| \Big| u(\boldsymbol{n} + \tbinom{0}{k}) = 0\Big]\rho + P\Big[\|\hat{v}(\boldsymbol{n} + \tbinom{0}{k/2})\| < \|\hat{v}(\boldsymbol{n} - \tbinom{0}{k/2})\| \Big| u(\boldsymbol{n} + \tbinom{0}{k}) \neq 0\Big](1 - \rho)} \geq 1$$

which indicates

$$P\Big[u(\boldsymbol{n} + \tbinom{0}{k}) = 0 \Big| \|\hat{v}(\boldsymbol{n} + \tbinom{0}{k/2})\| < \|\hat{v}(\boldsymbol{n} - \tbinom{0}{k/2})\|\Big] \geq \rho.$$

□

3 Modified Bilateral Filter

The autocorrelation function (Equation 11 in the paper) is defined by:

$$R_v(\boldsymbol{n}, \ell) \approx \frac{\sum_{\boldsymbol{m} \in \boldsymbol{\Lambda}} a(\boldsymbol{n} + \boldsymbol{m}, \ell) v(\boldsymbol{n} + \boldsymbol{m} + \tbinom{0}{\ell/2}) v(\boldsymbol{n} + \boldsymbol{m} - \tbinom{0}{\ell/2})}{\sum_{\boldsymbol{m} \in \boldsymbol{\Lambda}} a(\boldsymbol{n} + \boldsymbol{m}, \ell)}$$

where $\boldsymbol{\Lambda}$ defines the local neighborhood, $\boldsymbol{n}$ is the center pixel location, and $a(\boldsymbol{n}, \ell)$ denotes the averaging weight at location $\boldsymbol{n}$:

$$a(\boldsymbol{n} + \boldsymbol{m}, \ell) = e^{\frac{-\boldsymbol{m}^2}{\sigma_d^2}} e^{\frac{-(L_1(\boldsymbol{n}+\boldsymbol{m},\ell)^2 + A_1(\boldsymbol{n}+\boldsymbol{m},\ell)^2 + B_1(\boldsymbol{n}+\boldsymbol{m},\ell)^2 + L_2(\boldsymbol{n}+\boldsymbol{m},\ell)^2 + A_2(\boldsymbol{n}+\boldsymbol{m},\ell)^2 + B_2(\boldsymbol{n}+\boldsymbol{m},\ell)^2)}{2\sigma_r^2}} \tag{3}$$

where σ_d^2 and σ_r^2 are supported spatial and intensity variance. And Intensity difference of left shifted:

$$L_1(\boldsymbol{n} + \boldsymbol{m}, \ell) = L(\boldsymbol{n} + \boldsymbol{m} - \tbinom{0}{\ell/2}) - L(\boldsymbol{n})$$

$$A_1(\boldsymbol{n} + \boldsymbol{m}, \ell) = A(\boldsymbol{n} + \boldsymbol{m} - \tbinom{0}{\ell/2}) - A(\boldsymbol{n})$$

$$B_1(\boldsymbol{n}+\boldsymbol{m},\ell) = B(\boldsymbol{n}+\boldsymbol{m}-\binom{0}{\ell/2}) - B(\boldsymbol{n})$$

Intensity difference of right shifted:

$$L_2(\boldsymbol{n}+\boldsymbol{m},\ell) = L(\boldsymbol{n}+\boldsymbol{m}+\binom{0}{\ell/2}) - L(\boldsymbol{n})$$

$$A_2(\boldsymbol{n}+\boldsymbol{m},\ell) = A(\boldsymbol{n}+\boldsymbol{m}+\binom{0}{\ell/2}) - A(\boldsymbol{n})$$

$$B_2(\boldsymbol{n}+\boldsymbol{m},\ell) = B(\boldsymbol{n}+\boldsymbol{m}+\binom{0}{\ell/2}) - B(\boldsymbol{n})$$

Notice L, A, B are corresponding channels in CIE-Lab space.

References

[1] J. Cai, H. Ji, and Z. Shen. Blind motion deblurring from a single image using sparse approximation. *CVPR*, 62:291–294, January 2009.

[2] A. Chakrabarti, T. Zickler, and W. T. Freeman. Analyzing spatially-varying blur. *CVPR*, 2010.

[3] S. Chan, R. Khoshabeh, K. Gibson, P. Gill, and T. Nguyen. An augmented Lagrangian method for total variation video restoration. *Image Processing, IEEE Transactions on*, 20(11):3097–3111, 2011.

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