Computing Diffeomorphic Paths for Large Motion Interpolation

Paper number : 1342

Supplementary material submission

The purpose of this paper

- To introduce a novel framework for interpolating diffeomorphisms.
- Problem: Given two diffeomorphisms, find a “taut” path of diffeomorphisms between them.

Motivation

- To look for a general framework for longitudinal study in computer vision such as evolution scenarios or shape transformations over time which require two different types of optimization: the registration of images and the interpolation of diffeomorphisms.
- Popular previous frameworks : LDDMM-like frameworks [1 - 6].
  - These frameworks perform the two optimization simultaneously.

Motivation (cont')

- Issues with LDDMM-like frameworks :
  - Formulations using the Sobolev metric on velocity fields may cause undesirable geometric distortion in the space of diffeomorphisms.
  - There is no guarantee that these formulations can give best correspondences regardless of the modality of images.
    - Each modality requires a uniquely formulated registration framework.
  - Tuning the set of parameters for a convergent result is heavily data dependent, is cumbersome and tedious.
- Alternative way is to separate registration and interpolation stages to maximize the efficiencies of the two optimizations.

Motivation (cont')

- While there are plenty of registration frameworks for various modalities, frameworks for the interpolation of diffemorphisms are few and far between.
- Due to the infinite dimensionality of the space of diffeomorphisms, it is hard to get a geodesic path on this space through direct computation.
- All these difficulties may be overcome by projecting diffeomorphisms onto a known space which has the closed form for the geodesic.

A novel framework for diffeomorphic paths

- Step 1 : mapping a given pair of diffeomorphisms to the space of densities and estimating its geodesic on the space. A recent theorem in Mathematics tells us that this can be done[7].

- Sept 2 : lifting the geodesic evaluated in the space of densities back to the space of diffeomorphisms.

A novel framework for diffeomorphic paths (cont')

- Notation:

M : a compact n-dimensional Riemannian manifold

$Diff(M)$: the infinite-dimensional group of diffeomorphisms of M

$Diff(M)_\mu$: its infinite-dimensional subgroup of volume-preserving diffeomorphisms

$Diff(M)/Diff(M)_\mu$: its quotient space by the subgroup of volume-preserving diffeomorphisms

$Dense(M)$: the space of densities

id and ρ : initial (identity map) and final points in the space of diffeomorphisms respectively

$\phi(t)$: a diffeomorphic path to be estimated with given boundaries as $\phi(t=0)=id$ and $\phi(t=t_f)=\rho$

$Jac_\mu\phi$: the determinant of the Jacobian of $\phi(t)$

A novel framework for diffeomorphic paths (cont')

- Step 1
  - Introducing a map, Φ which projects diffeomorphisms onto $Dense(M)$ which is equal to $Diff(M)/Diff(M)_\mu$.

$$\Phi : \phi(t) \to f(t) = \sqrt{Jac_\mu \phi(t)/\mu(M)}$$

$$\int_M Jac_\mu \phi = \mu(M)$$

  - $Dense(M)$: the unit sphere in a Hilbert space
  - $f(t)$: a great circle connecting $f(0)$ and $f(t_f)$

A novel framework for diffeomorphic paths (cont')

- Step 2
  - $f(t)$ is bilinear equation.
  - $\phi(t)$ cannot be solved from $f(t)$ directly.
  - Additional constraint is required to lift $f(t)$ to the space of diffeomorphisms.
    - L_2 smoothness constraint of the deformation vector fields over time is adopted.
  - It can be posed as a quadratic program with bilinear constraints :

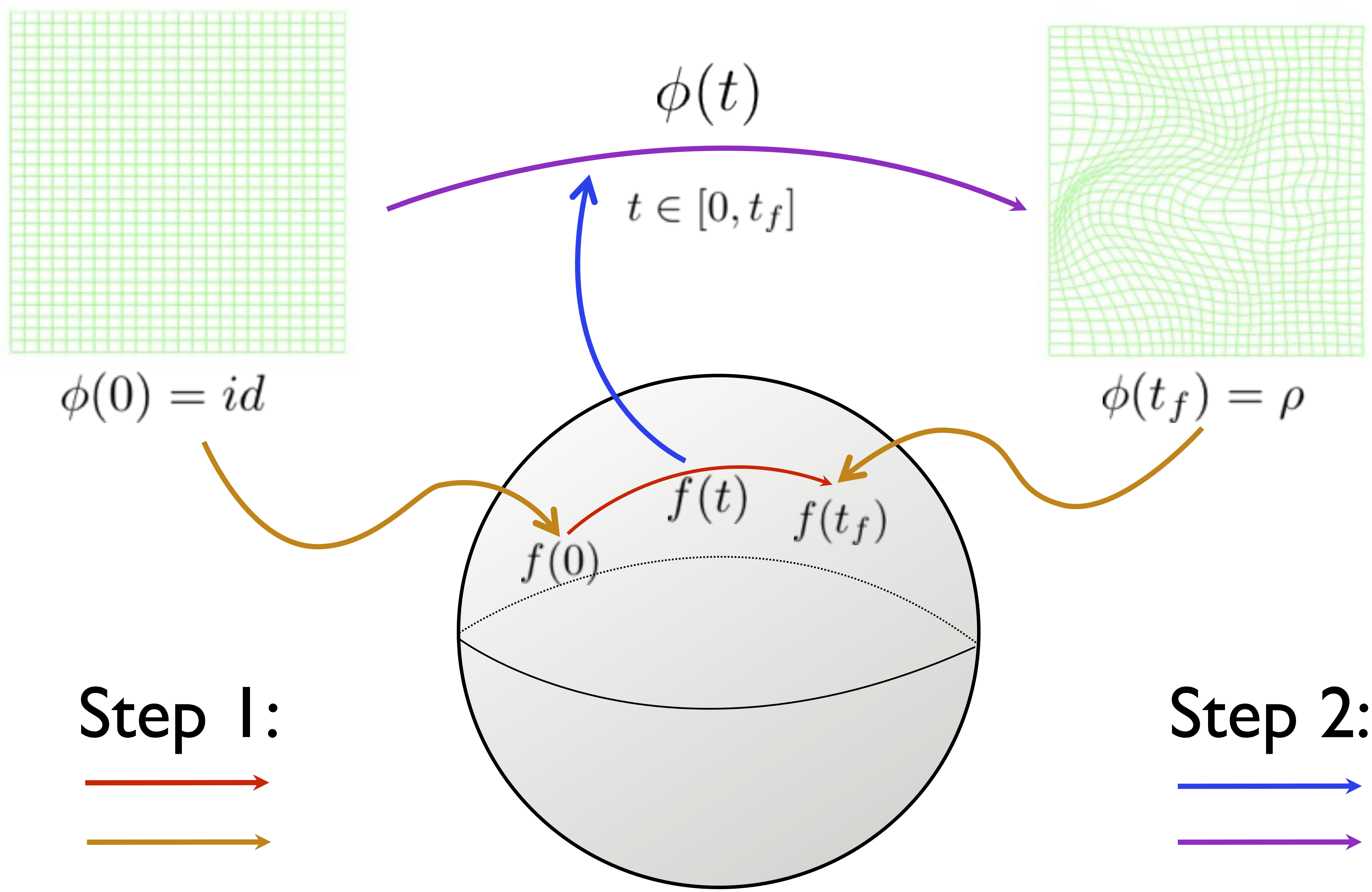
$$\min \int \left| \frac{dU(t)}{dt} \right|^2 + \left| \frac{dV(t)}{dt} \right|^2 d\mu dt$$

$$s.t. \quad Jac_\mu \phi(t) = f(t)^2 \mu(M).$$

with the definition of $\phi(t) = (x + U(t), y + V(t))$

A novel framework for diffeomorphic paths (cont')

- Diagrammatic view :



Step 1:

Step 2:

A novel framework for diffeomorphic paths (cont')

- Numeric solver :
  - The augmented Lagrangian method with penalties is adopted.
  - Details are provided in the section 3 of the paper number 1342.

Experiment

- Purpose : to show why the diffeomorphic paths evaluated with our framework are special.
  - Diffeomorphisms in the path yield locally shape-preserving deformations.
  - Features in a source image are preserved along its diffeomorphic path.
- Registration is NOT a part of our framework.
- The two end points of diffeomorphisms, (id and ρ) are computed via a registration step as preprocessing.
- Our results are compared with the ground truths and results from the popular LDDMM [1] method.

Experiment (cont')

- Data preparation :
  - A source and target pair of images are given :

    I_{source} and I_{target}

  - A diffeomorphic map which deforms the source image to the target image is evaluated :

    $$I_{target} = I_{source} \circ \rho$$

  - The boundary values are set as

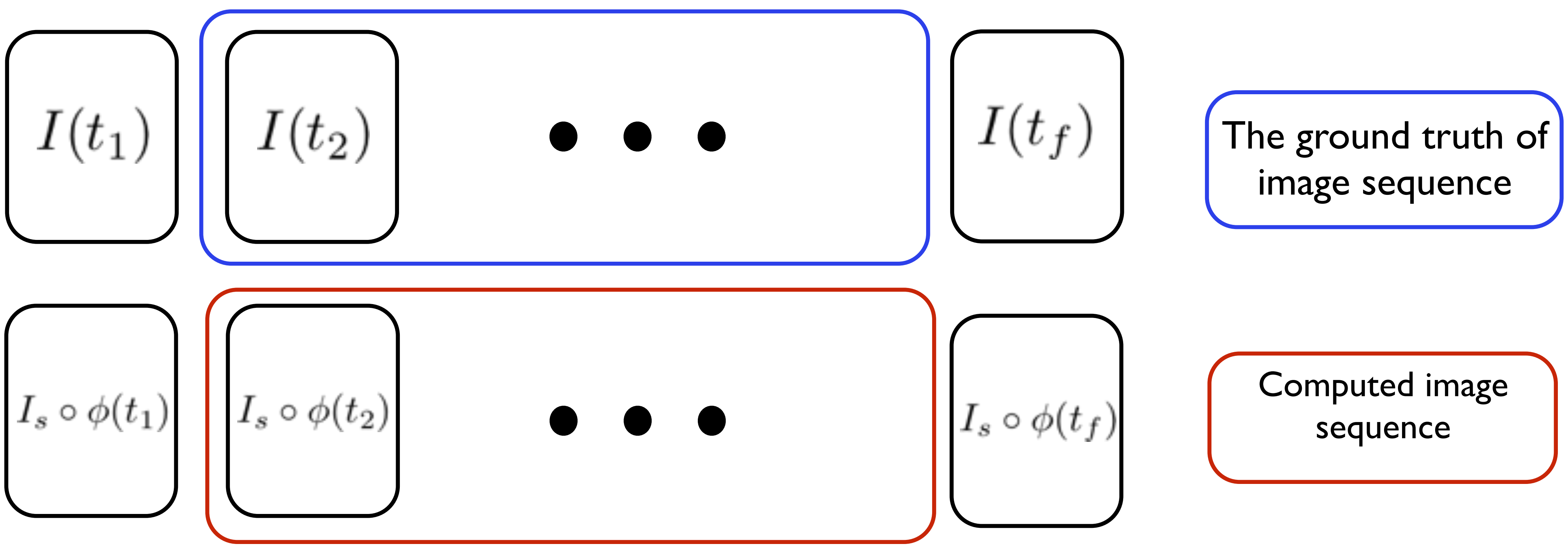
    $\phi(t = 0) = id$ and $\phi(t = t_f) = \rho$

Experiment (cont')

- Main test :
  - $\phi(t)$ is evaluated with the novel framework
  - Results are validated by comparing $I_{source} \circ \phi(t)$ with from $I(t)$ of the ground truth and LDDMM
    - pSNR scores are computed w.r.t the ground truth.
    - Visual comparisons are provided.

$$I_{souce} = I_s = I(t_1)$$

$$I_{target} = I_{t_f}$$



Experiment (cont')

- Example 1: rotating a cylinder
  - Data preparation :
    - The target image is formed from the source by rotating it by 50 degrees.

I_{source}

I_{target}

$\phi(t=0) = id$

$\phi(t=t_f) = \rho$

    - Ground truth : A sequence of 20 images from the source to the target are created by rotating the source with a regular angle difference.

Experiment (cont’)

- Example 1: rotating a cylinder
  - Results : visual comparison (movie files:)

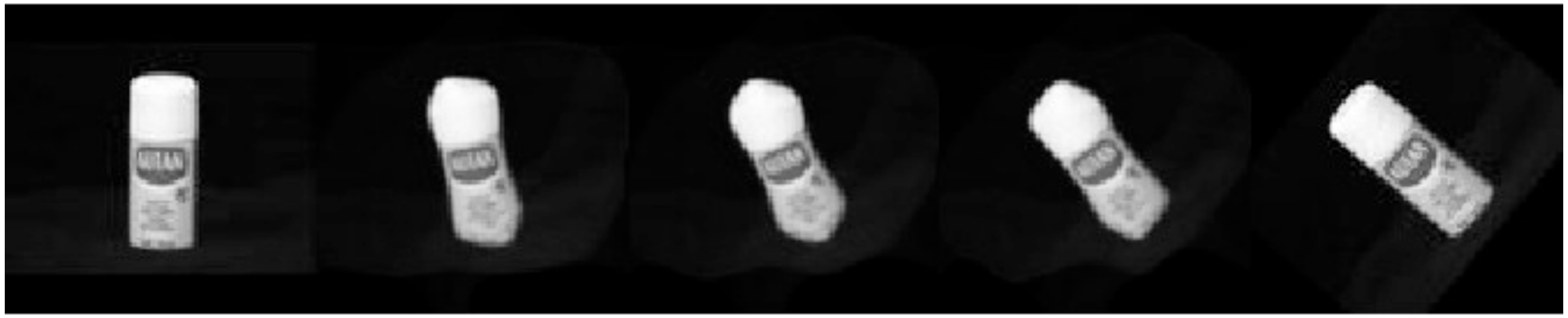
The Ground Truth :

Movie file: pill_org.mov

$I_{target} \circ \phi(t)$

Movie file: pill_dense.mov

LDDMM :

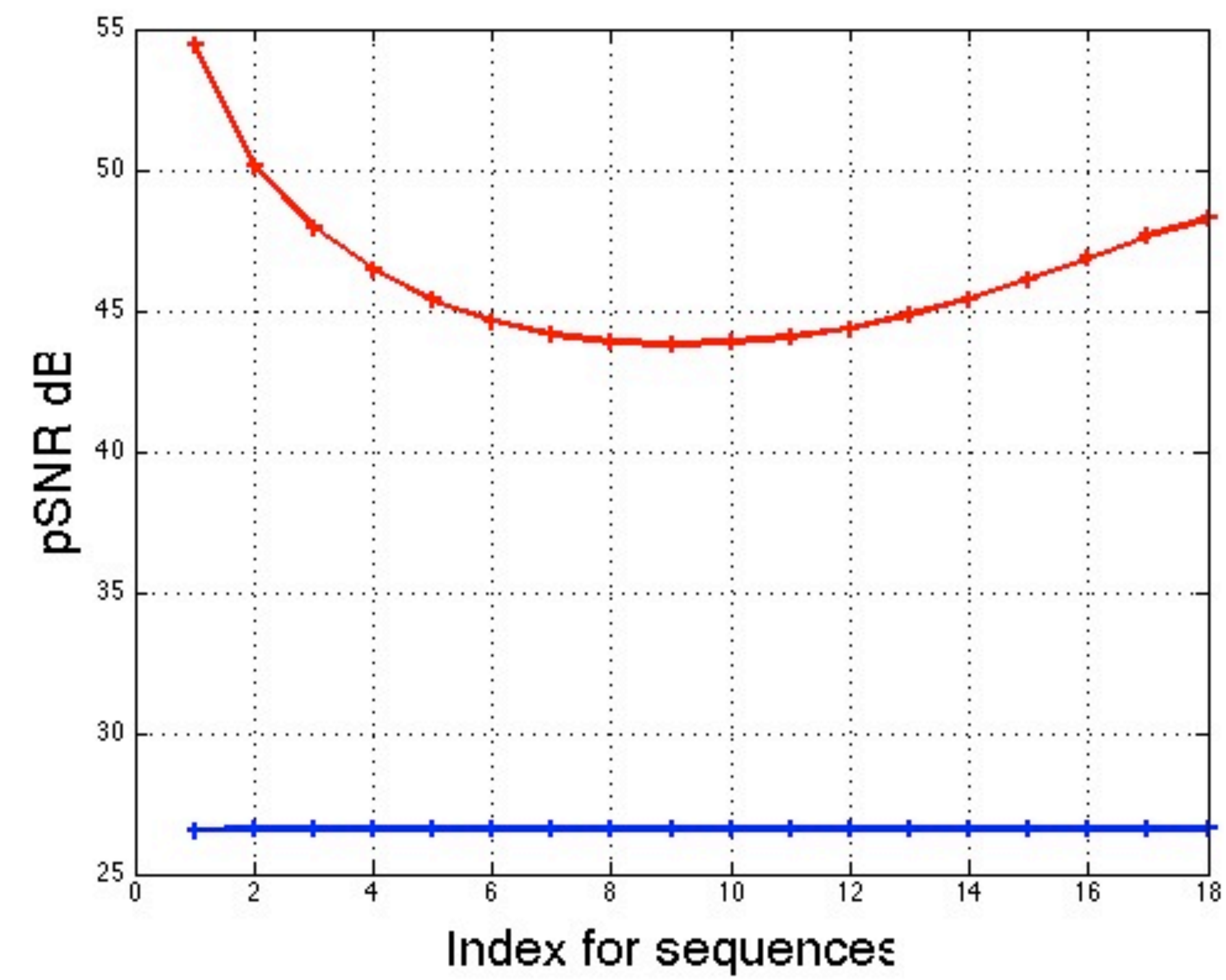


Movie file: pill_lddmm.mov

$t = 0^\circ$ $t = 25^\circ$ $t = 50^\circ$

Experiment (cont')

- Example 1: rotating a cylinder
  - Results : pSNR scores.



Our method :

LDDMM :

	Our method	LDDMM
Average of pSNR scores	46.24dB	26.59dB

Experiment (cont')

- Example 1: rotating a cylinder
  - Results : movie files

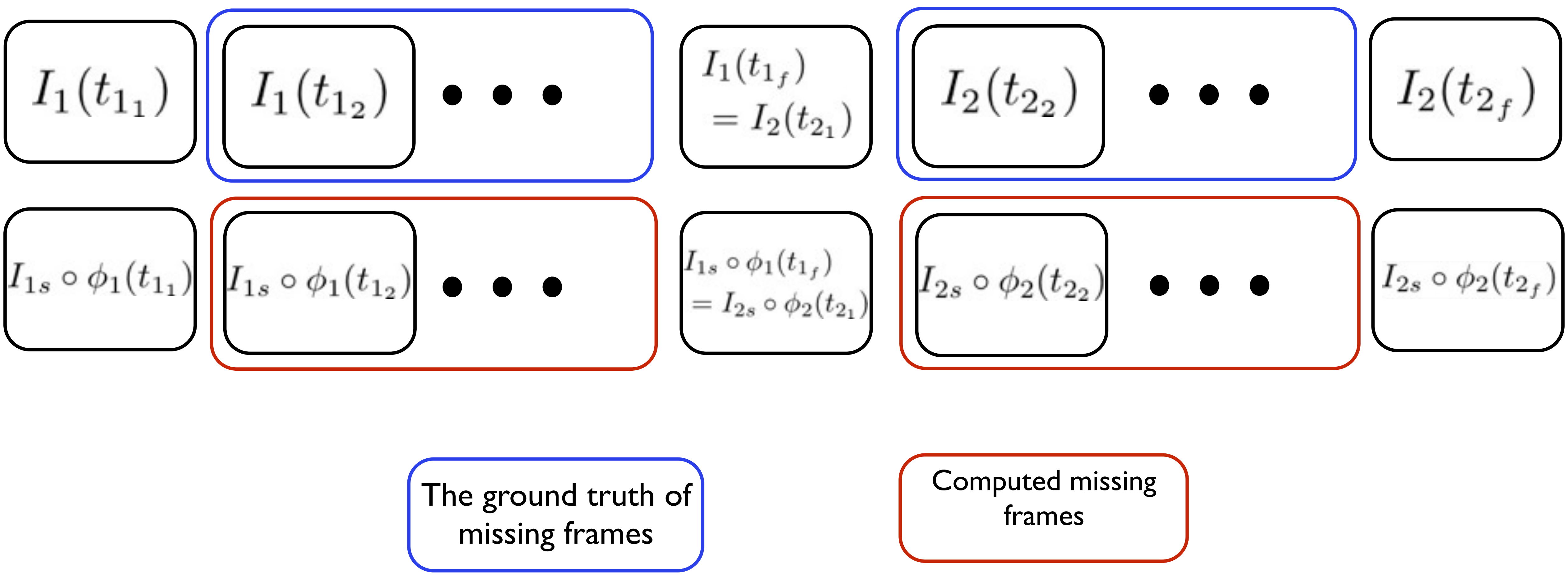
Rotation of the ground truth

Computed rotation by our method

Computed rotation by LDDMM

Experiment (cont')

- The novel framework is applied to estimate the missing frames from video footages, because it can provide easy comparison with ground truths.
- The video footages contain simple human motions.



Experiment (cont')

- Example 2: bending motion
  - The original video footage is provided in .mov format.
  - The footage is sampled in 30 fps, and it produces 34 image frames.
  - All frames except the first, last and 17th frames are removed.
  - The missing frames are then estimated using our framework.

The first frame : I_{first}

The 17th frame : I_{17th}

The last frame : I_{last}

$\rho 1$: a map from the first frame to the 17th frame

$\rho 2$: a map from the 17th frame to the last frame

Experiment (cont')

- Example 2: bending motion
  - There are two diffeomorphic paths to be estimated : between the first and 17th frames, and between the 17th and last frames.
  - Results : estimated missing frames ,

- Between the first and 17th frames
- Between the 17th and last frames


The ground truth Movie file: bending_org.mov The ground truth

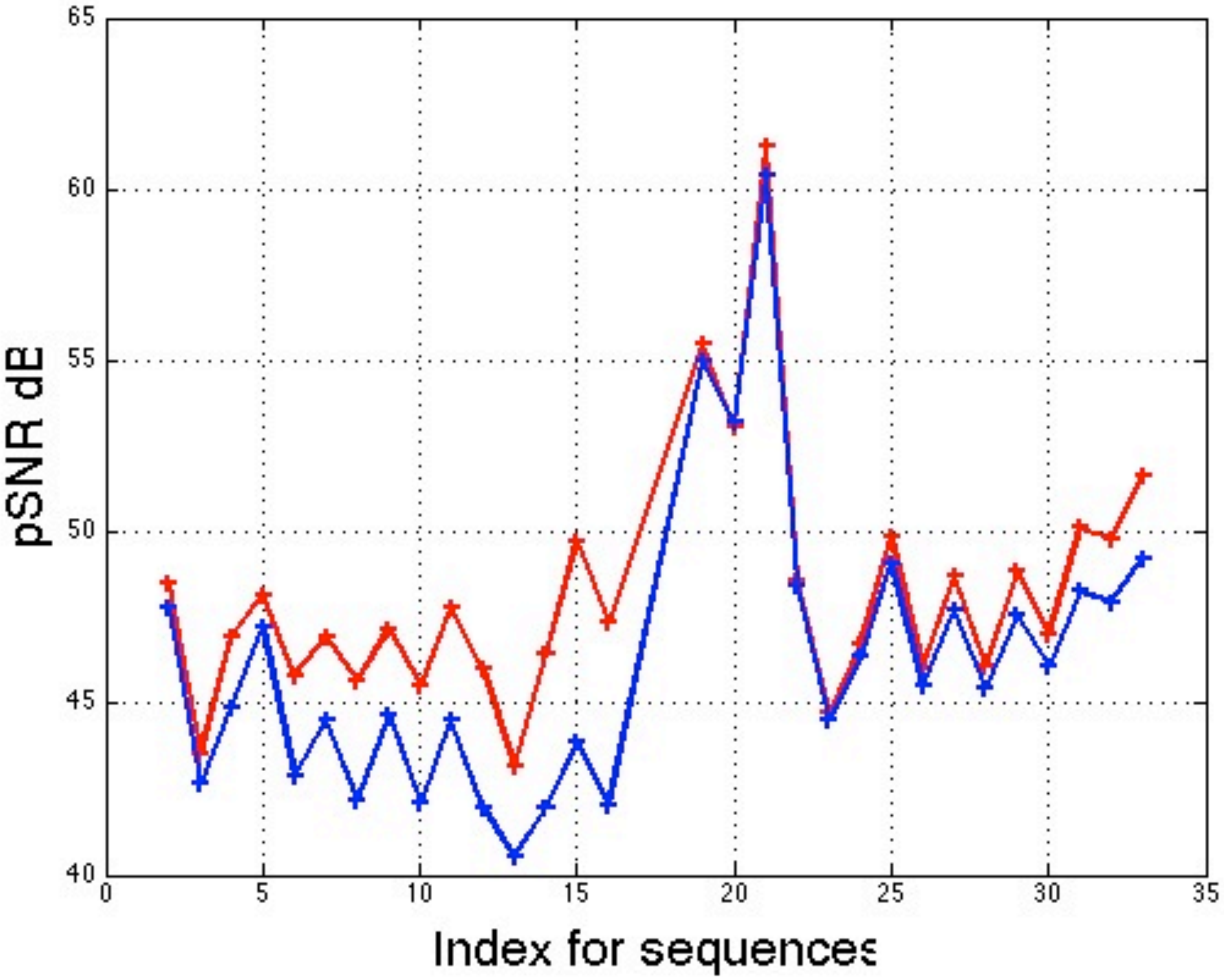

$I_{first} \circ \phi_1(t)$ Movie file: bending_dense.mov $I_{17th} \circ \phi_2(t)$


LDDMM Movie file: bending_lddmm.mov LDDMM

✳Results are down-sampled by half to save space
✳Movies are provided in .mov format.

Experiment (cont')

- Example 2: bending motion
  - Results : pSNR scores



	Our method	LDDMM
Average of pSNR scores	48.25dB	46.2dB

Experiment (cont')

- Example 2: bending motion
  - Results : movie files

Motion of the ground truth

Computed motion by our method

Computed motion by our LDDMM

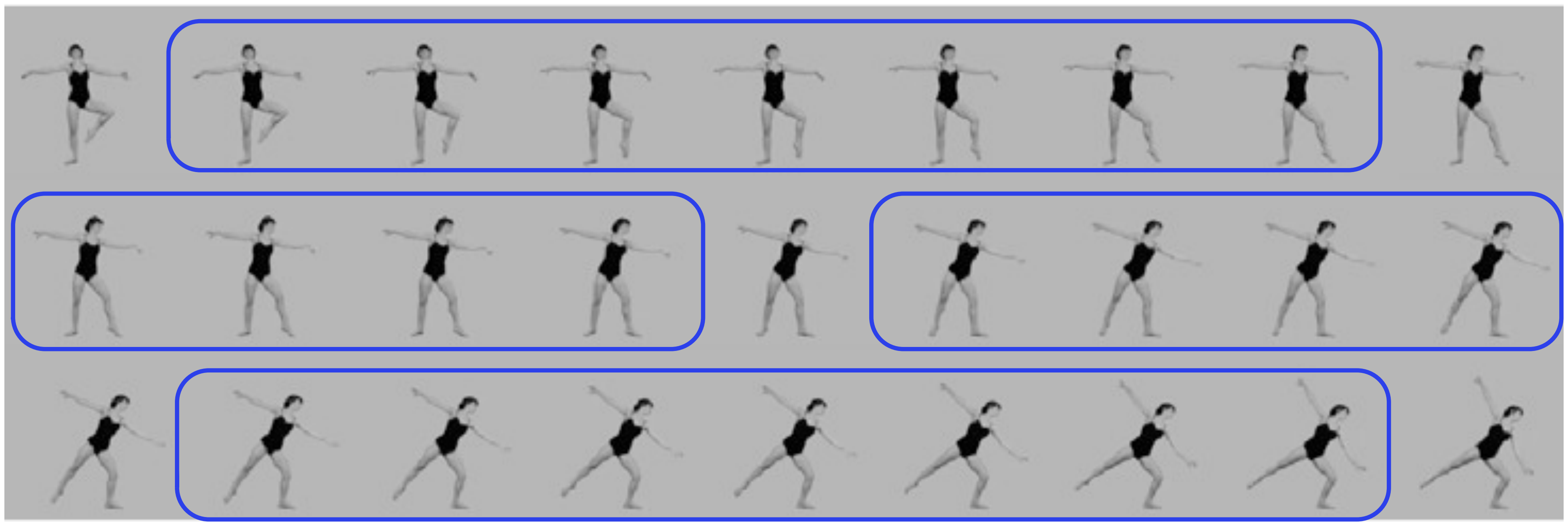
Experiment (cont')

- Example 3: dancing woman
  - The original video footage is provided in .mov format.
  - The footage is sampled in 30 fps, and it produces 29 image frames.
  - All frames are dropped out remaining five intermediate frames including the first and last ones.

  - Missing frames are computed with our framework as done in the previous example.

Experiment (cont')

- Example 3: dancing woman
  - There are five diffeomorphic paths to be estimated.
  - Results : estimated missing frames ,



Ground truth frames

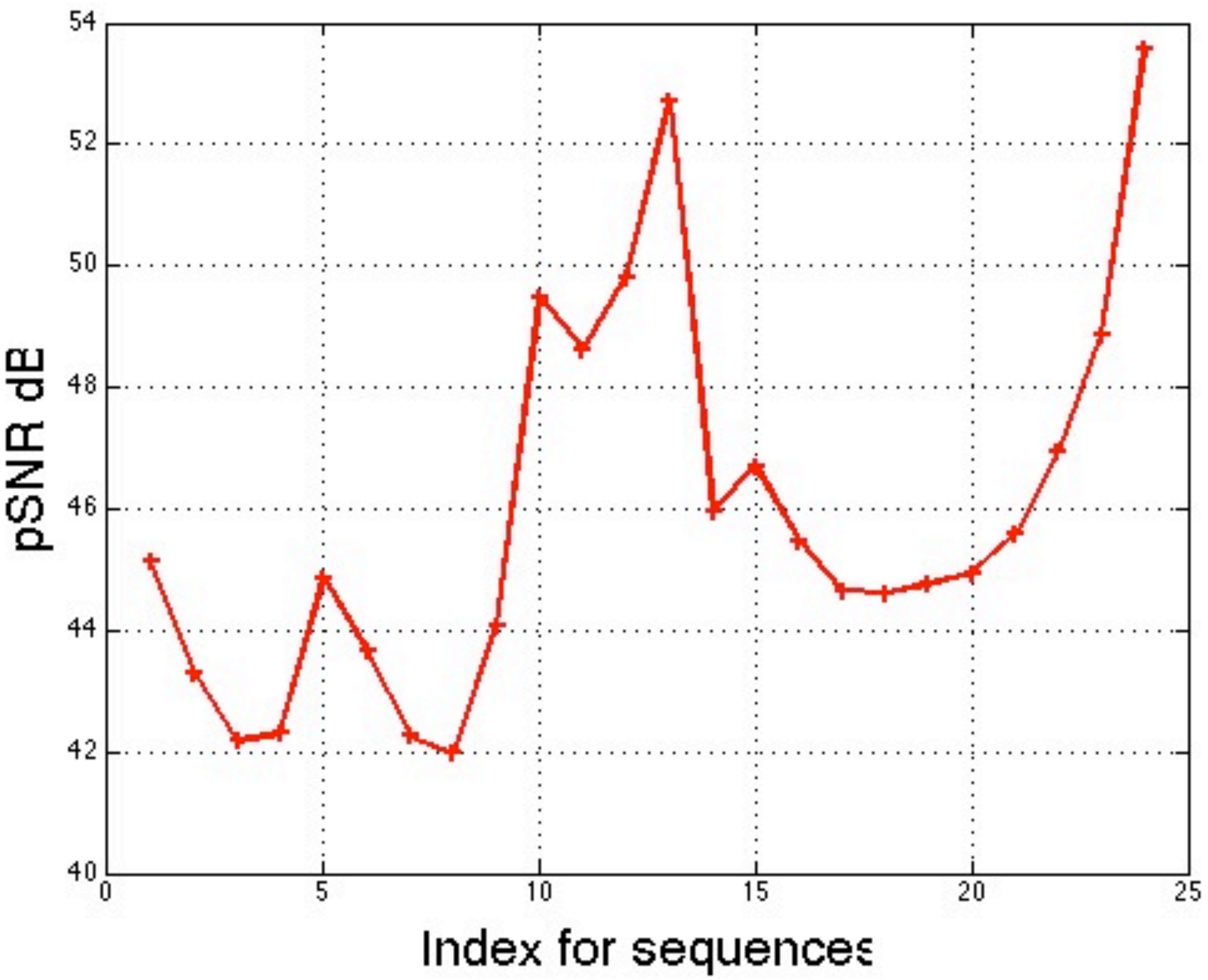
Movie file: dancing_org.mov

Computed frames

Movie file: dancing_dense.mov

Experiment (cont')

- Example 3: dancing woman
  - Results : pSNR scores
    - No LDDMM score : Despite enormous amount of parameter tuning, algorithm failed to converge.



Our method :

	Our method	LDDMM
Average of pSNR scores	45.94dB	

Experiment (cont')

- Example 3: dancing woman
  - Results : movie files

Motion of the ground truth

Computed motion by our method

Discussion

- A novel framework is introduced for the interpolation of two given diffeomorphisms.
- It provides more general approach than LDDMM-like frameworks by separating the registration stage and the interpolation stage of diffeomorphisms.
- While it is hard to find out the set of parameters for convergence in LDDMM, the novel framework is less sensitive to parameters.
  - In our algorithm, $1 \leq c^0 \leq 3$ and $\beta \simeq 1.1$ work well.
  - In the bending motion example, more reference images should be introduced between the first and 17th frame to make LDDMM converge.
  - In the dancing woman example, parameter tuning failed to make LDDMM converge.

Discussion

- In the experiments, diffeomorphisms along the interpolated path yield locally shape-preserving deformations.
- In this paper, only 2D formulation is considered, however, it can be easily extended to 3D.
- This framework is applicable to motion analysis, video analysis, statistical study of longitudinal dataset and etc.

References

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- [2] M. I. Miller, A. Trouv\'{e} and L. Younes, Geodesic shooting for computational anatomy, J. Math. Imaging Vis., 24 (2), 209-228, (2006)
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