Alg.1 describes full optimization method with the bilinear constraints.

Algorithm 1 Optimization of the augmented Lagrangian of the original Lagrangian with bilinear constraint

Set $k \leftarrow 0$
Set $c^k > 0$
Set $\lambda^k = \mathbf{0} \in \mathbb{R}^m$
Set $\beta > 1$
Initialize $\mathbf{V}^0$
while $||\mathbf{h}||^2 < \epsilon$ **do**
  $\mathbf{H}_u^k \leftarrow \mathbf{D}_U(\mathbf{V}^k) + \mathbf{C}_x$
  $\mathbf{A}(c^k) \leftarrow \mathbf{R} + c^k \mathbf{H}_U^{k\intercal} \mathbf{H}_U^k$
  $\mathbf{B}(c^k, \lambda^k) \leftarrow \mathbf{B}_U + 0.5c^k((\mathbf{G}_U^\intercal \mathbf{H}_U^k)^\intercal + \mathbf{H}_U^{k\intercal} \mathbf{G}_U) + (\lambda^{k\intercal} \mathbf{H}_U^k)^\intercal.$
  Solve $\mathbf{A}(c^k)\mathbf{U}^k = -\mathbf{B}(c^k, \lambda^k)$ for $\mathbf{U}^k$
  $\lambda^k \leftarrow \lambda^k + c^k(\mathbf{H}_U^k \mathbf{U}^k + \mathbf{G}_U)$
  $c^k \leftarrow \beta c^k$
  $\mathbf{H}_v^k \leftarrow \mathbf{D}_V(\mathbf{U}^k) + \mathbf{C}_y$
  $\mathbf{A}(c^k) \leftarrow \mathbf{R} + c^k \mathbf{H}_V^{k\intercal} \mathbf{H}_V^k$
  $\mathbf{B}(c^k, \lambda^k) \leftarrow \mathbf{B}_V + 0.5c^k((\mathbf{G}_V^\intercal \mathbf{H}_V^k)^\intercal + \mathbf{H}_V^{k\intercal} \mathbf{G}_V) + (\lambda^{k\intercal} \mathbf{H}_V^k)^\intercal.$
  Solve $\mathbf{A}(c^k)\mathbf{V}^k = -\mathbf{B}(c^k, \lambda^k)$ for $\mathbf{V}^k$
  $k \leftarrow k + 1$
  $\lambda^{k+1} = \lambda^k + c^k(\mathbf{H}_V^k \mathbf{V}^k + \mathbf{G}_V)$
  $c^{k+1} = \beta c^k$
end while
$\mathbf{U}^* \leftarrow U^k$, $\mathbf{V}^* \leftarrow V^k$
$\lambda^* \leftarrow \lambda^k$