Supplementary Material of "Minimum Uncertainty Gap for Robust Visual Tracking"

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Appendix

To minimize the KL divergence in (5), $q(\theta|\gamma, \mathbf{X}_t)$ is assumed to be exponential and $D(p \parallel q)$ is a linear functional of $q(\theta|\gamma, \mathbf{X}_t)$. Then, $\ln q(\theta|\gamma, \mathbf{X}_t)$ is convex with respect to γ. Hence, the global minimum of the KL divergence can be found. As aforementioned, we design $q(\theta|\gamma, \mathbf{X}_t)$ as the exponential family of distributions with the following form:

$$q(\theta|\gamma, \mathbf{X}_t) = g(\gamma)h(\theta)exp(\gamma^T v(\theta)), \tag{17}$$

where $h(\theta)$ and $v(\theta)$ are functions from the space of possible values of θ to the real numbers and $g(\gamma)$ is a normalization factor:

$$g(\gamma)\int_{\Theta} h(\theta)exp(\gamma^T v(\theta))d\theta = 1. \tag{18}$$

By taking the gradient of both sides with respect to γ, the following can be taken:

$$\begin{aligned}\nabla g(\gamma)&\int_{\Theta} h(\theta)exp(\gamma^T v(\theta))d\theta \\ &+ \int_{\Theta} g(\gamma)h(\theta)exp(\gamma^T v(\theta))v(\theta)d\theta = 0,\end{aligned} \tag{19}$$

Since $g(\gamma)h(\theta)exp(\gamma^T v(\theta)) = q(\theta|\gamma, \mathbf{X}_t)$ from (17) and $\int_{\Theta} h(\theta)exp(\gamma^T v(\theta))d\theta = \frac{1}{g(\gamma)}$ from (18), (19) is changed into

$$-\nabla \ln g(\gamma) = \mathbb{E}_{q(\theta|\gamma,\mathbf{X}_t)}\left[v(\theta)\right]. \tag{20}$$

By substituting $q(\theta|\gamma, \mathbf{X}_t)$ in (17) into the KL divergence in (5), we get

$$D(p \parallel q) = -\ln g(\gamma) - \gamma^T \mathbb{E}_{p(\theta|\mathbf{Y}_{1:t},\mathbf{X}_t)}\left[v(\theta)\right] + const. \tag{21}$$

To minimize KL divergence in (21), the gradient of $D(p \parallel q)$ is taken with respect to γ to zero. And we get

$$-\nabla \ln g(\gamma) = \mathbb{E}_{p(\theta|\mathbf{Y}_{1:t},\mathbf{X}_t)}\left[v(\theta)\right]. \tag{22}$$

Since $-\nabla \ln g(\gamma)$ in (20) is equal to $-\nabla \ln g(\gamma)$ in (22), we finally get

$$\mathbb{E}_{q(\theta|\gamma,\mathbf{X}_t)}\left[v(\theta)\right] = \mathbb{E}_{p(\theta|\mathbf{Y}_{1:t},\mathbf{X}_t)}\left[v(\theta)\right]. \tag{23}$$