1 Projective Transformations

This document contains details for locally approximating a projective transformation, that could not be included in the full paper. Recall,

$$x_2 = \frac{h_1 x_1 + h_2 y_1 + h_3}{h_7 x_1 + h_8 y_1 + h_9},$$

$$y_2 = \frac{h_4 x_1 + h_5 y_1 + h_6}{h_7 x_1 + h_8 y_1 + h_9}.$$

The Jacobian is simply a matrix containing four partial derivatives.

$$J_{\mathbf{x}_1} = \begin{bmatrix} \frac{\partial x_2}{\partial x_1} & \frac{\partial x_2}{\partial y_1} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial y_1} \end{bmatrix}$$

These are easily evaluated.

$$\frac{\partial x_2}{\partial x_1} = \frac{y_1(h_1 h_8 - h_7 h_2) + h_1 h_9 - h_7 h_3}{(h_7 x_1 + h_8 y_1 + h_9)^2}$$

$$\frac{\partial x_2}{\partial y_1} = \frac{x_1(h_2 h_7 - h_8 h_1) + h_2 h_9 - h_8 h_3}{(h_7 x_1 + h_8 y_1 + h_9)^2}$$

$$\frac{\partial y_2}{\partial x_1} = \frac{y_1(h_4 h_8 - h_7 h_5) + h_4 h_9 - h_7 h_6}{(h_7 x_1 + h_8 y_1 + h_9)^2}$$

$$\frac{\partial y_2}{\partial y_1} = \frac{x_1(h_5 h_7 - h_8 h_4) + h_5 h_9 - h_8 h_6}{(h_7 x_1 + h_8 y_1 + h_9)^2}$$

Hence, for any given point $\mathbf{x}_1 = [x_1, y_1]^T$, we can calculate the local affine approximation.