Shape of Gaussians as Feature Descriptors*

Liyu Gong, Tianjiang Wang and Fang Liu
Intelligent and Distributed Computing Lab, School of Computer Science and Technology
Huazhong University of Science and Technology, Wuhan, Hubei, 430074, P.R.China
gongliyu@smail.hust.edu.cn tjwang@hust.edu.cn fang.liu@hust.edu.cn

Abstract

*This paper introduces a feature descriptor called Shape of Gaussian (SOG), which is based on a general feature descriptor design framework called Shape of Signal Probability Density Function (SOSPDF). SOSPDF takes the shape of a signal's probability density function (*pdf*) as its feature. Under such a view, both histogram and region covariance often used in computer vision are SOSPDF features. Histogram describes SOSPDF by a discrete approximation way. Region covariance describes SOSPDF as an incomplete parameterized multivariate Gaussian distribution. Our proposed SOG descriptor is a full parameterized Gaussian, so it has all the advantages of region covariance and is more effective. Furthermore, we identify that SOGs form a Lie group. Based on Lie group theory, we propose a distance metric for SOG. We test SOG features in tracking problem. Experiments show better tracking results compared with region covariance. Moreover, experiment results indicate that SOG features attempt to harvest more useful information and are less sensitive against noise.*

1. Introduction

Feature descriptor is one of the most important factor for computer vision. Finding a good solution for most computer vision problems such as object detection and tracking often means finding an effective description of image signals.

Histogram is a kind of feature description scheme often used in computer vision. Different histogram based features have been utilized to address different problems. For example, there are gray scale histograms and color histograms for tracking[2][1][6], histogram of oriented gradients for object detection[3] and histogram of local binary pattern for texture analysis[7]. Although histograms are successfully used in these applications, they have several disadvantages. Firstly, dimensionality of a histogram increases exponentially with number of its signal channels. For example, to describe gray scale information of an image, we may just need a 10 bins histogram, which yields a 10 dimensional feature vector. However, to describe the image in RGB color space, we need a $10^3 = 1000$ bins histogram if we use 10 bins to describe each color channel. Secondly, since histograms do not lie on a vector space, finding an effective distance measurement for histogram is difficult [9]. Finally, because common machine learning algorithms treat feature as a point in vector space, it is hard to classify on histograms optimally.

Recently, Tuzel *et al*. [10] propose a novel feature descriptor called region covariance. The basic idea is describing a signal's feature using its covariance matrix. For a n channel signal, its covariance is a $n \times n$ matrix $C^{n\times n}$. Because covariance matrix is symmetric, $C^{n\times n}$ has only $\frac{1}{2}n(n+1)$ different values. That's to say, dimensionality of region covariance is $\frac{1}{2}n(n+1)$, which is significantly lower than histogram. Using region covariance, it is very convenient to fusion more signal channels to describe an image. For example, Tuzel *et al*. [8] use 7 channels (x-coordinate, y-coordinate, RGB values, horizontal gradients and vertical gradients) in total to track an object, the resulting feature vector is only of $\frac{1}{2} \times 7 \times (7+1) = 28$ dimensions. It is also possible to combine different modalities (*e.g*. color and infrared image) by using region covariance. As covariance matrix can be represented as a connected Riemannian Manifold, distance of region covariance can be measured in Riemannian space. Furthermore, since Riemannian Manifold is locally Euclidean, learning on region covariances can be solved by locally mapping [12].

Although region covariance has been successfully applied to many computer vision fields. A theory analysis of it is necessary. Under a theory framework, we can understand why and how it works. A theory analysis can also guide us to choose proper signal channel for specific application. Moreover, a theory analysis gives us the direction to extend region covariance itself. In this paper, we present

*Correspondence author: Tianjiang Wang. This research was supported by the National High Technology Research and Development (863) Program of China (2007AA01Z161) and the Natural Science Foundation of China (60873226, 60803112)

978-1-4244-3991-1/09/$25.00 ©2009 IEEE

SOSPDF to understand region covariance. Under the view of SOSPDF, histograms and region covariances are unified theoretically. Then we propose a new feature descriptor SOG, which inherits all the advantages of region covariance and can harvest more useful information. There are three major contributions in this paper: Firstly, we give a theory analysis of region covariance and propose the SOSPDF framework. Secondly, we propose a new feature descriptor called SOG. Finally, we identify that SOGs form a Lie group and derive a distance measurement for SOGs based on Lie group theory. We test image SOG features in visual tracking problem. Experiments show better tracking results compared with region covariance.

2. Shape of Signal *pdf*

We present shape of signal probability density function (SOSPDF) as a general framework for feature descriptor design. The term shape here means characteristic of a function. We consider a probability density function (*pdf*) as a geometry object (a curve or a surface), then the shape of this geometry object is the characteristic of the *pdf*.

We model a n channel signal as a n dimension random vector $X \in \Re^n$. The signal contains raw original data or results of some computation. For images, X may contains three color channel (RGB) data. If we want spatial information of pixels, horizontal position (x-coordinate) and vertical position (y-coordinate) can be added to X. If we want to describe image gradient, gradient values can be one channel of X. For example, a typical X may have a form as

$$X = [x, y, R, G, B, |I_x|, |I_y|, \sqrt{I_x^2 + I_y^2}] \tag{1}$$

where x and y are horizontal and vertical positions respectively. R G and B are values of the three color channel. I_x and I_y are the gradients in two direction.

SOSPDF describes a signal based on its *pdf*. Histogram is a discrete approximation of a *pdf*. So it is a kind of SOSPDF. However structure of the space formed by histograms is difficult to analyze. Therefore, it is hard to design effective distance measures and machine learning algorithms for histograms.

Region covariance is also a kind of SOSPDF. It describe a signal using covariance matrix, which is also a characteristic of the signal's *pdf*. Different covariance matrices produce different shapes of curves or surfaces representing the signal's *pdf*s. Covariance matrices, which are symmetric positive definite (SPD), forms a connected Riemannian manifold. Therefore, effective distance measure and machine learning algorithm for region covariance can be designed by analyzing its structure using Riemannian geometric theory.

However, region covariance is an incomplete parameterized multivariate Gaussian, which ignores the important mean vector parameter. If we approximate SOSPDF of X with full parameterized multivariate Gaussian, we can design feature descriptors which are more effective than region covariance. A full parameterized multivariate Gaussian approximation of $f(X)$ is:

$$f(X) = \frac{1}{(2\pi)^{\frac{n}{2}} |\Sigma|^{\frac{1}{2}}} e^{-\frac{1}{2}(X-\mu)^T \Sigma^{-1} (X-\mu)} \tag{2}$$

Where n is the dimensionality of X. The shape of $f(X)$ are controlled by mean vector μ and covariance matrix Σ together. We call this kind of SOSPDF shape of Gaussians (SOG).

In the next section, we present the definition of SOG descriptors and analyze the structure of the space they formed. We identify that SOGs form a Lie group. Then we can measure distance of SOGs based on Lie group theory.

3. Shape of Gaussians

3.1. Transformation of Multivariate Gaussians

Let X_0 be a n dimensional standard multivariate Gaussian distributed random vector. That's to say, each element of X_0 is independent and have mean 0 and standard variance 1. Then n dimensional random vector $X = PX_0 + \mu$ is also multivariate Gaussian distributed and has mean μ and covariance matrix $\Sigma = PP^T$. In reverse, given an X with μ and Σ, finding such a transformation is to find a solution of factorization $\Sigma = PP^T$. Because covariance matrices are SPD matrices, the factorization is always exist. We restrict P to a lower triangular matrix (*i.e.* nonzero elements are found only in the lower triangle of P, including the main diagonal). Then the solution of factorization $PP^T = \Sigma$ is unique and it is the solution of Cholesky factorization indeed. We call transformation $Z = PZ_0 + \mu$ with such kind of P *positive definite lower triangular affine transformation* (PDLTAT). So each multivariate Gaussian distribution can be transformed from a standard multivariate Gaussian distribution by a PDLTAT. The corresponding PDLTAT also represents the SOSPDF of the Gaussian distribution. PDLTAT can be written in a matrix form

$$\begin{bmatrix} X \\ 1 \end{bmatrix} = \begin{bmatrix} P & \mu \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_0 \\ 1 \end{bmatrix} \tag{3}$$

where P is a positive definite lower triangular matrix. We use matrix

$$M = \begin{bmatrix} P & \mu \\ 0 & 1 \end{bmatrix} \tag{4}$$

to represent the PDLTAT. PDLTAT matrices are close under matrix multiplicational and inverse operation, which means the product of two PDLTAT matrices and inverse of a PDLTAT matrix are also PDLTAT matrices. For any two n dimensional multivariate Gaussian distribution X_1 and X_2, let M_1 and M_2 be their PDLTAT matrices, then we have:

$$\begin{bmatrix} X_2 \\ 1 \end{bmatrix} = M_2 \begin{bmatrix} X_0 \\ 1 \end{bmatrix} = M_2 M_1^{-1} \begin{bmatrix} X_1 \\ 1 \end{bmatrix} \tag{5}$$

We can see that PDLTAT from X_1 to X_2 is also an PDLTAT $\Delta M = M_2 M_1^{-1}$. So shape of multivariate Gaussians and their distance measure can be derived by analyzing structure of PDLTAT matrix M.

3.2. Definition of SOG

Let x_i be the i_{th} sample of n dimensional random vector X. Note that x_i and X are both vectors. Then SOG of X is defined as:

$$S(X) = \begin{bmatrix} R & \mu \\ 0 & 1 \end{bmatrix} \tag{6}$$

Where R is the Cholesky factorization of X's covariance matrix Σ. That's to say, $RR^T = \Sigma$ and R is a lower triangle matrix. μ is the mean vector of X. μ and Σ can be computed as follows:

$$\mu = \frac{1}{N} \sum_{i=1}^{N} x_i \tag{7}$$

$$\Sigma = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \mu)(x_i - \mu)^T \tag{8}$$

It can be proved that $S(X)$ forms a Lie group, the group multiplicational and inverse operation is matrix multiplication and inverse respectively. So we can analyze structures of SOGs using Lie group theory.

3.3. Distance Calculation on SOGs

As PDLTAT matrices form a Lie group, we measure distance of two SOGs as distance between their corresponding PDLTAT based on Lie group theory. Lie group is also a differentiable manifold. Distance of two points on the manifold (*i.e.* elements of Lie group) can be measured by length of curve connecting these two points. The curve with minimum length between two points is called geodesic. The length of geodesic can be measured in tangent space.

The tangent space of Lie group G to its identity element forms a Lie algebra $\mathfrak{g}$. We can map between Lie group and its tangent space from identity element I using $\exp$ and $\log$ map.

$$m = \log(M) \tag{9}$$
$$M = \exp(m) \tag{10}$$

where $m \in \mathfrak{g}$ and $M \in G$ are elements of Lie algebra and Lie group respectively.

Specially, Lie algebra of n dimensional PDLTAT is the set of matrices

$$m = \begin{bmatrix} U & v \\ 0 & 0 \end{bmatrix} \tag{11}$$

where U is a $n \times n$ lower triangular matrix, $v \in \Re^n$ is a $n \times 1$ vector. We often unfold m to a $\frac{1}{2}n(n+3)$ dimensional vector.

For matrix Lie group such as PDLTAT matrices, a $\log$ map in equation (9) is a common matrix $\log$ operation with a following matrix to vector unfolding manipulation (*i.e.* to complete a $\log$ map operation, we should complete a matrix $\log$ operation firstly, then unfold the resulting $(n+1) \times (n+1)$ matrix to a $\frac{1}{2}n(n+3)$ dimensional vector since it has only $\frac{1}{2}n(n+3)$ variables). In the same way, a $\exp$ map in equation (10) is a vector to matrix manipulation with a following common matrix $\exp$ operation.

The geodesic length d between two group elements M_1 and M_2 of PDLTAT can be computed as

$$d(M_1, M_2) = \| \log(M_1^{-1} M_2) \| \tag{12}$$

where $\| \boldsymbol{.} \|$ is L_2 norm of a vector. Geodesic length is an effective distance measure of points on a manifold. So we can measure distance between two SOGs using geodesic length of their correspondence PDLTAT matrices.

4. Image SOG features

SOG is a general feature description scheme, which can describe images, audios and other signals even combination of different signals. In this section, we discuss SOG for images.

To describe an image using SOG, the first step is to choose signal channels we need. Signals may contain raw data in an image as well as data produced from raw data. For example, sometimes we just need RGB values to represent color information of an image. But if we need to describe how color is distributed in image plane, spatial coordinates should be added to signals. In some applications, gradients should be included in signals.

Let n be the number of signal channel we choose, for each pixel in an image or a region of the image we can get a $n \times 1$ vector. Let N be number of pixels, we can get N samples $x = \{x_i | i = 1, 2 \ldots N\}$ of signal X. Using formula (7),(8) and (6), we can compute SOG of the image (or region of the image).

In object detection and tracking, we usually need to compute SOG of many regions which are heavily overlapped in the same image. Under such situation, the mean vector μ and covariance matrix Σ which are needed by SOG can be computed using a fast algorithm based on the integral image idea. Tuzel *et al*. [12] present the fast computation algorithm for covariance matrix. Fast mean vector computation can be derived easily in the same manner.

5. Experiments

To compare SOGs with region covariances, we apply them in visual tracking. The main goal of our experiments is to test the effectiveness of SOGs. So we do not focus on designing sophisticated tracking algorithms. In all of our experiment, we using a local exhaustive search algorithm to track manual initialized object.

We conduct experiments on the image sequences of OTCBVS dataset [4]. We use seven channels signal for color image sequences. The signal vector is chosen as

$$X = [x, y, R, G, B, |I_x|, |I_y|] \tag{13}$$

where x and y are horizontal and vertical positions respective. R, G and B are values of three color channel. I_x and I_y are gradients in two directions. Both our SOG descriptors and region covariance are calculated on the same signals.

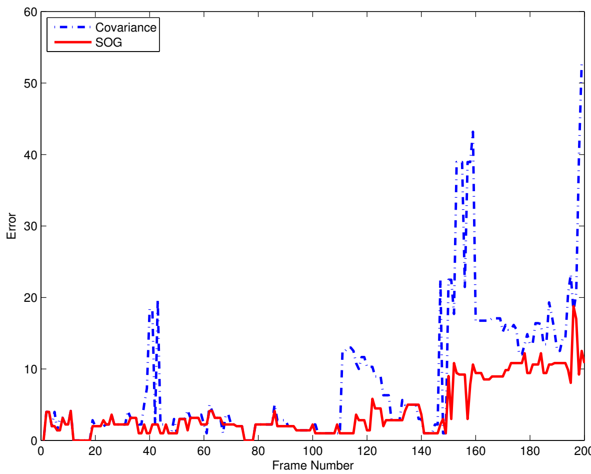
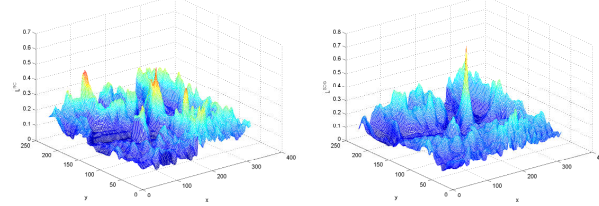


Figure 2. Tracking results on OTCBVS OSU 1b color image sequence. The horizontal axis is frame number of images, The vertical axis is tracking errors. The red solid line and blue dash-dot line are plotted by our proposed SOG tracker and region covariance tracker.

The tracked object is represented as an rectangle region. Initial position of the object is set manually. Features of different positions are computed on the corresponding rectangle regions. For SOG feature, we set the mean of x-coordinate and y-coordinate to zero. Because same sized boundary box at different position contain different x-coordinate and y-coordinate values are region related, setting their mean to zero can eliminate the affect. Features of initial position in the first frame are used as model of object appearance. In the following frames, we locally search the region around the position of the preceding frame. We use a fixed search radius, which is 40 pixels, and a fixed search step, which is 2 pixels. The size of the boundary box is fixed too. In other words, we do not search object at different scales. For each position in the search range, we compute a feature descriptor. Then we calculate distances from these descriptors to the object appearance model. At each frame, we update object position to the best matched position.



(a) (b)

Figure 3. Similarity map of region covariance (a) and SOG (b). The mode of SOG's similarity map is more apparent than region covariance's.

Figure 2 shows error curves of both SOG based and region covariance based tracker. We use the Euclidean distances from positions produced by the two tracking algorithms to ground truth positions as tracking errors. The results prove that our SOG feature descriptor is more effective than region covariance. Figure 1 gives some tracking examples.

To give an illustration of the effectiveness of SOG, we plot a similarity map in figure 3. The similarity map was computed on the 120_{th} frame of OTCBVS OSU 1b image sequence. Both similarity of region covariance and SOG are computed. We exhaustively scan the image with window of object size. The scan step is one pixel horizontal and vertical. At each position (x, y) we compute a region covariance $RC_{x,y}$ and a SOG feature vector $S_{x,y}$. Note that the mean of x-coordinate and y-coordinate is set to zero in SOG features as discussed in previous paragraph. Then distance of region covariance $\rho^{RC}_{x,y}$ and distance of SOG feature $\rho^{SOG}_{x,y}$ from the associate object appearance model are measured. At last we calculate the similarity measurement $L^{RC}_{x,y}$ and $L^{SOG}_{x,y}$ as follows

$$L^{RC}_{x,y} = \frac{1}{\rho^{RC}_{x,y}} \tag{14}$$

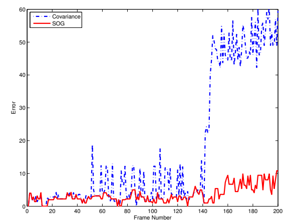
$$L^{SOG}_{x,y} = \frac{1}{\rho^{SOG}_{x,y}} \tag{15}$$

The whole map L^{RC} and L^{SOG} are show in figure 3. We observed that the global maximum value of SOG's similarity map is much higher than local maximums. In another word, the mode of SOG's similarity map is very apparent. For region covariance, differences between the largest similarity value and the second largest values are much smaller. This property proves that SOG harvest more useful information than region covariance.

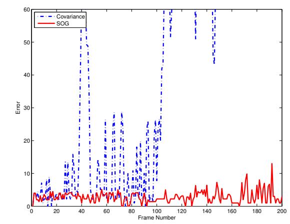
To test sensitivity against noise, we contaminated the image color values with additive zero mean Gaussian noise with variance σ^2. We use four different values for σ^2, which is 0.001, 0.01, 0.1 and 0.3. The error curves are show in figure 4. We observed that SOG is less sensitive against noise. Actually, when variance of noise increases, region covariance based tracker lost the object but SOG based tracker

Figure 1. Tracking example on OTCBVS OSU 1b color image sequence. From left to right and top to down, the frame numbers are 1,19,32,39,121,171,184,194 respectively. Results of region covariance and SOG are marked by blue and red ellipses respectively.

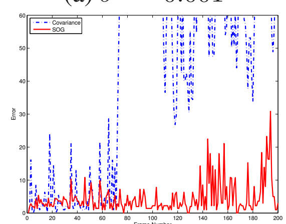
still have very low error. Even when the variance is 0.3, the image is blurred heavily, SOG based tracker still works stably. This can be explained that zero mean Gaussian noise affects the covariance of signal but does not change the mean of signal, by adding mean to feature descriptor, SOG is less sensitive against noise than region covariance. Actually, region covariance can not discriminate two distribution with different mean but same covariance. Moreover, because region covariance dose not take the mean of a distribution into consideration, two pair of distributions which have same distance under SOG feature will have different distance under region covariance. Just as points in a circle have same distance to center point if we use x-coordinates and y-coordinates together to compute distance, but if we compute distance using x-coordinates only, they have different distances.



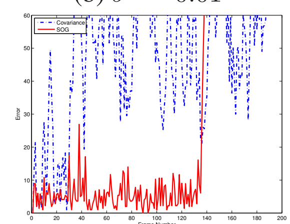
(a) $\sigma^2 = 0.001$



(b) $\sigma^2 = 0.01$



(c) $\sigma^2 = 0.1$



(d) $\sigma^2 = 0.3$

Figure 4. Tracking results on OTCBVS OSU 1b color image sequence with zero-mean gaussian noise added. The horizontal axis is frame number of images, The vertical axis is tracking errors. The red solid line and blue dash-dot line are plotted by our proposed SOG tracker and region covariance tracker. The four curves show errors when gaussian noise with different variance σ^2 contaminated. In (a) (b) (c) and (d), σ^2 is 0.001 0.01 0.1 and 0.3 respectively.

6. Discussion

One may argue that SOG is just a slight extension of region covariance. But the most important advantage of region covariance is that its structure can be well analyzed using Riemannian geometry, which means effective distance measure and classification algorithms can be designed for it. Simply adding a mean vector to region covariance will destroy its Riemannian structure. So the corresponding advantages will not exist. To analyze the structure of SOG in a way like region covariance, more work need to do. In this paper, we form SOGs as PDLTAT matrices utilizing Cholesky factorization. Then we identify that PDLTAT matrices form a Lie group. So we analyze the structure of SOG using Lie group theory.

Just as histogram and region covariance, SOG is not a specific descriptor, but a scheme for designing descriptors. We can choose different signal channel to design different descriptors according to application requirement. Whether a SOG descriptor is sensitive to illumination change or not depends on choice of signal channel. As we know, we have color histograms which are sensitive to illumination change and histogram of oriented gradients (HOG) which are not. For SOG, it can be done in the same way. If we chose signal channels which are insensitive to illuminate change, then the corresponding SOG descriptor will be insensitive to illumination change too.

Although it is possible to use mixture of Gaussian (MoG) in our SOSPDF framework, there are some issues remained. SOG can be extracted in a very fast way as discussed in section 4. But algorithms such as EM for estimating MoG are very slow. Furthermore, it is difficult to analyze MoG's structure, so to design an effective distance measure. Nevertheless, as discussed in section 7, integrate MoG with SOSPDF is a promising direction for our future work.

This paper focuses on image feature descriptor, but not sophisticated tracking algorithm. Object tracking itself is an open problem which has many challenges such as partial and full occlusions. Solving all these problems in object tracking involves a lot of technologies such as Kalman filter, particle filter, mean shift *et al*. However, it is possible to integrate SOG feature into sophisticated tracking algorithms.

The most important contribution of this paper is the identification of Gaussian function space's Lie group structure. Therefore, we can analyzing Gaussian function space using Lie group theory. Utilizing Lie group structure of Gaussian function space, we can derive more effective distance measures, mean computation and machine learning algorithms. As Gaussian *pdf* is very useful in computer vision, especially in feature description, analyzing the structure of its space have many promising application. For example, image feature space analyzing gains more and more attention in recent years [12] [5] [13]. But all these related works focus on region covariance. To the best of our knowledge, no one has analyzed structures of other feature descriptors. In this paper, we analyze structure of SOG, which is a more general and more effective feature descriptor than region covariance.

Tuzel *et al*. [11] also measure distance of two affine matrices using Lie group theory. But we note that the idea and problem in this paper is very different from theirs. We focus on image feature descriptor. But they focus on object motion information. Although both of us use the geodesic length as a distance measure, it is an already existent well-known mathematical theory. More precisely, we make a connection between Gaussian pdfs and PDLTAT matrices using Cholesky factorization. PDLTAT matrices is a subspace of affine matrices.

7. Conclusion

In this paper, we present a novel feature descriptor called Shape of Gaussians (SOG) and a feature descriptor framework called Shape of Signal Probability Density Function (SOSPDF). SOG, region covariance and histogram are theoretically unified under SOSPDF. Using transformations of multivariate Gaussians, we form SOGs as positive definite lower triangular affine transform matrices. Then we identify that SOGs form a Lie group. Based on Lie group theory, we derive a distance measure of SOGs. Experiments are conducted on visual tracking problem. Tracking results show that SOG is more effective and less sensitive against noise than region covariance.

There are several interesting issues for future work, some of which we are currently working on. The first is to design a more sophisticated tracking algorithm using SOG. For tracking, the update of object appearance model is important. As SOG does not lie on Euclidean space, we should complete the update procedure by mean computation on Lie group. A second issue is to classify on SOGs, such as designing an object detection algorithm based on SOG. Finally, we may also consider extending SOG to more discriminative descriptors based on more sophisticated distribution approximate model, such as Gaussian Mixtures.

References

[1] A. Adam, E. Rivlin, and I. Shimshoni. Robust fragments-based tracking using the integral histogram. In *IEEE Conference on Computer Vision and Pattern Recognition(CVPR)*, 2006.

[2] D. Comaniciu, V. Ramesh, and P. Meer. Kernel-based object tracking. *IEEE Transactions on Pattern Analysis and Machine Intelligence(PAMI)*, 25(5):564–577, 2003.

[3] N. Dalal and B. Triggs. Histograms of oriented gradients for human detection. In *IEEE Conference on Computer Vision and Pattern Recognition(CVPR)*, 2005.

[4] J. W. Davis and V. Sharma. Background-subtraction using contour-based fusion of thermal and visible imagery. *Computer Vision and Image Understanding(CVIU)*, 106(2-3):162–182, 2007.

[5] X. Li, W. Hu, Z. Zhang, X. Zhang, M. Zhu, and J. Cheng. Visual tracking via incremental log-euclidean riemannian subspace learning. In *IEEE Conference on Computer Vision and Pattern Recognition(CVPR)*, 2008.

[6] K. Nummiaro, E. Koller-Meier, and L. V. Gool. An adaptive color-based particle filter. *Image and Vision Computing*, 21(1):99–110, January 2003.

[7] T. Ojala, M. Pietikainen, and T. Maenpaa. Gray scale and rotation invariant texture classification with local binary patterns. In *European Conference on Computer Vision(ECCV)*, 2000.

[8] F. Porikli, O. Tuzel, and P. Meer. Covariance tracking using model update based on lie algebra. In *IEEE Conference on Computer Vision and Pattern Recognition(CVPR)*, 2006.

[9] Y. Rubner, C. Tomasi, and L. J. Guibas. The earth mover's distance as a metric for image retrieval. *International Journal of Computer Vision(IJCV)*, 40(2):99–121, 2000.

[10] O. Tuzel, F. Porikli, and P. Meer. Region covariance: A fast descriptor for detection and classification. In *European Conference on Computer Vision(ECCV)*, 2006.

[11] O. Tuzel, F. Porikli, and P. Meer. Learning on lie groups for invariant detection and tracking. In *IEEE Conference on Computer Vision and Pattern Recognition(CVPR)*, 2008.

[12] O. Tuzel, F. Porikli, and P. Meer. Pedestrian detection via classification on riemannian manifolds. *IEEE Transactions on Pattern Analysis and Machine Intelligence(PAMI)*, 30(10):1713–1727, 2008.

[13] A. Tyagi and J. W. Davis. A recursive filter for linear systems on riemannian manifolds. In *IEEE Conference on Computer Vision and Pattern Recognition(CVPR)*, 2008.