

Abnormal Crowd Behavior Detection using Social Force Model

Ramin Mehran
Computer Vision Lab,
University of Central Florida
ramin@cs.ucf.edu

Alexis Oyama
University of Nevada at Reno
oyamaa@unr.nevada.edu

Mubarak Shah
Computer Vision Lab,
University of Central Florida
shah@cs.ucf.edu

Abstract

In this paper we introduce a novel method to detect and localize abnormal behaviors in crowd videos using Social Force model. For this purpose, a grid of particles is placed over the image and it is advected with the space-time average of optical flow. By treating the moving particles as individuals, their interaction forces are estimated using social force model. The interaction force is then mapped into the image plane to obtain Force Flow for every pixel in every frame. Randomly selected spatio-temporal volumes of Force Flow are used to model the normal behavior of the crowd. We classify frames as normal and abnormal by using a bag of words approach. The regions of anomalies in the abnormal frames are localized using interaction forces. The experiments are conducted on a publicly available dataset from University of Minnesota for escape panic scenarios and a challenging dataset of crowd videos taken from the web. The experiments show that the proposed method captures the dynamics of the crowd behavior successfully. In addition, we have shown that the social force approach outperforms similar approaches based on pure optical flow.

1. Introduction

One of the most challenging tasks in computer vision is analysis of human activity in crowded scenes. While understanding of actions performed by individuals is a problem yet to be fully solved, crowd scene analysis faces even more challenges like emergent behaviors and self-organizing activities [11].

Crowd behavior analysis in computer vision is a new area of interest in the research community which could potentially lend itself to a host of new application domains, such as automatic detection of riots or chaotic acts in crowds and localization of the abnormal regions in scenes for high resolution analysis.

Crowd behavior analysis is thoroughly studied in the field of transportation and public safety where some well-established models have been developed for describing the

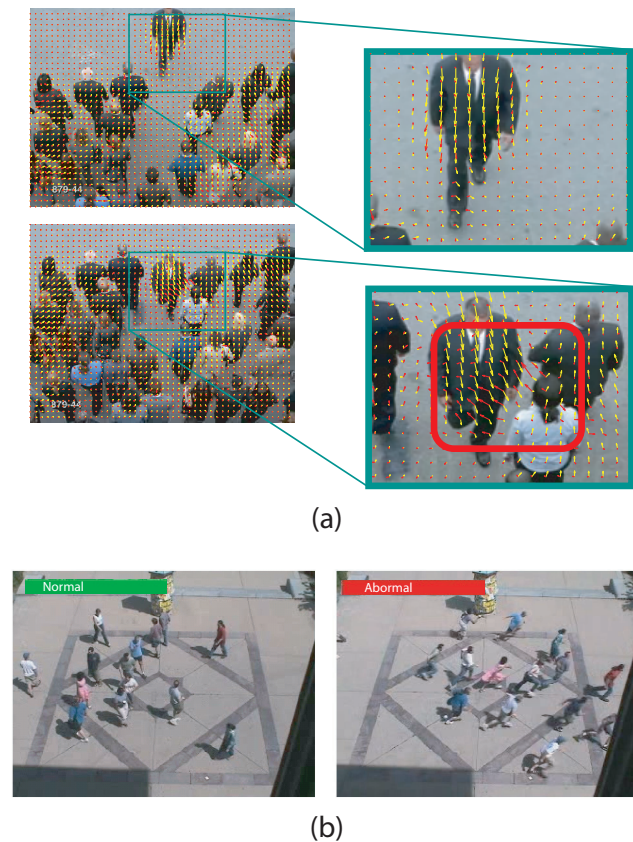


Figure 1. (a) The Optical flow (yellow) and the computed interaction force (red) vectors of two sampled frames. Note that the interaction force is computed accordingly for pedestrians who are approaching each other (red box). (b) An example of detection of escape panic using the proposed approach. Green denotes the normal and red denotes the abnormal frame.

individual and group behaviors in crowded scenes [17][18]. At high level, there are three main approaches in modeling the crowds in this community. (1) Microscopic approach which defines pedestrians' motivation in movement and treats crowd behaviors as a result of a self-organization process. Social Force Model by Helbing *et al.* in [17] is the best known example of this approach. (2) Macroscopic

approach which focuses mainly on goal-oriented crowds. In this approach, a set of group-habits is determined based on the goals and destinations of the scene. Pedestrians are then partitioned into different groups to follow the predetermined habits. Therefore, instead of determining the motion of individuals the group behaviors are modeled [18][13]. (3) Hybrid methods which inherit from macroscopic models as well as microscopic ones [28].

Based on socio-psychological studies, Helbing *et al.* in [17] originally introduced Social Force model to investigate the pedestrian movement dynamics. The social force captures the effect of the neighboring pedestrians and the environment on the movement of individuals in the crowd. Later, Helbing published his popular [10] work in combining the collective model of social panic with social force model to create a generalized model. In this model, both psychological and physical effects are considered in formulating the behavior of the crowd.

Recently, the computer vision community has focused on crowd behavior analysis. In [6] a review of the latest research trends and approaches from different research communities is provided. There are two main approaches in solving the problem of understanding crowd behaviors. In the conventional approach, which we refer as the “object-based” methods, a crowd is considered as a collection of individuals [22][19]. Therefore, to understand the crowd behavior it is necessary to perform segmentation or detect objects to analyze group behaviors [7]. This approach faces considerable complexity in detection of objects, tracking trajectories, and recognizing activities in dense crowds where the whole process is affected by occlusions. On the other hand, “holistic” approaches [15][2] consider the crowd as a global entity in analysis of medium to high density scenes. In related works by Avidan *et al.* in [23] and Chan and Vasconcelos in [8], instead of tracking individual objects, scene modeling techniques are used to capture features for the crowd behavior and car traffic respectively. These are top-down approaches which directly tackle the problem of dense occluded crowds in contrast to the object-based methods. In addition, there are some works that mix the bottom-up view of object-based methods with top-down methods such as Ali and Shah’s [3] for tracking humans in very dense crowds.

Meanwhile, crowd behavior analysis has been an active research topic in simulation and graphic fields where the main goal is to create realistic crowd motions. The real crowd motion exhibits complex behaviors like line forming [18], laminar and turbulent flow [14][29], arching and clogging at exits, jams around obstacles [17], and panic [10]. Exact simulation of a crowd using behavior modeling leads to design of proper public environments that minimize the possibility of the hazardous events. Furthermore, in the graphics community, accurate mod-

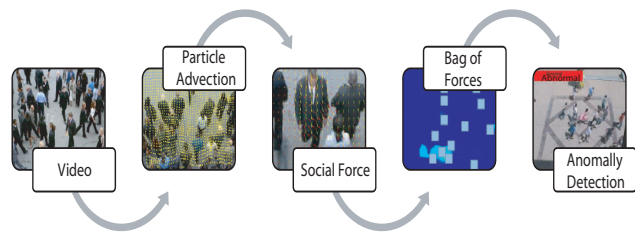


Figure 2. The summary of the proposed approach for abnormal behavior detection in the crowd videos.

eling of the crowd movements is used to create realistic special effects of crowds without the need for human actors[25][9][20][27].

1.1. Overview of the Method

In this paper, we introduce a computer vision method to detect and localize abnormal crowd behavior using the *Social Force* model [17]. Social force model describes the behavior of the crowd as the result of interaction of individuals. Therefore, the abnormal crowd behavior is essentially an eccentric state of the crowd interactions. Since social force model in [17] emulates the crowd dynamics with a high degree of accuracy, we conclude that abnormal social forces in the crowd portray abnormal behaviors. We estimate the social force parameters to create a model of likely behaviors in the crowd.

Figure 2 summarizes the main steps of the algorithm. In our method, we avoid tracking of objects to avert typical problems in tracking of high density crowds such as extensive clutter and dynamic occlusions. Instead, we incorporate a holistic approach to analyze videos of crowds using the particle advection method similar to [2]. In this approach, we place a grid of particles over the image and move them with the underlying flow field. We compute the social force between moving particles to extract interaction forces. In a crowd scene, the change of interaction forces in time determines the on going behavior of the crowd. We capture this by mapping the interaction forces to image frames. The resulting vector field is denoted as *force flow*, which is used to model the normal behaviors in a bag of words approach [12].

Andrade *et al.* [15] proposed a method for event detection in the crowd scene using HMM. However, the principal contribution of our work is to capture dynamics of the interaction forces in the crowd in addition to optical flow. Antonini *et al.* [16] reported a model for describing pedestrian behaviors to enhance tracking and detection. On the contrary, our primary goal is to introduce a holistic method independent of object tracking to detect abnormal crowd behaviors. Ali and Shah in [2] proposed a method for segmentation of high density crowds by introducing a method based on Coherent Structures from fluid dynamics and particle ad-

vection. Their method is capable of detecting instabilities in the crowd by identifying changes in the segmentation. Even though our work uses the same framework for particle advection, we use a completely different course by estimating the interaction forces of people in the crowd and detect anomalies directly without segmentation.

The organization of this paper is as follows. In the next section we introduce Social Force model for modeling the crowd movement. In Section 3 we introduce our method to estimate the social forces in the crowd. Section 4 describes the proposed method to detect abnormal behaviors in the crowd. Finally, in Section 5 we demonstrate abilities of the approach to detect and localize abnormal behaviors on a publicly available dataset.

2. Social Force Model

In the following, we describe social force model for pedestrian motion dynamics by considering personal motivations and environmental constraints. In this model, each of N pedestrians i with mass of m_i changes his/her velocity v_i as

$$m_i \frac{dv_i}{dt} = F_a = F_p + F_{int}, \quad (1)$$

as a result of actual force F_a , and due to individualistic goals or environmental constraints. This force consists of two main parts: (1) personal desire force F_p , and (2) interaction force F_{int} .

People in crowds generally seek certain goals and destinations in the environment. Thus, it is reasonable to consider each pedestrian to have a desired direction and velocity v_i^p . However, the crowd limits individual movement and the actual motion of pedestrian v_i would differ from the desired velocity. Furthermore, individuals tend to approach their desired velocity v_i^p based on the personal desire force

$$F_p = \frac{1}{\tau} (v_i^p - v_i), \quad (2)$$

where τ is the relaxation parameter.

The interaction force F_{int} consists of the repulsive and attraction force F_{ped} based on psychological tendency to keep a social distance between pedestrians and an environment force F_w to avoid hitting walls, buildings, and other obstacles. Therefore, the *interaction force* is defined as

$$F_{int} = F_{ped} + F_w. \quad (3)$$

It is logical to model pedestrians such that they keep small distances with people they are related or attracted to and keep far distances from discomforting individuals or environments. In social force model, these forces are defined based on potential fields functions. Further elaboration of this issue is not in the interest of this paper and readers are referred to [17] and [10] for detailed discussion of these

functions. In this paper, we focus our attention to estimate the *interaction force* F_{int} between pedestrians as a single quantity.

Generalized social force model considers the effect of *panic* where herding behaviors appear in event like escaping from a hazardous incident. In this model, personal desire velocity v_i^p is replaced with

$$v_i^q = (1 - p_i)v_i^p + p_i \langle v_i^c \rangle, \quad (4)$$

where p_i is the panic weight parameter and $\langle v_i^c \rangle$ is the average velocity of the neighboring pedestrians. The pedestrian i exhibits individualistic behaviors as $p_i \rightarrow 0$ and herding behaviors as $p_i \rightarrow 1$. Overall, generalized social force model can be summarized as

$$m_i \frac{dv_i}{dt} = F_a = \frac{1}{\tau} (v_i^q - v_i) + F_{int}. \quad (5)$$

Generalized social force model is the cornerstone for many studies in simulation of crowd behavior [14] [29][26] in addition to the studies in computer graphics [21][5][25] for creating realistic animations of the crowd. Furthermore, estimation of parameters of the model provides valuable information about the governing dynamics of the crowd [4].

3. Estimation of Interaction Forces in Crowds

In this section, we describe the process of estimation of interaction forces F_{int} from a video of a crowd using social force model. The ideal case for computing the social force is to track all objects in the crowd and estimate the parameters as in [4]. However, tracking of individuals in a high density crowd is still a challenging problem in computer vision [3]. In a nutshell, low resolution images of the objects in the dense crowd, dynamic and static occlusions, and similarity of the objects have made the tracking of individuals in the crowd a daunting task. Therefore, in the crowded scenes, object-based methods fall short in accurate estimation of social force parameters.

It has been observed that when people are densely packed, individual movement is restricted and members of the crowd can be considered granular particles [3]. Thus, in the process of estimating the interaction forces, we treat the crowd as a collection of interacting particles. Similar to [2], we put a grid of particles over the image frame and move them with the flow field computed from the optical flow. To analyze the scene, we treat moving particles as the main cue instead of tracking individual objects. As the outcome, the proposed method does not depend on tracking of objects; therefore, it is effective for the high density crowd scenes as well as low density scenes. Furthermore, the particle advection captures the continuity of the crowd flow which neither optical flow nor any instantaneous measure could capture [24] [2].

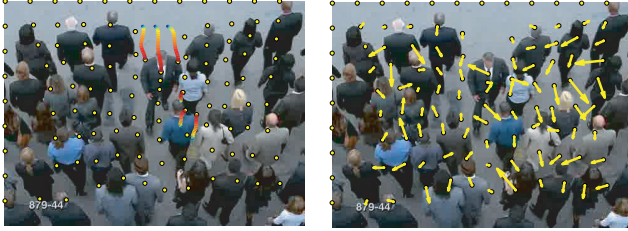


Figure 3. An example of particle advection using the average optical flow field and the corresponding interaction forces. (Left) The trajectories of a small set of particles are depicted for demonstration. (Right) The set of computed interaction forces of particles.

In the next section we describe a modification of social force model to operate on moving particles instead of pedestrians and we discuss the advection of particles using the optical flow. In Section 3.2, we introduce the modification of the generalized social force model for particle advection.

3.1. Particle Advection

To advect particles, we compute the average optical flow field O_{ave} , which is the average of the optical flow over a fixed window of time and as well as space. The spatial average is done by a weighted average using a gaussian kernel. To start the particle advection process, we put a grid of N particles over the image and move the particles with the corresponding flow field they overlay. The effective velocity of particles is computed using a bilinear interpolation of the neighboring flow field vectors.

Using the described particle advection process, particles move with the average velocity of their neighborhood. This resembles the collective velocity of a group of people in the crowd. Figure 3 illustrates an example of particle advection.

3.2. Computing the Social Force

As a tangible analogy, the particles moving by optical flow resemble the motion of the leaves over a flow of water. This notion helps in understanding the modification of social force model for the particle grid. In the case of leaves, wherever there is an obstacle, joining, or branching of the fluid, the leaves have different velocities than the average flow. By analogy, we conclude that particles are also capable of revealing divergent flows in the regions that their desired movement is different from the average flow.

We modify Equation 5 for particle advection by defining the actual velocity of the particle v_i as

$$v_i = O_{ave}(x_i, y_i), \quad (6)$$

where $O_{ave}(x_i, y_i)$ is the effective spatio-temporal average of optical flow for the particle i and in the coordinate (x_i, y_i) . We write the desired velocity of the particle v_i^q as

$$v_i^q = (1 - p_i)O(x_i, y_i) + p_iO_{ave}(x_i, y_i), \quad (7)$$

where $O(x_i, y_i)$ is the optical flow of particle i in the coordinate (x_i, y_i) . The effective average flow field and effective optical flow of particles are computed using linear interpolation.

Using the above modification, particles move with the collective velocity of the flow of the crowd. Furthermore, each particle has a desired velocity which depends on the current optical flow. Hence, any difference between the desired velocity of the particle and its actual velocity relates to interaction of the particle with the neighboring particles or the environment. Figure 3 demonstrates an example of the computed interaction force for a sub-sample set of particles.

Without loss of generality, for a given scene or certain type of crowd with consistently similar sizes of objects, we assume that $m_i = 1$. Hence, we can simply estimate interaction force, F_{int} , from equation 5 for every particle as

$$F_{int} = \frac{1}{\tau}(v_i^q - v_i) - \frac{dv_i}{dt}. \quad (8)$$

4. Event Detection

The computed interaction forces determine the synergy between advecting particles. However, discrete value of forces is not a clear evidence of abnormal behaviors. For instance, in a normal scene of a stock market, the interaction force of stock brokers would be quite higher than the interaction forces of walking pedestrians in a street scene. In other words, the instantaneous forces in a scene do not discriminate the abnormalities but the pattern of forces over a period of time does. In the following, we propose a method to model the normal patterns of forces over time.

In this method, we map the magnitude of the interaction force vectors to the image plane such that for every pixel in the frame there is a corresponding force vector. As a result, for a stream of image frames $I(t)$ of m pixels, we construct a feature matrix of *force flow* $S_f(t)$ of the same resolution. Figure 5 illustrates force flow for a sample of frames of a video stream.

The process of identifying the likely patterns in the $S_f(t)$ is a special case of scene modeling which is considerably studied in computer vision. The bag of words [12] method is one of the typical candidates for such an analysis. In this paper, we consider using bag of words method to estimate the likelihood force flow $S_f(t)$ and we use only normal videos for training LDA.

To use LDA, we partition the force flow into blocks of T frames which we refer as *Clips*. Next, from each clip D_j , K visual words Z_j are extracted. We randomly pick visual words of size $n \times n \times T$ from locations in *force flow* where corresponding optical flow is not zero. Finally, a code book of size C is formed using K-means clustering. Figure 4 illustrates the process of computing *force flow* and the extraction of visual words.

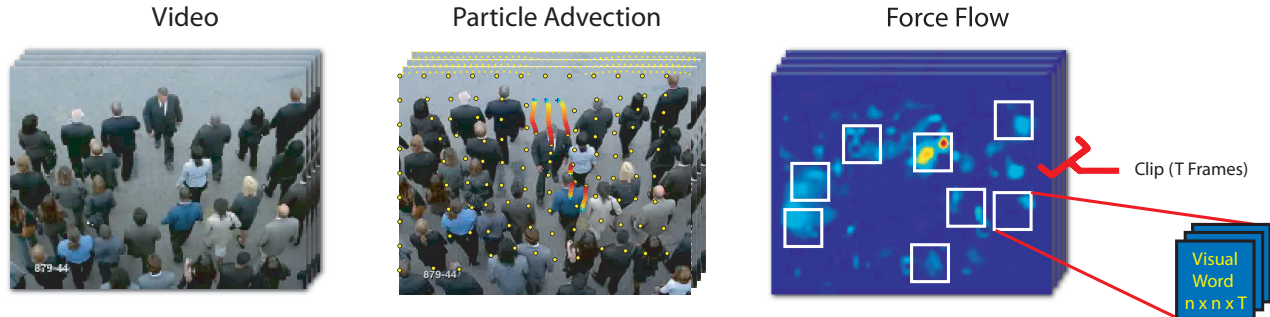


Figure 4. The overall demonstration of the algorithm. Using the average optical flow field, a grid of particles is updated and the interaction forces between particles are computed. The forces are mapped back to the image space to construct the force flow. Visual words are randomly picked as 3D volumes of features from the force flow to use in LDA model.

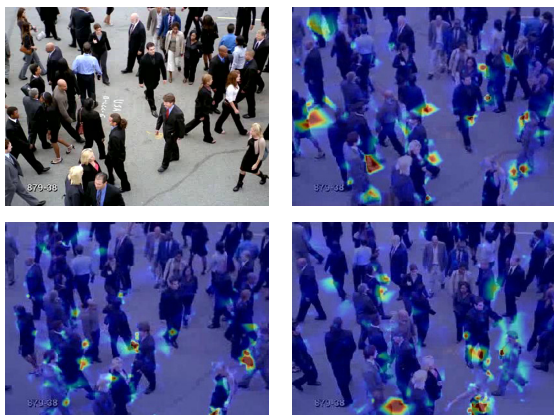


Figure 5. Examples of the computed force field for one example video sequence. The image on the top left is the first frame, and the rest are sample frames of the sequence with alpha channel of forces overlaid. The color map Jet is used so red values represent higher forces where as blue values represent low force flow.

Therefore, for a set of normal force flows of a given scene or a group of similar scenes, we construct the corpus $D = \{D_1, D_2, D_3, \dots, D_M\}$ and we use Latent Dirichlet Allocation (LDA) [12] to discover the distribution of L topics for the normal crowd behavior. Using the modified Expectation Maximization (EM) algorithm in [12], we approximate the bag of words model to maximize the likelihood of corpus as

$$\ell(\alpha, \beta) = \sum_{j=1}^M \log p(D_j | \alpha, \beta), \quad (9)$$

where α and β are the learned model parameters. By using the model, we estimate the likelihood $\log p(D_j | \alpha, \beta)$ for every clip from the video sequence. Based on a fixed threshold on the estimated likelihood, we label frames as

normal or as abnormal.

4.1. Localization of Abnormalities

Using LDA model with force flows, we distinguish abnormal frames from the normal frames. Although it is really helpful to localize regions in the frame that correspond to the abnormalities, the bag of words method does not implicitly provide a method to localize the unlikely visual words. As we discussed earlier, the force flow reveals the interaction forces in the scene, which correspond to the activities in the scene. In an abnormal scene, we expect the anomalies to occur in active regions or the regions with higher social interactions. Therefore, we localize abnormalities in the abnormal frame by locating the regions of high force flow.

5. Experiments and Discussion

5.1. The UMN Dataset

The approach is tested on the publicly available dataset of normal and abnormal crowd videos from University of Minnesota [1]. The dataset comprises the videos of 11 different scenarios of an escape event in 3 different indoor and outdoor scenes. Figure 6 shows sample frames of these scenes. Each video consists of an initial part of normal behavior and ends with sequences of the abnormal behavior.

In the particle advection phase, the resolution of the particle grid is kept at 25% of the number of pixels in the flow field for computational simplicity. For computation of the interaction forces, the panic parameter is kept fixed as $p_i = 0$. Therefore, the interaction forces are computed by assuming that the crowd is not in panic in normal motion. As a result, any high magnitude interaction force relates to activities different from the collective movement of the crowd. The force flow is computed by linear mapping of the force field into an image of the same resolution as the video frame. For construction of visual words, we used 3D

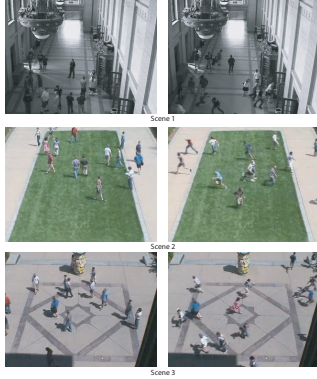


Figure 6. Sample frames in three different scenes of the UMN dataset: Normal (left) and abnormal (right).

volumes of $5 \times 5 \times 10$. $K = 30$ visual words are extracted from block of $T = 10$ frames of force flow with one frame overlap. The final codebook contains $C = 10$ clips. The LDA is used to learn $L = 30$ latent topics.

To evaluate the approach, 5 different video sequences of the first scene are selected and LDA model is created for visual words from the frames with normal behavior. The trained model is used to estimate the likelihood of being normal for blocks of T frames. Therefore, the method chops any input video into clips of T frames and labels all frames in each clip as normal or abnormal. Figure 7 shows some of the qualitative results for detection of abnormal scenes. In each row, the figure depicts the first frame of the sequence on the left and a detected abnormal frame on the right. The black triangles on the horizontal bars identify the timing of the shown abnormal frames. The false positive detections in Figure 7 are result of incorrect estimation of social forces. Overall, these results show that estimated social force model is capable of detecting the governing dynamics of the abnormal behavior, even in the scenes that it is not trained. All videos in the dataset exhibit behavior of escape panic and the proposed approach successfully models the dynamics of the abnormal behavior regardless of the scene characteristics.

In addition, we demonstrate the power of the proposed social force model in capturing the abnormal behaviors in contrast to use of optical flow. In this experiment, instead of force flow, we use spatio-temporal patches of optical flow as visual words. Thus, we create a codebook from optical flow information to learn a LDA model. We use the same parameters for LDA training in the experiment with optical flow. Therefore, the blocks of 10 frames of the magnitude of the optical flow are used as clips to learn the distribution of latent topics and to compute the likelihood of frames. We use the same dataset for this experiment with the same set of parameters for learning LDA model. The ROC curves in Figure 9 illustrate that the the proposed method outperforms the method based on pure optical flow in detecting

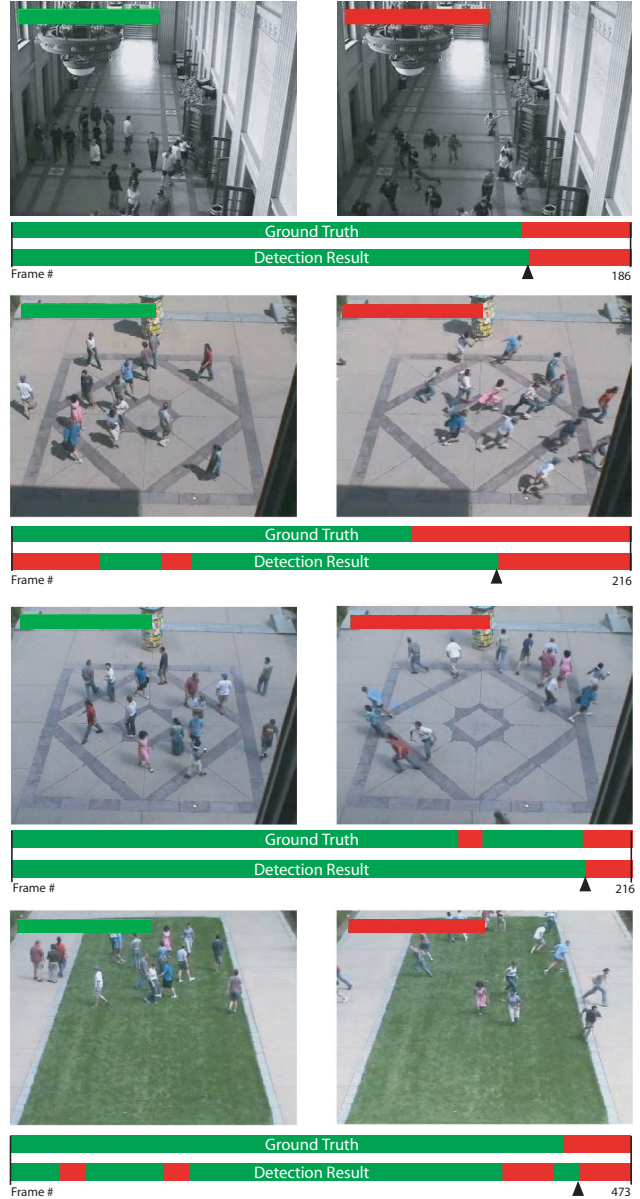


Figure 7. The qualitative results of the abnormal behavior detection for four sample videos of UMN dataset. Each row represents the results for a video in the dataset. The ground truth bar and the detection bar represent the labels of each frame for that video. Green color represents the normal frames and red corresponds to abnormal frames. The left column shows the first frame of the video and the right column is the first frame of the detected abnormal block (black triangles).

abnormalities, and Table 1 provides the quantitative results of the comparison.

In Figure 8, we demonstrate the qualitative results of localization of abnormal behaviors in the crowd, where the escaping individuals are highlighted as abnormal areas of frames. The results show that the interaction forces are ca-

Method	Area under ROC
Social Force	0.96
Pure Optical Flow	0.84

Table 1. The comparison of the use of the proposed social force method and pure optical flow for detection of the abnormal behaviors in the UMN dataset.

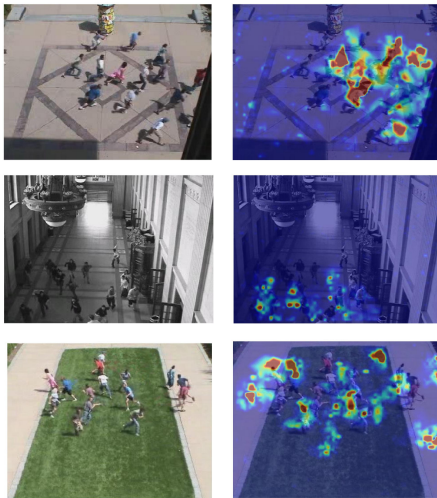


Figure 8. The localization of the abnormal behaviors in the frames using the interaction force. Original frames (left), Localized abnormal behaviors(right). Red pixels correspond to the the highly probable abnormal regions.

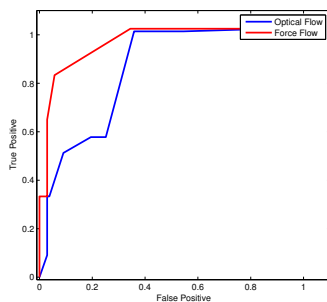


Figure 9. The ROCs for detection of abnormal frames in the UMN dataset. Proposed method (Red) outperforms use of pure optical flow (Blue).

pable of locating the abnormalities in the regions that are occupied by the crowd. As the figure shows, the proposed method provides regions of abnormality and does not label individuals.

5.2. The Web Dataset

To evaluate our method in practical applications, we conduct an experiment on a challenging set of videos which has been collected from the sites like Getty Images and ThoughtEquity.com which contain documentary and high



Figure 10. Sample frames of 6 sequences of our web dataset. (Left Column) Normal samples. (Right column) Abnormal samples.

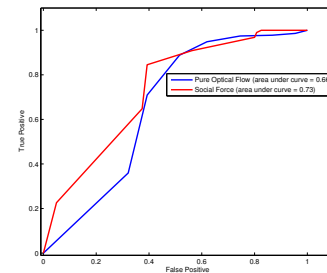


Figure 11. The ROCs of abnormal behavior detection in the web dataset.

quality videos of crowds in different urban scenes. The dataset comprises 12 sequences of normal crowd scenes such as pedestrian walking, marathon running, and 8 scenes of escape panics, protesters clashing, and crowd fighting as abnormal scenes. All the frames are resized to the fixed width of 480 pixels. Figure 10 shows sample frames of the normal and abnormal sequences.

In this experiment, the resolution of the particle grid is kept at 10% of the number of original pixels. For construction of visual words, we extracted $K = 30$ similar $5 \times 5 \times 10$ volumes from a block of $T = 10$ frames of force flow. The codebook for this experiment contains $C = 30$ clips and the LDA is used to learn $L = 50$ latent topics. To learn the LDA model, we used the normal sequences in a 2-fold fashion. We randomly excluded 2 sequences from the normal set and trained on the rest. In the testing phase we added the excluded sequences to the test set. We did this experiment 10 times and constructed the ROC by averaging the results of these experiments.

The ROC in Figure 11 demonstrates that the proposed method outperforms optical flow method to distinguish abnormal sequences.

6. Conclusion

Using social force model, we introduce a method to detect abnormal behaviors in crowd scenes. We address the ability of the method to capture the dynamic of crowd behavior based on the interaction forces of individuals without the need to track objects individually or perform segmentation. The results of our method, indicates that the method is effective in detection and localization of abnormal behaviors in the crowd.

References

- [1] Unusual crowd activity dataset of University of Minnesota, available from <http://mha.cs.umn.edu/movies/crowd-activity-all.avi>.
- [2] S. Ali and M. Shah. A lagrangian particle dynamics approach for crowd flow segmentation and stability analysis. *Computer Vision and Pattern Recognition, 2007. CVPR '07. IEEE Conference on*, pages 1–6, June 2007.
- [3] S. Ali and M. Shah. Floor fields for tracking in high density crowd scenes. *ECCV*, 2008.
- [4] D. H. Anders Johansson and P. K. Shukla. Specification of the social force pedestrian model by evolutionary adjustment to video tracking data. *Advances in Complex Systems*, 10(2):271–288, December 2007.
- [5] E. A. S. G. M. L. Avneesh Sud, Russell Gayle and D. Manocha. Real-time navigation of independent agents using adaptive roadmaps. *VRST 07: Proceedings of the 2007 ACM symposium on Virtual reality software and technology*, pages 99–106, 2007.
- [6] P. R. S. V. Beibei Zhan, Dorothy Monekosso and L.-Q. Xu. Crowd analysis: a survey. *Machine Vision and Applications*, 19(5-6), October 2008.
- [7] G. Brostow and R. Cipolla. Unsupervised bayesian detection of independent motion in crowds. *Computer Vision and Pattern Recognition, 2006 IEEE Computer Society Conference on*, 1:594–601, June 2006.
- [8] A. B. Chan and N. Vasconcelos. Mixtures of dynamic textures. In *ICCV '05: Proceedings of the Tenth IEEE International Conference on Computer Vision Volume 1*, pages 641–647, Washington, DC, USA, 2005. IEEE Computer Society.
- [9] N. Courty and T. Corpetti. Crowd motion capture. *Comput. Animat. Virtual Worlds*, 18(4-5):361–370, 2007.
- [10] I. F. D. Helbing and T. Vicsek. Simulating dynamical features of escape panic. *Nature*, pages 487–490, 2000.
- [11] I. J. F. D. Helbing, P. Moln r and K. Bolay. Self-organizing pedestrian movement. *Environment and Planning B: Planning and Design*, 28:361–383, 2001.
- [12] A. Y. N. David M. Blei and M. I. Jordan. Latent dirichlet allocation. *Journal of Machine Learning Research*, 3:993–1022, 2003.
- [13] S. S. Dietmar Bauer and N. Brändle. Macroscopic pedestrian flow simulation for designing crowd control measures in public transport after special events. *SCSC: Proceedings of the 2007 summer computer simulation conference*, pages 1035–1042, 2007.
- [14] A. J. Dirk Helbing and H. Z. Al-Abideen. The dynamics of crowd disasters: An empirical study. *Physical Review E*, 75:046109, 2007.
- [15] S. B. Ernesto L. Andrade and R. B. Fisher. Modelling crowd scenes for event detection. *Pattern Recognition, International Conference on*, 1:175–178, 2006.
- [16] M. B. Gianluca Antonini, Santiago Venegas Martinez and J. P. Thiran. Behavioral priors for detection and tracking of pedestrians in video sequences. *Int. J. Comput. Vision*, 69(2):159–180, 2006.
- [17] D. Helbing and P. Molnar. Social force model for pedestrian dynamics. *Physical Review E*, 51:4282, 1995.
- [18] R. L. Hughes. A continuum theory for the flow of pedestrians. *Transportation Research Part B: Methodological*, 36(6):507–535, July 2002.
- [19] A. J. A. Jorge S. Marques, Pedro M. Jorge and J. M. Lemos. Tracking groups of pedestrians in video sequences. *Computer Vision and Pattern Recognition Workshop*, 9:101, 2003.
- [20] A. Lerner, Y. Chrysanthou, and D. Lischinski. Crowds by example. *Computer Graphics Forum (Proceedings of Eurographics)*, 26(3), 2007.
- [21] J. M. A. N. Pelechano and N. I. Badler. Controlling individual agents in high-density crowd simulation. *SCA 07: Proceedings of the 2007 ACM SIGGRAPH/Eurographics symposium on Computer animation*, pages 99–108, 2007.
- [22] G. D. N. K. J. R. T. Y. P. Tu, T. Sebastian. Unified crowd segmentation. *ECCV*, 2008.
- [23] P. Reisman, S. A. O. Mano, and A. Shashua. Crowd detection in video sequences. *Intelligent Vehicles Symposium, 2004 IEEE*, pages 66–71, June 2004.
- [24] P. Sand and S. Teller. Particle video: Long-range motion estimation using point trajectories. *Computer Vision and Pattern Recognition*, 02:2195–2202, 2006.
- [25] W. Shao and D. Terzopoulos. Autonomous pedestrians. *Graph. Models*, 69(5-6):246–274, 2007.
- [26] D. J. K. Taras I. Lakoba and N. M. Finkelstein. Modifications of the helbing-molnar-farkas-vicsek social force model for pedestrian evolution. *Simulation*, 81(5):339–352, 2005.
- [27] A. Treuille, S. Cooper, and Z. Popović. Continuum crowds. *ACM Trans. Graph.*, 25(3):1160–1168, 2006.
- [28] K. D. Xiaoshan Pan, Charles S. Han and K. H. Law. Human and social behavior in computational modeling and analysis of egress. *Automation in Construction*, 15(4):448–461, 2006.
- [29] W. Yu and A. Johansson. Modeling crowd turbulence by many-particle simulations. *Physical Review E (Statistical, Nonlinear, and Soft Matter Physics)*, 76(4):046105, 2007.