Regularization of Diffusion Tensor Field Using Coupled Robust Anisotropic Diffusion Filters

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Abstract

This paper presents a method to simultaneously regularize diffusion weighted images and their estimated diffusion tensors, with the goal of suppressing noise and restoring tensor information. We enforce a data fidelity constraint, using coupled robust anisotropic diffusion filters, to ensure consistency of the restored diffusion tensors with the regularized diffusion weighted images. The filters are designed to take advantage of robust statistics and to be adopted to the anisotropic nature of diffusion tensors, which can effectively keep boundaries between piecewise constant regions in the tensor volume and also the diffusion weighted images during the regularized process. To facilitate Euclidean operations on the diffusion tensors, log-Euclidean metrics are adopted when performing the filtering. Experimental results on simulated and real image data demonstrate the effectiveness of the proposed method.

1. Introduction

Diffusion tensor imaging (DTI) has been widely used to delineate tissue micro-structures, particularly in white matter regions of the brain [1, 2]. The estimation of diffusion tensors (DTs) from diffusion-weighted images (DWIs) is often limited by imaging noise that leads to uncertainty in the computation of tensors and their derived quantities, such as principal directions. Since most fiber tracking algorithms rely on principal directions estimated from these tensors to reconstruct fiber pathways, it is crucial to reduce the effect of noise.

Existing methods for regularizing diffusion tensors fall in two categories: two-step methods and one-step methods. Two-step methods perform diffusion tensor estimation and smoothing separately. Some of them focus on the smoothing of DWIs prior to the estimation of diffusion tensors. To smooth the vector valued DWIs, Perona-Malik nonlinear filtering, Weickert anisotropic filtering and a T-V norm minimization based method have been proposed [3-5]. In these methods, the problem of DWI smoothing is formulated in Partial Differential Equation (PDE) based frameworks. Recently, an adaptive smoothing method is proposed to smooth DWIs with weights inferred from properties of the estimated tensor field rather than from the DWIs themselves [6]. Other two-step methods perform regularization directly on the estimated diffusion tensors. In [7], the well known bilateral filtering method for scale images is extended to smooth diffusion tensors while preserving edges by means of weighted averaging of nearby image values. Markov model based method, vector based diffusion PDEs, median filtering and Fermat median filtering based on tensor images have also been proposed to regularize the tensor fields [8-12]. These methods regularize the diffusion tensors or their principal diffusion directions as a whole after the diffusion tensors are estimated from the DWIs. The major limitation of these methods is that the positive definiteness of the regularized tensors cannot be guaranteed.

In contrast to the above-mentioned methods, one-step methods simultaneously regularize and estimate diffusion tensors from DWIs using energy minimization based frameworks [13-15]. The energy function to be minimized typically includes a tensor smoothness term and a data fidelity term, which ensure the consistency of the smoothed diffusion tensors with respect to the DWIs. The energy minimization problem can be solved using iterative algorithms which constrain the estimated and smoothed diffusion tensors in a valid tensor manifold. In [13], the diffusion tensor is parameterized by its Cholesky decomposition, which can ensure that positive definiteness of final regularized tensors. However, as pointed out in [14], a better solution for the problem is to process tensors using Riemannian metrics [16]. Using log-Euclidean Riemannian metrics, operations on the diffusion tensors are identical to those in the Euclidean space, which makes computation simple and fast [17, 18]. Based on log-Euclidean metrics, an anisotropic filter was proposed for regularizing diffusion tensors that are simultaneously estimated from DWIs [14]. These one-step methods provide a neat solution for smoothing diffusion tensor;

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however the problem of how to make such smoothing stop at edge of tensors remains unsolved, causing unnecessary smoothing on important anatomical details.

To achieve robust smoothing of diffusion tensors, we propose to simultaneously smooth the diffusion tensors and the DWIs using coupled robust anisotropic diffusion filters, which are subject to a data fidelity constraint in the Gaussian diffusion tensor model based on the Stejskal and Tanner equations [19]. An "edge-stopping" function based on Tukey's biweight robust estimator is adopted to preserve sharp boundaries in both DWI space and diffusion tensor space [20], which will in the end ensure piecewise smoothing of diffusion tensors. Using the Log-Euclidean metrics, the robust anisotropic diffusion filters can be implemented using simple Euclidean operations. The proposed method is validated using simulated DTI data and also in vivo human brain data. Quantitative and qualitative comparisons with state-of-the-art algorithms demonstrate the superiority of the proposed method.

2. Method

DTI makes use of the anisotropic nature of water diffusivity in structured tissues to capture white matter structural information. Motion of water molecules is favored along the fiber direction and hindered in the orthogonal directions. Measurement of such water diffusivity at each voxel location provides an effective way of estimating the local fiber orientation. Water diffusivity can be represented by a 3×3 symmetry tensor **D**, which has six independent elements. In DTI, instead of a homogeneous magnetic field, the magnetic field is varied linearly by a pulsed field gradient so as to capture the water diffusivity in different directions.

To calculate the tensor, at least 6 DWIs corresponding to different magnetic gradients and a baseline image (no magnetic gradient) is required. If the diffusion process is assumed to be Gaussian, the *i*-th DWI I_i and diffusion tensor **D** are related by the Stejskal-Tanner equation as follows.

$$I_i = I_0 e^{-b\vec{g}_i^T \mathbf{D}\vec{g}_i}$$

or

$$logI_i = logI_0 - b\vec{g}_i^T \mathbf{D}\vec{g}_i \quad (1)$$

where b is LeBihan's b-factor, $\vec{g}_i$ is the normalized magnetic diffusion gradient vector corresponding to the *i*-th DWI, and I_0 is the image acquired with $\|\vec{g}_i\| = 0$. The diffusion tensor **D** is a 3×3 symmetric tensor matrix, and has six degrees of freedom. Usually, a least-squares procedure is used to estimate the DTs. Obviously, the quality of the DTs hinges on that of the DWIs. It is therefore better to simultaneously restore the DWIs and diffusion tensors.

2.1. Simultanous Estimation and Restoration of Diffusion Tensors

To simultaneously restore the DWIs and the DTs, we adopt the framework proposed in [13] to optimize the energy minimization problem:

$$\min E = \alpha\, Smooth(\mathbf{D}) + \beta \sum_{i=1}^{N} Smooth(I_i(\mathbf{D})) + \textstyle\sum_{i=1}^{N} Fidelity(I_i(\mathbf{D})) \quad (2)$$

where N is the total number of diffusion gradients, α and β are regularization parameters. The optimization problem can be solved iteratively by a gradient descent approach.

$$\mathbf{D}_{t+1} = \mathbf{D}_t - \Delta t \left[\alpha \nabla Smooth(\mathbf{D})_t + \beta \textstyle\sum_{i=1}^{N} \nabla Smooth\big(I_i(\mathbf{D})\big)_t + \sum_{i=1}^{N} \nabla Fidelity\big(I_i(\mathbf{D})\big)_t\right] \quad (3)$$

where Δt is the step for each iteration, and parameters α and β balance the smoothness terms of DWIs and diffusion tensors. The performance of such a solution is often limited if the smoothing functions are not properly defined, resulting in over-smoothing.

2.2. Log-Euclidean space

In Log-Euclidean space, the DT, **D**, is represented as $\mathbf{D}^{LE} = \log(\mathbf{D})$. The $\mathbf{D}^{LE}$ can be calculated from **D** as follows. First, diagonalization of **D** is performed:

$$\mathbf{D} = \mathbf{R}^T \mathbf{S} \mathbf{R} \quad (4)$$

where **R** is a rotation matrix, and **S** a diagonal matrix with the eigenvalues of **D** at its diagonal. Then each diagonal element of **S** is converted into its natural logarithm in order to obtain a new diagonal matrix $\hat{\mathbf{S}}$. Finally, using $\hat{\mathbf{S}}$ instead of **S** in equation (4), we obtain:

$$\mathbf{D}^{LE} = \mathbf{R}^T \hat{\mathbf{S}} \mathbf{R} \quad (5)$$

2.3. Coupled Robust Anisotropic Diffusion Filters

To restore the diffusion tensors, and at the same time be robust to over-smoothing, we adopt a set of smoothing functions which are based on robust statistics [21]. Specifically, coupled robust anisotropic diffusion filters are used to simultaneously smooth DT and DWIs in the log-Euclidean space.

In the log-Euclidean space, the corresponding terms of the energy function are represented as:

$$Smooth(\mathbf{D}^{LE}) = \int_\Omega \rho_1(\|\nabla \mathbf{D}^{LE}(I_i)\|, \sigma_1)\, d\mathbf{X} \quad (6)$$

$$Smooth(I_i(\mathbf{D}^{LE})) = \int_\Omega \rho_2(\|\nabla (I_i(\mathbf{D}^{LE}))\|, \sigma_2) d\mathbf{X} \quad (7)$$

$$Fidelity(I_i(\mathbf{D}^{LE})) = \frac{1}{2}\int_{\Omega}\sum_{i=1}^{N}\left(\left(log\left(\frac{I_0}{I_i}\right) - b\vec{g}_i^T \exp\,(\mathbf{D}^{LE})\vec{g}_i\right)^2\right)d\mathbf{X} \quad (8)$$

where Ω is the image domain, $\rho_1(\cdot)$ and $\rho_2(\cdot)$ are the robust error norm, σ_1 and σ_2 are the scale parameters. The effect of the outliers is minimized by making appropriate choices of ρ_1 and ρ_2.

For the robust estimation of function ρ_1, and ρ_2, we employ the Tukey's biweight algorithm since it has proven to robustness for image denoising [20]. It is defined as:

$$\rho_j(x,\sigma_j) = \begin{cases} \frac{x^2}{\sigma_j^2} - \frac{x^4}{\sigma_j^4} + \frac{x^6}{3\sigma_j^6} & |x| \le \sigma_j, j = 1{,}2 \\ \frac{1}{3} & \text{otherwise} \end{cases} \quad (9)$$

By defining $\psi_j(x,\sigma_j) \doteq \frac{\rho_j'(x,\sigma_j)}{x}$, and $G = b\vec{g}_i^T \exp(\mathbf{D}^{LE})\,\vec{g}_i$, with

$$\psi_j(x,\sigma_j) = \begin{cases} x\left[1 - (\frac{x}{\sigma_j})\right]^2 & |x| \le \sigma_j \\ 0 & \text{otherwise} \end{cases}, \text{j} = 1{,}2,$$

we can compute the data gradient (divergence):

$\int_{\Omega}\rho_1(\|\nabla\mathbf{D}^{LE}(I_i),\sigma_1\|)\,d\mathbf{X}$ and $\int_{\Omega}\rho_2(\|\nabla(I_i)\|,\sigma_2)d\mathbf{X}$ by

$$\nabla Smooth(\mathbf{D}^{LE}) = -2div(\psi_1(\|\nabla\mathbf{D}^{LE}\|)\nabla\mathbf{D}^{LE}), \quad (10)$$

$$\nabla Smooth\big(I_i(\mathbf{D}^{LE})\big) = -2 \times div[\psi_2(\|\log(I_0) - G\|) \times \nabla(\log(I_0) - G)], \quad (11)$$

$$\nabla Fidelity = -\sum_{i=1}^{N}\left(log\left(\frac{I_0}{I_i}\right) - G\right)\frac{\partial G}{\partial\mathbf{D}^{LE}}. \quad (12)$$

With these gradients, the diffusion tensor smoothing problem can be solved iteratively by:

$$(\mathbf{D}^{LE})_{t+1} = (\mathbf{D}^{LE})_t - \Delta t[\alpha\nabla Smooth(\mathbf{D}^{LE})_t + \beta\sum_{i=1}^{N}\nabla Smooth\big(I_i(\mathbf{D}^{LE})\big)_t + \sum_{i=1}^{N}\nabla Fidelity\big(I_i(\mathbf{D}^{LE})\big)_t\Big] \quad (13)$$

The details on the numerical implementation of $\frac{\partial G}{\partial\mathbf{D}^{LE}}$, $\nabla\mathbf{D}^{LE}$ and ∇G, can be found in [13, 14].

To use the Tukey's biweight algorithm, we need to determine how large the image gradient or diffusion tensor gradient can be before we consider it to be an outlier, we automatically estimate the "robust scale" as [22]:

$$\sigma_1 = \sqrt{5}\sigma_{1e} \text{ and } \sigma_2 = \sqrt{5}\sigma_{2e} \quad (14)$$

$$\sigma_{1e} = 1.4826\text{median}[\|\nabla\mathbf{D}^{LE}\| - \text{median}(\|\nabla\mathbf{D}^{LE}\|)] \quad (15)$$

$$\sigma_{2e} = 1.4826\text{median}[\|\nabla I((\mathbf{D}^{LE})\| - \text{median}(\|\nabla I(\mathbf{D}^{LE})\|)] \quad (16)$$

To quantitatively evaluate the performance of the proposed method, we compute a summary statistic of the angle differences between principal diffusion directions (PDDs) of the original and restored tensors. The measure used is defined as the mean of angle difference:

$$\text{Angle}_{\text{PDD}} = \frac{\sum_{\mathbf{x}\in\Omega}\frac{<p(\mathbf{x}),\hat{\mathrm{p}}(\mathbf{x})>}{\|\mathrm{p}(\mathbf{x})\|\|\hat{\mathrm{p}}(\mathbf{x})\|}}{\sum_{\mathbf{x}\in\Omega}1} \quad (17)$$

where p, $\hat{\mathrm{p}}$ are the principal diffusion directions of the smoothed data and the noise-free data. A small value for $\text{Angle}_{\text{PDD}}$ indicates good result.

3. Experiment Results

A set of experiments on simulated and in vivo DTI data have been performed to validate the proposed method. We have also compared the proposed method with two representative methods: adaptive smoothing of DWIs [6], and simultaneous smoothing and estimating diffusion tensors [14] using the R software and MedINRIA [6, 23], with the default parameters provided by the authors. The experimental results are displayed using ExploreDTI [24].

3.1. Simulated data

A set of simulated data, each consisting of concentric cylindrical shells, are generated to model 4 fiber bundles using R software [6]. Each of the dataset consists of 25 diffusion weighed images in 25 directions and one image with a null gradient. The dimension of each of these images is 64×64×26. Fig. 1(a) shows one representative slice of the diffusion tensor image, depicted in its ellipsoidal representation map. Four rings, two closed and two unclosed can be observed.

To test the performance of regularization algorithms, Rician noise is added to the original DWIs by first transforming the DWIs into k space and then adding Gaussian noise with a standard deviation of 25 to both the real and the imaginary parts. The resulting noise distorted diffusion ellipsoidal representation map is shown in Fig.1 (b).

Applying the regularization algorithms, the results shown in Fig.1 (c-e) are obtained. As shown in Fig. 1(c), MedINRIA does not distinguish the ring, as pointed by the arrow (A), with other portions of the image, which might be due to the possibility that the smoothing did not stop at the edges. This result is consistent with that observed in [20], and hence indicates robust anisotropic diffusion filters are needed.

As shown in Fig. 1(d), the adaptive smoothing of DWIs method yields a better result and effectively recovers the rings. However, the fiber directions in the region pointed by the arrow are not correct, as indicated by the colors of ellipsoidal representation map. They should be consistent with those of Fig. 1(a).

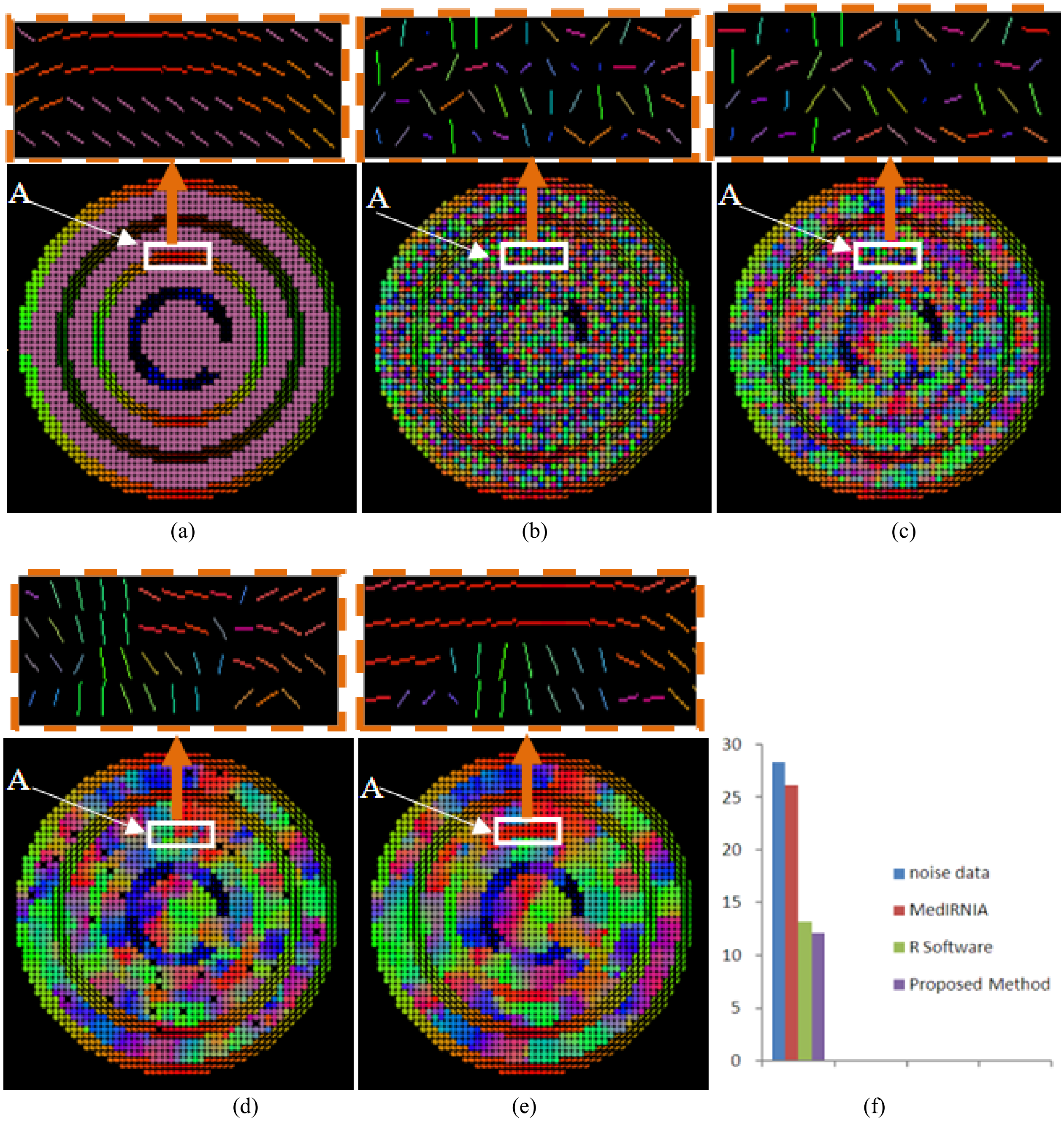

Fig.1. Tensor ellipsoidal representation maps for (a) simulated diffusion tensors, (b) diffusion tensors distorted by Rician noise, (c) diffusion tensors restored by MedIRNIA, (d) diffusion tensors restored by adaptive smoothing of DWIs, and (e) diffusion tensors restored by the proposed method. Means of angle differences between PDDs of noisy and restored tensors with respect to the original tensors are shown in (f).

The result obtained by the proposed method is shown in Fig.1 (e). The consistency of the tensor directions between Fig.1 (a) and (e) indicates that our method yields the best result.

As shown in Fig. 1(f), all of the restoration algorithms can recover the original diffusion tensor to some extent. Using $Angle_{PDD}$ measure, the proposed method obtains the best results.

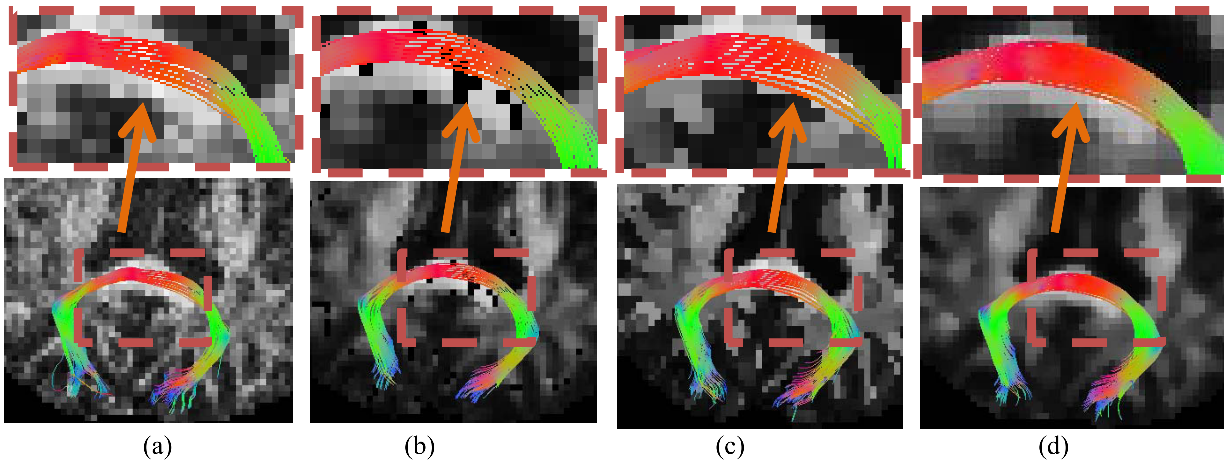

Fig.2. Fiber tracts in (a) original DT data, (b) DT data smoothed by MedIRNIA, (c) DT data smoothed by R software, and (d) DT data smoothed by the proposed method.

The PDDs of the rectangle region pointed by arrow (A) also indicate that our result is the best.

3.2. In vivo brain data

A set of in vivo DWIs were acquired from a healthy subject. It was scanned with 30 gradient directions with a b value of 1000 sec/mm^2. The image size is $128 \times 128 \times 58$. After processing with MedINRIA, R software and the proposed method, fiber tracts are generated using ExploreDTI with the FA threshold value set at 0.2 and the maximum angle of deviation limited by 30-degree. Although all methods yield seemingly similar results, the fiber tracts given by the proposed method is much more uniform, as shown in Fig. 2.

4. Conclusion

We have developed a DTI smoothing method, utilizing coupled robust anisotropic diffusion filtering, to estimate and smooth diffusion tensors. Tukey's biweight algorithm is adopted to make smoothing stop effectively at edges. Compared with the results achieved by MedINRIA and R software, for the simulated data, both visual inspection and measurement of mean of PDD angle difference indicate that the proposed method gives the best performance. For in vivo human brain data, the tractography result shows that the proposed method obtains much more uniform fibers. In the future, we will examine the influence of the regularization parameters, and also validate the proposed method using larger datasets.

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