Simulations and Trajectory Tracking of Two Manipulators Manipulating a Flexible Payload

Peng Zhang
College of Communication Engineering
Jilin University
Changchun, China

Yuan-chuan Li
College of Communication Engineering
Jilin University
Changchun, China

***Abstract*—Manipulating flexible payloads are being extensively studied because of the potential applications. So far, the modeling method isn't perfect and there are few studies about the trajectory tracking of them. In this paper the modeling and trajectory tracking of manipulating flexible payloads by robot manipulators are studied. The finite element method (FEM) is used to approximate the vibration of flexible payload. In order to restrain the disturbance of vibration to trajectory tracking the adaptive sliding mode control law is designed. The stability of the system is proven by stability theory. Simulation results validate the modeling method and control law.**

***Keywords*—manipulators, flexible payload, adaptive, trajectory**

I. INTRODUCTION

Because of the potential applications of manipulating flexible payload in industry, such as in automobile industry and shipbuilding, it stirs a great deal of interest. A flexible beam which one end is fixed manipulated by robot manipulator was studied by T. Yukawa, J.Q. Wu[1],[2]. In industry some assembly tasks, such as inserting a flexible part into a hole, require to consider the flexibility of objects and these can found in [3],[4],[5]. During the production of printed wiring boards(PWB), a large number of elastic sheets need to be assembled to form a book[6],[7]and the trajectory planning was studied in them. J.K. Mills[8],[9]firstly proposed the rigid control method and studied the robotic fixtureless assembly of sheet metal by this method. Sun and Liu[10],[11],[12] who used the rigid control method have done a great deal of work in manipulating flexible payload by multiple robot manipulators which advanced the studies of manipulating flexible payload.

At present, most of work about manipulating flexible payload only considered the position control which moves a flexible payload from an original position/orientation to prearranged position/orientation. In[13] the trajectory tracking of two manipulators manipulating flexible payload was studied using sliding mode control and a fixed estimated item replaced all the elements except the rigid elements. In this paper the trajectory tracking will be studied and the adaptive sliding mode control law will be used. Compared to the assumed mode method, the finite element method (FEM) has some advantages that every coordinate in FEM has its physical meaning and the numbers of coordinate can be chosen according to the precision so the FEM is used to approximate the vibration dynamics here and the configuration of pinned-pinned is used. The model of the system is got in task space because compared to the joint space and end-effector space, it doesn't require to plan the trajectory of single robot manipulator in the task space so it can reduce the calculation. Because of the choice of system coordinates the definition of vibration coordinates will have some difficulty and it's solved properly in this paper. In the trajectory tracking of manipulating flexible payload the disturbance of vibration can't be neglected. In order to restrain the effect of vibration the adaptive sliding control law is designed. Compared to [14] the regressor matrix doesn't require to be calculated and the chatering when the control law traverses the sliding surface will reduce. The stability of the system is proven by stability theory and simulation results validate the modeling method and control law.

The organization of the paper is as follows. Section 2 the model of the system is derived and the decomposition of it is done. Section 3 the adaptive sliding control law is designed and the stability of the system is proven. The simulation results are shown in section 4. The conclusions are given at last.

II. MODEL

A. Model of the system

In [16], Sun indicated that besides the 'clamped-free' model, the other model can be also used if the rigid body of the flexible payload can be determined. In this paper the pinned-pinned model will be used. The virtual link is defined as the dot line as in Fig.1 and it is also the rigid body of the flexible payload. In the process of motion the manipulators and virtual link form a closed-loop as cooperating rigid payload.

Two assumptions are made.

1) There is no deformation in the process of motion so the virtual link and the last link of manipulators will keep parallel;

2) The payload is grasped rigidly.

The position and velocity of any point in flexible payload are as follows

$$R_{ri} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} (i-1)l\cos(\theta) \\ (i-1)l\sin(\theta) \end{bmatrix} + T\begin{bmatrix} x_i \\ y_i \end{bmatrix}$$

$$\dot{R}_{ri} = \begin{bmatrix} \dot{x}_0 \\ \dot{y}_0 \end{bmatrix} + \begin{bmatrix} -(i-1)l\sin(\theta) \\ (i-1)l\cos(\theta) \end{bmatrix}\dot{\theta} + \dot{T}\begin{bmatrix} x_i \\ y_i \end{bmatrix} + T\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} \quad (1)$$

$$R_{li}=\begin{bmatrix}x_0\\y_0\end{bmatrix}-\begin{bmatrix}il\cos(\theta)\\il\sin(\theta)\end{bmatrix}+T\begin{bmatrix}x_i'\\y_i'\end{bmatrix}$$

$$\dot{R}_{li}=\begin{bmatrix}\dot{x}_0\\\dot{y}_0\end{bmatrix}+\begin{bmatrix}il\sin(\theta)\\-il\cos(\theta)\end{bmatrix}+\dot{T}\begin{bmatrix}x_i\\y_i\end{bmatrix}+T\begin{bmatrix}\dot{x}_i\\\dot{y}_i\end{bmatrix} \quad (2)$$

$$T=\begin{bmatrix}\cos(\theta) & -\sin(\theta)\\\sin(\theta) & \cos(\theta)\end{bmatrix}$$

The figure is as following

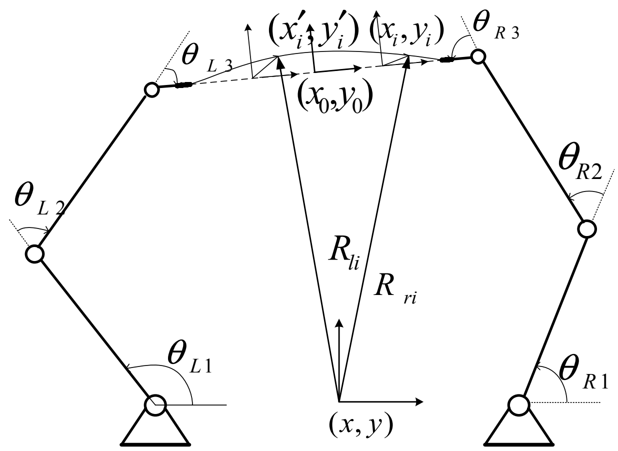


Fig.1 Configuration of cooperative manipulating

It can be seen that the expressions of are different and it will be explained later. The expressions of is

$$y_i=\sum_{j=1}^{4}\varphi_i(x)q_{2(i-1)+j} \quad (3)$$

is as follows

$$\varphi_1=1-\frac{3x_i^2}{l^2}+\frac{2x_i^3}{l^3} \qquad \varphi_2=x_i-\frac{2x_i^2}{l}+\frac{x_i^3}{l^2}$$

$$\varphi_3=\frac{3x_i^2}{l^2}-\frac{2x_i^3}{l^3} \qquad \varphi_4=-\frac{x_i^2}{l}+\frac{x_i^3}{l^2} \quad (4)$$

R is the displacement of some point of the payload corresponding to Cartesian coordinates, is the position of the payload's origin in Cartesian coordinate, x_0,y_0 is the angle of payload corresponding to Cartesian coordinates, the subscript l,r express left and right of the payload's origin, l is the length of single element, i is ith element from the origin, T is the Jacobian matrix between object coordinate and Cartesian coordinate, x_i,y_i are local coordinates.

The kinetic energy of the flexible payload is

$$E_k=E_{kl}+E_{kr} \qquad E_{ki}=\frac{1}{2}\rho\int_0^l \dot{R}_i^T\dot{R}_i dx_i \quad (5)$$

The potential of it is

$$E_p=E_{pr}+E_{pl} \qquad E_{pi}=\frac{1}{2}EI\int_0^l(\frac{\partial^2 y_i}{\partial x_i^2})^2dx_i \quad (6)$$

The flexible payload is divided into two elements. We define that q_{l1},q_{l2} are q_1,q_2 , p_{l3},p_{l4} are equal to q_{r1},q_{r2} are q_3,q_4 , q_{r3},q_{r4} are q_5,q_6 respectively because of the definition of R . The coordinates of vibration are $q_1\sim q_6$.

Ignoring the gravity, the Lagrange equation is as follows

$$\frac{d}{dt}(\frac{\partial L}{\partial\dot{p}})-(\frac{\partial L}{\partial p})=f \quad (7)$$

$$L=E_k-E_p$$

The model of the flexible payload is

$$A\ddot{p}+B\dot{p}+G_p p=f \quad (8)$$

$$p=[x_0,y_0,\theta,q]$$

f only acts to x_0,y_0,θ . Because the two ends of flexible payload are fixed, the q_1,q_5 are 0. Dividing the model of flexible payload into two parts: rigid motion and vibration, the following equations are got

The equations of rigid motion

$$A_{11}\ddot{x}+A_{12}\ddot{q}+B_{11}\dot{x}+B_{12}\dot{q}=f \quad (9)$$

The equations of vibration

$$A_{21}\ddot{x}+A_{22}\ddot{q}+B_{21}\dot{x}+B_{22}\dot{q}+K_F q=0 \quad (10)$$

The model of single manipulator is following

$$M_i(\theta)\ddot{\theta}_i+C_i(\theta,\dot{\theta})\dot{\theta}_i+G_i=\tau_i-J_i^T(\theta)f_i \quad (11)$$

The relations between are

$$\dot{x}_i=J_i(\theta)\dot{\theta}_i=J_i(x)\dot{x}$$

$$\dot{\theta}_i=J_i^{-1}(\theta)J_i(x)\dot{x}=J_i\dot{x}$$

$$\ddot{\theta}_i=J_i\ddot{x}+\dot{J}_i\dot{x} \quad (12)$$

In (11) the θ_i is expressed by x and each side of (11) is left-multiplied by J_i . The dynamic model of manipulator in task space is as follows

$$J_i^T M_i(\theta)J_i\ddot{x}+(J_i^T C_i(\theta,\dot{\theta})J_i+J_i^T M_i(\theta)\dot{J}_i)\dot{x}+J_i^T G_i$$

$$=J_i^T\tau_i-J_i^T J_i^T(\theta)f_i$$

$$\Rightarrow \overline{M}_i\ddot{x}+\overline{C}_i\dot{x}+\overline{G}_i=u_i-J_i^T(x)f_i \quad (13)$$

The relations between the f_i are as follows

$$\sum J_i^T(x)f_i=f \quad (14)$$

The f_i can be divided into two parts moving force f_{mi} and internal force f_{Ii}. The sum of internal forces is 0, so we can get

$$f_i = f_{im} + f_{iI} \qquad \sum J_i^T(x) f_{iI} = 0 \tag{15}$$

Form (9), (13), (14)and(15)the model of system is as follows

$$M\ddot{p} + C\dot{p} + G = \begin{bmatrix} \sum u_i \\ 0 \end{bmatrix} \tag{16}$$

It can be shown that the dynamics of the system derived using Euler-Lagrange formulation has the following properties.

1) M is symmetrical and positive definite and the sub-items made up of M are also symmetrical and positive definite;

2) $p^T(\dot{M} - 2C)p = 0$ if we choose C properly;

3) K_F is symmetrical and positive definite.

B. The decomposition of model

From the previous derivations the rigid coordinates and vibration are in coupling. In this section model of rigid motion will be decomposed.

From the equation (10) we can get

$$\ddot{q} = -A_{22}^{-1}(A_{21}\ddot{x} + B_{21}\dot{x} + K_F q) \tag{17}$$

$$A_{22}\ddot{q} + K_F q = -A_{21}\ddot{x} - B_{21}\dot{x} \tag{18}$$

From (16) and (18) the following equation will be got

$$\overline{M}\ddot{x} + \overline{C}\dot{x} + \overline{G} + \widetilde{M}\ddot{x} + \widetilde{C}\dot{x} + f(q,\dot{q}) = u \tag{19}$$

$$f(q,\dot{q}) = B_{12}\dot{q} - A_{12}A_{22}^{-1}K_F q$$

In (19) the first three items are made up of rigid coordinates and the latter three items is made up of rigid and vibration coordinates. The relations $x^T(\dot{\overline{M}} - 2\overline{C})x$ =0 is still satisfied from the derivations.

Theorem1. From (18) and given the $x, \dot{x}, \ddot{x}$ being bounded, the states of vibration will be bounded.

Proof. We can see the $x, \dot{x}, \ddot{x}$ are bounded because the inputs are bounded. The x can be expressed by trigonometric function from the transition of Fourier. Assuming the x has the simple expression as follows

$$x = \eta \sin(\omega t) \tag{20}$$

So the right side of (18) can be expressed as $\eta' \sin(\omega t + \sigma)$ and it is bounded.

From [15] the following equations will be got

$$q = \eta_0 \sin(\omega_0 t + \sigma_0) + \eta_1 \sin(\omega t + \varsigma) \tag{21}$$

The dimensions of vibration are more than one so the (21) is analytic expression. $\eta_0 \sin(\omega_0 t + \sigma_0)$ is related to original value and physical characteristic of the material. $\eta_1 \sin(\omega t + \varsigma)$ is compulsive vibration because of x . Because A_{22}, K_F are symmetrical and positive definition and $x, \dot{x}, \ddot{x}$ are bounded the q are bounded and it can be got that $\dot{q}, \ddot{q}$ are bounded similarly. From the proof the $f(q,\dot{q})$ will be bounded.

From [14] there are b_0, b_1, b_2 that are satisfied with the following relation.

$$\left\| -(\widetilde{M}\ddot{x} + \widetilde{C}\dot{x} + f(q,\dot{q})) \right\| = \|\Psi\|$$

$$\|\Psi\| < b_0 + b_1\|x\| + b_2\|x\|^2 \tag{22}$$

III. TRAJECTORY TRACKINGY

In order to track a planning trajectory and avoid the disturbance of vibration the adaptive sliding mode control law is designed. This control law needn't compute the regressor matrix and the chatering will reduce.

Let us define the state error and sliding surface as follows

$$r = \dot{x}_d - \Lambda e \qquad s = \dot{x} - r$$

$e = x - x_d$ and Λ is positive diagonal matrix.

Let us assume $\hat{b}_0, \hat{b}_1, \hat{b}_2$ are the estimations of b_0, b_1, b_2. $\tilde{b}_0 = b_0 - \hat{b}_0$, $\tilde{b}_1 = b_1 - \hat{b}_1$, $\tilde{b}_2 = b_2 - \hat{b}_2$. Because b_0, b_1, b_2 are fixed we get $\dot{\tilde{b}}_0 = -\dot{\hat{b}}_0, \dot{\tilde{b}}_1 = -\dot{\hat{b}}_1, \dot{\tilde{b}}_2 = -\dot{\hat{b}}_2$.

Theorem 2. The system is stable if the control law is following

$$u = \overline{u}_1 + \overline{u}_2 + \overline{u}_3$$

$$\overline{u}_1 = \overline{M}\dot{r} + \overline{C}r + \overline{G} \qquad \overline{u}_2 = -Ks$$

$$\overline{u}_3 = \begin{cases} -(\hat{b}_0 + \hat{b}_1\|x\| + \hat{b}_2\|x\|^2)\dfrac{s}{\|s\|} & \|s\| > \varepsilon \\ -(\hat{b}_0 + \hat{b}_1\|x\| + \hat{b}_2\|x\|^2)\dfrac{s}{\varepsilon} & \|s\| \le \varepsilon \end{cases}$$

$$\dot{\hat{b}}_0 = \frac{\|s\|}{k_0}, \dot{\hat{b}}_1 = \frac{\|s\|}{k_1}\|x\|, \ \dot{\hat{b}}_2 = \frac{\|s\|}{k_2}\|\dot{x}\|^2 \tag{23}$$

K , Λ is positive diagonal matrix. k_0, k_1, k_2 are positive. ε is the accurate range.

Proof. We start by defining the nonnegative function

$$V = \frac{1}{2}s^T\overline{M}s + \frac{1}{2}k_0\tilde{b}_0^2 + \frac{1}{2}k_1\tilde{b}_1^2 + \frac{1}{2}k_2\tilde{b}_2^2$$

From (19)and (23) we get

$$\dot V = s^T \overline{M}\dot s + \frac{1}{2}s^T \dot{\overline{M}}s + k_0\tilde b_0\dot{\tilde b}_0 + k_1\tilde b_1\dot{\tilde b}_1 + k_2\tilde b_2\dot{\tilde b}_2$$

$$= s^T(u_1 + u_2 + \Psi) + k_0\tilde b_0\dot{\tilde b}_0 + k_1\tilde b_1\dot{\tilde b}_1 + k_2\tilde b_2\dot{\tilde b}_2$$

$$= -s^T Ks + s^T u_2 + s^T \Psi$$

$$+ k_0\tilde b_0\dot{\tilde b}_0 + k_1\tilde b_1\dot{\tilde b}_1 + k_2\tilde b_2\dot{\tilde b}_2$$

When $\|s\| > \varepsilon$ and from (22) and (23) the following equation is satisfied

$$\dot V \le -s^T Ks + s^T u_2 + \|s\|(b_0 + b_1\|x\| + b_2\|x\|^2)$$

$$+ k_0\tilde b_0\dot{\tilde b}_0 + k_1\tilde b_1\dot{\tilde b}_1 + k_2\tilde b_2\dot{\tilde b}_2$$

$$= -s^T Ks - \|s\|(\hat b_0 + \hat b_1\|x\| + \hat b_2\|x\|^2)$$

$$+ \|s\|(b_0 + b_1\|x\| + b_2\|x\|^2)$$

$$+ k_0\tilde b_0\dot{\tilde b}_0 + k_1\tilde b_1\dot{\tilde b}_1 + k_2\tilde b_2\dot{\tilde b}_2$$

It used $s^T s = \|s\|^2$ here.

$$\dot V \le -s^T Ks + \|s\|(\tilde b_0 + \tilde b_1\|x\| + \tilde b_2\|x\|^2)$$

$$+ k_0\tilde b_0\dot{\tilde b}_0 + k_1\tilde b_1\dot{\tilde b}_1 + k_2\tilde b_2\dot{\tilde b}_2$$

From (23) we get

$$\dot V \le -s^T Ks < 0 \quad (24)$$

The proof is over and so the system is stable.

In the paper the amount of manipulators is two so $i = 1,2$. It should be seen from the previous derivation that in the process of controller designing the u is provided by two manipulators so the force/ torque provided by single manipulator is as following

$$u_i = (\overline{M}_i + \frac{1}{2}A_{11} - \frac{1}{2}A_{12}A_{22}^{-1}A_{21})\dot r + \overline{C}_i r + \overline{G}_i$$
$$+ \frac{1}{2}\overline{u}_2 + \frac{1}{2}\overline{u}_3 + J_i^T(x)f_{iI} \quad (25)$$

Here we assume the force/ torque offering to the flexible payload is provided by two manipulators averagely.

It's defined $J_i^T(x)f_{iI} = F_I$. From (13), (16) and (25) the internal force can be described as following

$$F_{iI} = -u_i - \frac{1}{2}A_{12}A_{22}^{-1}(C_{21}\dot x + C_{22}\dot q + K_F q)$$

$$+ (\overline{C}_i + \frac{C_{11}}{2})\dot x + \frac{C_{12}}{2}\dot q + \overline{G}_i$$

$$+ (\overline{M}_i + \frac{A_{11}}{2} - \frac{1}{2}A_{12}A_{22}^{-1}A_{21})$$

$$(\overline{M}_1 + \overline{M}_2 + A_{11} - A_{12}A_{22}^{-1}A_{21})^{-1}$$

$$[u_1 + u_2 - (\overline{C}_1 + \overline{C}_2 + C_{11})\dot x - C_{12}\dot q - \overline{G}_1 - \overline{G}_2$$

$$+ A_{12}A_{22}^{-1}(C_{21}\dot x + C_{22}\dot q + K_F q)] \quad (26)$$

It has been proven all the states on the right side of (26) are bounded[17] so the internal is bounded.

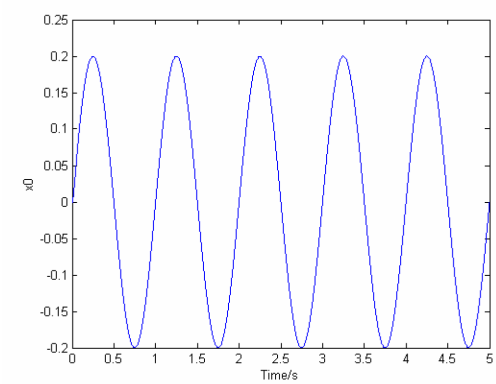
IV. SIMULATION

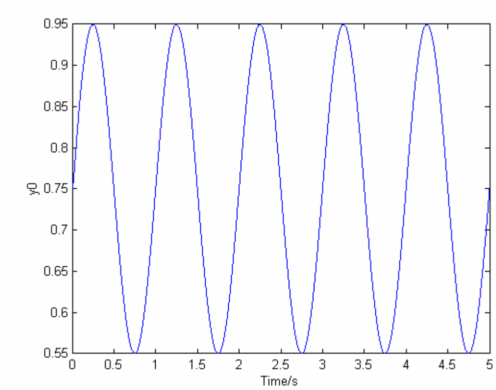
To verify the modeling method and control law the simulation was done on two identical three DOF planar manipulators (Fig.1). The parameters of the robot manipulators and the payload are that the masses of three links are $m_1 = m_2 = 8kg$ and $m_3 = 4kg$; the length of three links are $L_1 = 0.7m$, $L_2 = 0.6m$ and $L_3 = 0.2m$; the centers of links are $L_{1g} = 0.4m, L_{2g} = 0.35m$, $L_{3g} = 0.1m$. The length of payload is $L = 0.5m$ and the density is $\rho = 2kg/m$. The parameters of control low are $EI = 100$, $\varepsilon = 0.1$, $C_{\dot p} = 0.1$, $K = 200$, $\Lambda = 50$, $k0 = k1 = k2 = 10$.

The trajectory is as follows

$$x_d = [0.2\sin(2\pi t), 0.75 + 0.2\sin(2\pi t), 0.2\sin(2\pi t)]^T$$

$$\dot x_d = [0.4\pi\cos(2\pi t), 0.4\pi\cos(2\pi t), 0.4\pi\cos(2\pi t)]^T$$





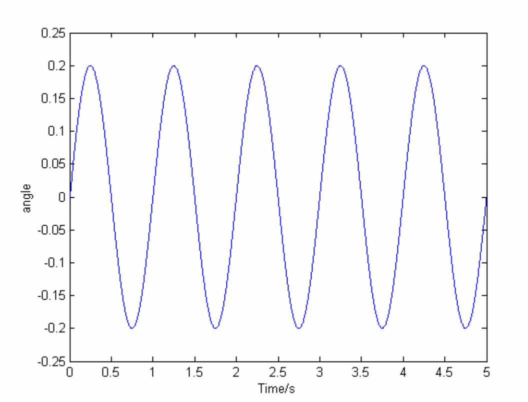


Fig.2 Trajectory of payload

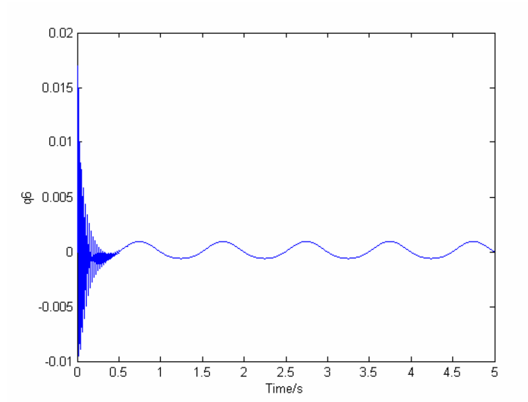
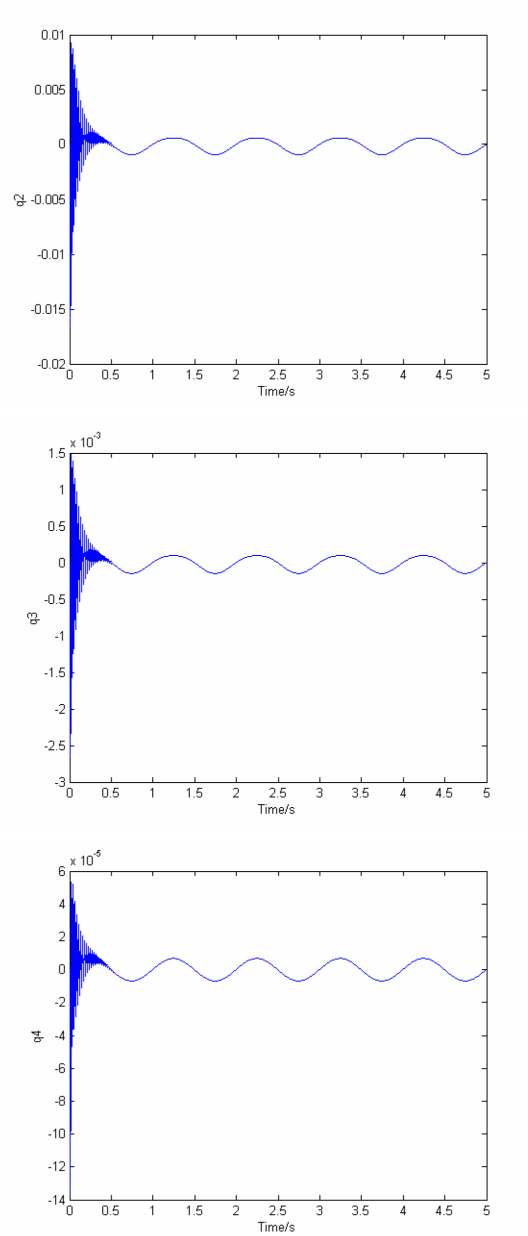


Fig.3 Vibration



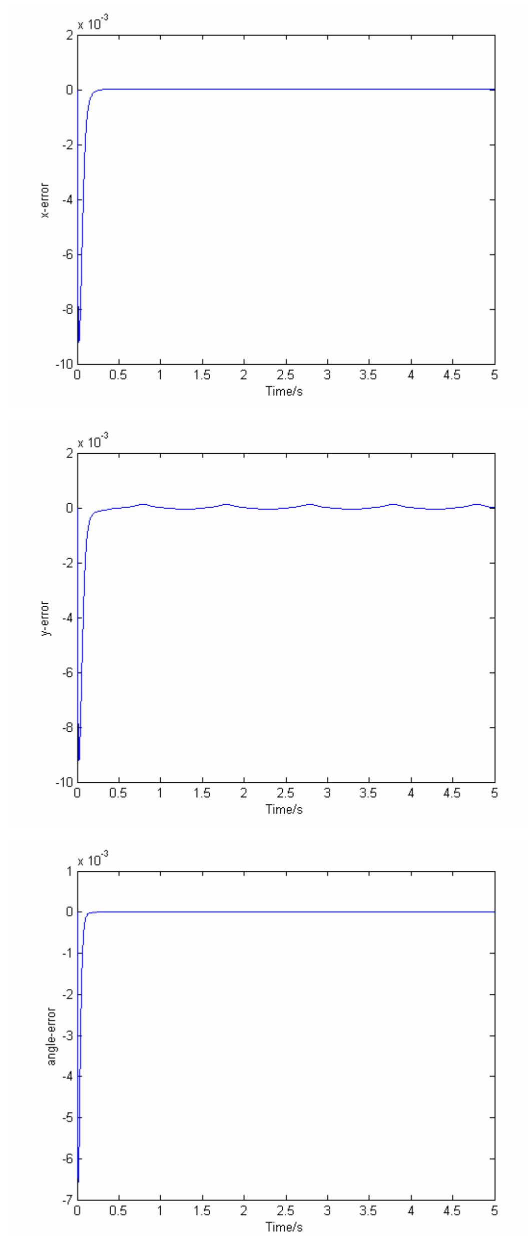


Fig.4 Error of trajectory

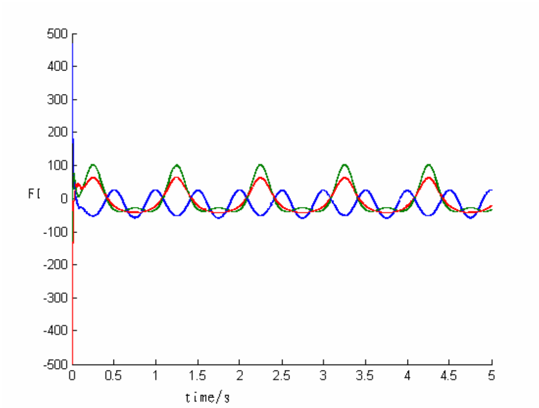


Fig.5 Internal force

From the simulation results it can be seen that the trajectory tracks the planning trajectory quickly. Firstly, the main influence to vibration is the physical characteristic of the flexible payload. At about t=0.5s the original vibration because of the physical characteristic disappears and then the vibration is compelled by the rigid motion. From the simulation results of internal force it can be seen the effect of vibration to internal is more distinct than to rigid position and this can be explained by the physical characteristic. The frequency of vibration is similar to the frequency of rigid motion. The simulation results prove the Theorem.1.

V. CONCLUSIONS

The modeling and trajectory tracking of manipulating flexible payloads by robot manipulators are studied in this paper. The finite element method (FEM) is used to approximate the vibration of flexible payload. Comparing the three spaces the system model is got in task space. In order to restrain the disturbance of vibration to trajectory tracking the adaptive sliding mode control law is designed. It is proven that the modeling method is accurate and the control law is valid through the simulation results. We didn't propose a direct control method in this paper that can guarantee the vibration converging zero and we will do this work next.

ACKNOWLEDGMENT

This work is supported by National Natural Science Foundation of China, grant number (60375030, 60674091).

REFERENCES

[1] T. Yukawa M. Uchiyama and H. Inooka. Handling of a Constrained Flexible Object by a Robot. Proceedings - IEEE International Conference on Robotics and Automation. 1995. 324-329

[2] J.Q. Wu, Z.W. Luo M. Yamakita K. Ito. Gain Scheduled Control of Robot Manipulators for Contact Tasks on Uncertain Flexible Objects. Proceedings of the IEEE International Conference on Systems, Man and Cybernetics. 1996.41-46.

[3] Y.F. Zhang, R. Pei and C.Y. Chen. Strategies for Automatic Assembly of Deformable Objects. Proceedings - IEEE International Conference on Robotics and Automation. 1991. 2598-2603.

[4] H. Nakagaki K. Kitagaki and H. Tsukune. Study of Insertion Task of a Flexible Beam into a Hole. Proceedings - IEEE International Conference on Robotics and Automation. 1995.330-335.

[5] Jr. Werner Kraus and B.J. McCarragher. Force Fields in the Manipulation of Flexible Materials. Proceedings - IEEE International Conference on Robotics and Automation. 1996. 2352-2357.

[6] J. K. Mills and Ing, Jerry G.-L. Robotic Fixtureless Assembly of Sheet Metal Parts Using Dynamic Finite Element Models: Modeling and Simulation. Proceedings - IEEE International Conference on Robotics and Automation. 2350-2357.

[7] W. Nguyen and J. K. Mills. Multi-Robot Control For Flexible Fixtureless Assembly of Flexible Sheet Metal Auto Body Parts. Proceedings - IEEE International Conference on Robotics and Automation. 1996. 2340-2345.

[8] Y.F. Zheng and M. Z. Chen. Trajectory planning for two manipulators to deform flexible beams. Proceedings - IEEE International Conference on Robotics and Automation. 1993. 1019-1024.

[9] M.Z. Chen and Y.F. Zheng. Vibration-free movement of deformable beams by robot manipulator. Proceedings - IEEE International Conference on Robotics and Automation. 1993. 456-461.

[10] Y.H. Liu and D. Sun. Stabilizing a Flexible Beam Handled by Two Manipulators Via PD Feedback. IEEE Transactions on Automatic Control. 2000,VOL. 45(11): 2159-2164,

[11] D Sun and Y.H. Liu. Position and Force Tracking of a Two-Manipulator System Manipulating a Flexible Beam. Journal of Robotic Systems 2001, Vol.18(4): 197-212.

[12] D. Sun and Y.H. Liu. Position and Force Tracking of a Two-Manipulator System Manipulating a Flexible Beam Payload. Proceedings - IEEE International Conference on Robotics and Automation. 2001.3483-3488.

[13] Al-Yahmadi, Amer S. Abdo, Jamil. etc. Modeling and control of two manipulators handling a flexible object [J]. Journal of the Franklin Institute. Vol.344(5), pp349-361.

[14] Sze San Chong and Xinghuo Yu. A Robust Adaptive Sliding Mode Controller for Robotic Manipulators. IEEE Workshop on Variable Structure Systems.1996. pp31－35

[15] Y.Z. Liu, W.L.Chen, L.Q. Chen. Mechanics of Vibration. Higher Education Press. China. 1998

[16] D. Sun. Cooperative Control of Two-Manipulator Systems Handling Flexible Objects. Ph.D. dissertation. Dept. Sys. Eng. & Eng. Management. Chinese Univ. Hong Kong, 1997

[17] P. Zhang and Y.C. Li, Position/Force Control of Two Manipulators Handling a Flexible Payload Based on Finite-element Model 2007 International Conference on Robotics and Biomimetics