Robust Guaranteed Cost Control for a Class of Two-dimensional Discrete Systems with Shift-Delays

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***Abstract*— This paper considers the problem of robust guaranteed cost control for uncertain two-dimensional (2-D) discrete shift-delayed systems in Fornasini-Marchesini model the second class (FMMⅡ). The parameter uncertainty is assumed to be norm-bounded. The problem to be addressed is the design of state feedback controllers such that the closed-loop system is quadratic stable and an adequate level of performance can be guaranteed for all admissible uncertainties. In terms of a linear matrix inequality (LMI), a sufficient condition for the solvability of the problem is obtained. A desired state feedback controller can be constructed by solving a certain LMI. A numerical example is provided to demonstrate the application of the proposed method.**

***Keywords*—2-D discrete systems; guaranteed cost control; quadratic stable; shift-delays.**

I. INTRODUCTION

In recent years, two-dimensional (2-D) discrete systems have received considerable attention since 2-D systems have extensive applications in many areas such as image date processing, seismographic date processing, thermal processes, gas absorption, water stream heating [1]. Many important results have been reported in the literatures. The stability of 2-D discrete systems has been investigated extensively [2-15]. The controller and filter design problems have been considered in [5-15]. Time delays frequently occur in practical systems and are often the source of instability. There are many examples containing inherent delays in practical 2-D discrete systems, the stability of 2-D discrete systems with time delay have been also studied in [16-21].

On the other hand, the guaranteed cost control is first reported in [22]. To this end, robust guaranteed cost control of 1-D systems has been widely considered [23-26]. The guaranteed cost control technique for the 2-D discrete system has been considered and a robust control design method has been established in [11]. In [12], some technical errors in the derivation of Theorem 4 in [11] have been pointed out and corrected. LMI-based criterion for the robust optimal guaranteed cost control for 2-D systems described by the Fornasini-Marchesini second model [13] has been investigated. An LMI approach of optimal guaranteed cost control for 2-D discrete uncertain systems [14] was considered. All of these results that guaranteed cost control for 2-D discrete are not involved with delays. For convenience, here we call the delays described by the 2-D indices (i, j) the shift-delay.

In this paper, we discuss the problem of robust guaranteed cost of 2-D discrete systems in the FMM Ⅱ setting with shift-delays. The parameter uncertainty is assumed to be unknown but norm-bounded.

The paper is organized as follows: Firstly, we provide the conditions for 2-D discrete systems with shift-delays and give the cost function. Next, robust guaranteed cost performance analysis is done and robust guaranteed cost control via static-state feedback is discussed. A numerical example is provided to illustrate the presented techniques. Finally, concluding remarks are given.

Notations: Throughout the paper, the superscript "T" and "-1" stands for matrix transposition and inversion, respectively; R^n denotes the n-dimensional Euclidean space, $R^{n\times m}$ is the set of all real matrices of dimension $n\times m$; An asterisk "*" represents a term that is induced by symmetry and diag$\{\cdots\}$ stands for a block-diagonal matrix.

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II. PROBLEM FORMULATION

This paper deals with the problem of robust guaranteed cost control of a class of 2-D discrete uncertain systems in the FMM Ⅱ setting with shift-delays. The system under consideration is given by

$$\begin{aligned} x(i+1,j+1) = &(A_1+\Delta A_1)x(i,j+1) \\ &+(A_2+\Delta A_2)x(i+1,j) \\ &+(A_{1d}+\Delta A_{1d})x(i-d,j+1) \\ &+(A_{2k}+\Delta A_{2k})x(i+1,j-k) \\ &+(B_1+\Delta B_1)u(i,j+1) \\ &+(B_2+\Delta B_2)u(i+1,j) \end{aligned} \tag{1a}$$

$$A=\begin{bmatrix}A_1 & A_2 & A_{1d} & A_{2k}\end{bmatrix},\ B=\begin{bmatrix}B_1 & B_2\end{bmatrix}$$

Here $x(i,j)\in R^n$ is the state. $u(i,j)\in R^m$ is the control input, respectively. The matrices A_1, A_2, A_{1d}, $A_{2k}\in R^{n\times n}$ and $B_1,B_2\in R^{n\times m}$ are known constant matrices. d,k are constant positive scalars representing delays along vertical direction and horizontal direction respectively. The matrices ΔA_1, ΔA_2, ΔA_{1d}, ΔA_{2k}, ΔB_1, ΔB_2 represent norm-bounded parameter uncertainties, which are assumed to be of the form

$$\begin{aligned}\Delta A&=\begin{bmatrix}\Delta A_1 & \Delta A_2 & \Delta A_{1d} & \Delta A_{2k}\end{bmatrix}\\ &=LF(i,j)\begin{bmatrix}M_{11} & M_{12} & M_{13} & M_{14}\end{bmatrix}\end{aligned}$$

$$\Delta B=\begin{bmatrix}\Delta B_1 & \Delta B_2\end{bmatrix}=LF(i,j)\begin{bmatrix}M_{21} & M_{22}\end{bmatrix}$$

In the above, L,M_1 and M_2 can be regarded as known structural matrices of uncertainty and $F(i,j)$ is an unknown matrix satisfying that

$$\|F(i,j)\|\le 1$$

The initial conditions are that there exist two positive integers r_1 and r_2 such that

$$\begin{aligned} &x(i,j)=0, \quad j\ge r_1, i=-d,-d+1,\cdots 1,0; \\ &or \quad i\ge r_2, j=-k,-k+1,\cdots 1,0 \end{aligned} \tag{1b}$$

The cost function is

$$J=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}[u^T(i,j+1)R_1u(i,j+1)+u^T(i+1,j)R_2u(i+1,j)] + \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\xi_{ij}{}^T W\xi_{ij} \tag{2}$$

where

$$\xi_{ij}=[x^T(i,j+1)\ x^T(i+1,j)\ x^T(i-d,j+1)\ x^T(i+1,j-k)]^T$$

$$0<R_1,R_2\in R^{m\times m}$$

$$W=\begin{bmatrix}S_1 & 0 & 0 & 0\\ 0 & S_2 & 0 & 0\\ 0 & 0 & S_3 & 0\\ 0 & 0 & 0 & S_4\end{bmatrix},\quad 0<S_1,\ S_2,\ S_3,\ S_4\in R^{n\times n}$$

The main objective of this paper is to derive a sufficient condition for the existence of static-state feedback robust controller for system (1a) with the cost function (2) such that the closed-loop system is asymptotically stable and the cost function of closed-loop system is lower than a specified upper bound.

III. MAIN RESULTS

A. Robust guaranteed cost performance analysis

Consider the 2-D discrete system with zero input

$$\begin{aligned}x(i+1,j+1)&=A_{\Delta 1}x(i,j+1)+A_{\Delta 2}x(i+1,j)\\ &+A_{\Delta 1d}x(i-d,j+1)+A_{\Delta 2k}x(i+1,j-k),\end{aligned} \tag{3}$$

where

$$A_{\Delta 1}=A_1+\Delta A_1=A_1+LF(i,j)M_{11},$$
$$A_{\Delta 2}=A_2+\Delta A_2=A_2+LF(i,j)M_{12},$$
$$A_{\Delta 1d}=A_{1d}+\Delta A_{1d}=A_{1d}+LF(i,j)M_{13},$$
$$A_{\Delta 2k}=A_{2k}+\Delta A_{2k}=A_{2k}+LF(i,j)M_{14}.$$

The associated cost function is

$$J_0=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\xi_{ij}{}^T W\xi_{ij} \tag{4}$$

Lemma 1 The uncertain system (3) is asymptotically stable for all admissible uncertainties if there exist $n\times n$ positive definite symmetric matrices $P>0$, $Q>0$, $Q_1>0$, $Q_2>0$ satisfying

$$\Gamma=H^TPH-\begin{bmatrix}P-Q-Q_1-Q_2 & 0 & 0 & 0\\ 0 & Q & 0 & 0\\ 0 & 0 & Q_1 & 0\\ 0 & 0 & 0 & Q_2\end{bmatrix}<0 \tag{5}$$

where $H=\begin{bmatrix}A_{\Delta 1} & A_{\Delta 2} & A_{\Delta 1d} & A_{\Delta 2k}\end{bmatrix}$

for all $\|F(i,j)\|\le 1$.

Remark 1 Lemma 1 shows that (5) is a sufficient condition of the robust asymptotical stability for system (3). Note that the left of (5) is of quadratic form. We therefore call the systems that satisfies (5), for convenience, the quadratic stable systems. When $\Delta A_1=\Delta A_2=\Delta A_{1d}=\Delta A_{2k}=0$, Lemma 1 is Theorem 3 in [16].

Remark 2 If there are matrices $P>0$, $Q>0$, $Q_1>0$, $Q_2>0$ satisfying

$$\Omega=\Gamma+W<0 \tag{6}$$

for all $\|F(i,j)\| \le 1$,

Then, the uncertain system (3) is said to be quadratic stable with a guaranteed cost.

In the following, we consider the bound of the cost function.

Lemma 2 If there exist matrices $P>0$, $Q>0$, $Q_1>0$, $Q_2>0$ satisfying (6) for system (3) with initial conditions (1b) and cost function (4), then system (3) is quadratic stable with a guaranteed cost and for all admissible uncertainties. The cost function satisfies the bound

$$J_0 < \sum_{i=1}^{r_2-1} x^T(i,0)(P-Q-Q_1-Q_2)x(i,0) + \sum_{j=1}^{r_1-1}\sum_{i=-d}^{0} x^T(i,j)Q_1x(i,j) + \sum_{j=1}^{r_1-1} x^T(0,j)Qx(0,j) + \sum_{i=1}^{r_2-1}\sum_{j=-k}^{0} x^T(i,j)Q_2x(i,j). \quad (7)$$

Proof. If there exist matrices $P>0$, $Q>0$, $Q_1>0$, $Q_2>0$ satisfying (6), the system (3) is quadratic stable with a guaranteed cost. In this case, we have that

$$\xi_{ij}^T\Omega\xi_{ij} = \xi_{ij}^T\Gamma\xi_{ij} + \xi_{ij}^T W\xi_{ij} < 0 \quad (8)$$

Hence

$$J_0 = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\xi_{ij}^T W\xi_{ij} < -\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\xi_{ij}^T\Gamma\xi_{ij}$$

Substituting Γ

$$\begin{aligned}
J_0 &< -\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\xi_{ij}^T\Gamma\xi_{ij} \\
&= -\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}[x^T(i+1,j+1)Px(i+1,j+1) \\
&\quad - x^T(i+1,j)(P-Q-Q_1-Q_2)x(i+1,j) \\
&\quad - x^T(i+1,j-k)Q_2x(i+1,j-k) \\
&\quad - x^T(i,j+1)Qx(i,j+1) \\
&\quad - x^T(i-d,j+1)Q_1x(i-d,j+1)] \\
&= -\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}[x^T(i+1,j+1)(P-Q \\
&\quad -Q_1-Q_2)x(i+1,j+1) \\
&\quad - x^T(i+1,j)(P-Q-Q_1-Q_2)x(i+1,j)] \\
&\quad -\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}[x^T(i+1,j+1)Qx(i+1,j+1) \\
&\quad - x^T(i,j+1)Qx(i,j+1)] \\
&\quad -\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}[x^T(i+1,j+1)Q_2x(i+1,j+1) \\
&\quad - x^T(i+1,j-k)Q_2x(i+1,j-k)] \\
&\quad -\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}[x^T(i+1,j+1)Q_1x(i+1,j+1) \\
&\quad - x^T(i-d,j+1)Q_1x(i-d,j+1)].
\end{aligned}$$

when $i,j\to\infty$, obtain the upper bound (7). This completes the proof.

Remark 3 The bound of the cost function obtained in the Lemma 3 depends on the initial conditions.

Now we are in the position to give a LMI-based sufficient condition for system (3) with the initial conditions（1b）and the cost function (4) is robust quadratic stable with a guaranteed cost.

Next, we have the following result.

Theorem 1 If there exist a scalar $\varepsilon>0$ and positive definite symmetric matrices P, $P_1=\varepsilon P^{-1}$, and Y_1, Y_2, Y_3, such that the following LMI is feasible,

$$Z=\left[\begin{array}{cccccccccc}
-P_1 & A_1P_1 & A_2P_1 & A_{1d}P_1 & A_{2k}P_1 & L & 0 & 0 & 0 & 0 & 0 \\
* & Z_{22} & 0 & 0 & 0 & 0 & P_1M_{11}^T & P_1S_1^{1/2} & 0 & 0 & 0 \\
* & * & -Y_1 & 0 & 0 & 0 & P_1M_{12}^T & 0 & P_1S_2^{1/2} & 0 & 0 \\
* & * & * & -Y_2 & 0 & 0 & P_1M_{13}^T & 0 & 0 & P_1S_3^{1/2} & 0 \\
* & * & * & * & -Y_3 & 0 & P_1M_{14}^T & 0 & 0 & 0 & P_1S_4^{1/2} \\
* & * & * & * & * & -I & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -I & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -\varepsilon I & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -\varepsilon I & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -\varepsilon I & 0 \\
* & * & * & * & * & * & * & * & * & * & -\varepsilon I
\end{array}\right] < 0 \quad (9)$$

where $Z_{22}=-P_1+Y_1+Y_2+Y_3$, then system (3) with initial condition（1b）and cost function (4) is robust quadratic stable with a guaranteed cost. Moreover the cost function satisfies the bound

$$J_0 < \varepsilon[\sum_{i=1}^{r_2-1} x^T(i,0)(P_1^{-1} - P_1^{-1}Y_3P_1^{-1} - P_1^{-1}Y_1P_1^{-1} - P_1^{-1}Y_2P_1^{-1})x(i,0)$$
$$+\sum_{j=1}^{r_1-1}\sum_{i=-k}^{0} x^T(i,j)P_1^{-1}Y_3P_1^{-1}x(i,j) + \sum_{j=1}^{r_1-1} x^T(0,j)P_1^{-1}Y_1P_1^{-1}x(0,j)$$
$$+\sum_{i=1}^{r_2-1}\sum_{j=-d}^{0} x^T(i,j)P_1^{-1}Y_2P_1^{-1}x(i,j)] \tag{10}$$

where $P_2 = P_1^{-1}$.

Proof. Using [11, Lemma 2], (6) can be rearranged as

$$\begin{bmatrix} \Gamma_{11} & A_1 & A_2 & A_{1d} & A_{2k} \\ A_1^T & \Gamma_{22} & \varepsilon M_{11}{}^T M_{12} & \varepsilon M_{11}{}^T M_{13} & \varepsilon M_{11}{}^T M_{14} \\ A_2^T & * & \Gamma_{33} & \varepsilon M_{12}{}^T M_{13} & \varepsilon M_{12}{}^T M_{14} \\ A_{1d}^T & * & * & \Gamma_{44} & \varepsilon M_{13}{}^T M_{14} \\ A_{2k}^T & * & * & * & \Gamma_{55} \end{bmatrix} < 0$$

Here,

$\Gamma_{11} = -P^{-1} + \varepsilon^{-1}LL^T$,

$\Gamma_{22} = -(P - Q_1 - Q - Q_2) + S_1 + \varepsilon M_{11}^T M_{11}$,

$\Gamma_{33} = -Q + S_2 + \varepsilon M_{12}^T M_{12}$,

$\Gamma_{44} = -Q_1 + S_3 + \varepsilon M_{13}^T M_{13}$,

$\Gamma_{55} = -Q_2 + S_4 + \varepsilon M_{14}^T M_{14}$,

Pre-multiplying and post-multiplying by the matrix $diag\{\varepsilon^{1/2} I, \varepsilon^{1/2} P^{-1}, \varepsilon^{1/2} P^{-1}, \varepsilon^{1/2} P^{-1}, \varepsilon^{1/2} P^{-1}\}$ and applying Schur complement, we can obtain (9).
where

$$P_1 = \varepsilon P^{-1},$$
$$Y_1 = \varepsilon^{-1} P_1 Q P_1, Y_2 = \varepsilon^{-1} P_1 Q_1 P_1, Y_3 = \varepsilon^{-1} P_1 Q_2 P_1 \tag{11}$$

Substituting (11) for (7), we can obtain (10). This completes the proof.

B. Robust guaranteed cost control via static-state feedback

In this section, we will design a static-state feedback $u(i,j) = Kx(i,j)$ for system (1) and cost function (2) such that the closed-loop system is quadratic stable and the cost function of closed-loop system is lower than a specified upper bound.

The closed-loop system (1a) with $u(i,j) = Kx(i,j)$ can be expressed as

$$x(i+1,j+1) = (A_{\Delta 1} + B_{\Delta 1}K)x(i,j+1) + (A_{\Delta 2} + B_{\Delta 2}K)x(i+1,j)$$
$$+ A_{\Delta 1d}x(i-d,j+1) + A_{\Delta 2k}x(i+1,j-k), \tag{12}$$

where

$$A_{\Delta 1} = A_1 + \Delta A_1 = A_1 + LF(i,j)M_{11},$$
$$A_{\Delta 2} = A_2 + \Delta A_2 = A_2 + LF(i,j)M_{12},$$
$$A_{\Delta 1d} = A_{1d} + \Delta A_{1d} = A_{1d} + LF(i,j)M_{13},$$
$$A_{\Delta 2k} = A_{2k} + \Delta A_{2k} = A_{2k} + LF(i,j)M_{14},$$
$$B_{\Delta 1} = B_1 + \Delta B_1 = B_1 + LF(i,j)M_{21},$$
$$B_{\Delta 2} = B_2 + \Delta B_2 = B_2 + LF(i,j)M_{22}$$

The cost function (2) can be reduced

$$J_0 = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \xi_{ij}{}^T W_1 \xi_{ij} \tag{13}$$

where

$$W_1 = \begin{bmatrix} S_1 + K^T R_1 K & 0 & 0 & 0 \\ 0 & S_2 + K^T R_2 K & 0 & 0 \\ 0 & 0 & S_3 & 0 \\ 0 & 0 & 0 & S_4 \end{bmatrix}.$$

Lemma 3 If there exist matrices $P > 0$, $W_1 > 0$, $Q > 0$, $Q_1 > 0$, $Q_2 > 0$ satisfying

$$H_1{}^T P H_1 - \begin{bmatrix} P - Q - Q_1 - Q_2 & 0 & 0 & 0 \\ 0 & Q & 0 & 0 \\ 0 & 0 & Q_1 & 0 \\ 0 & 0 & 0 & Q_2 \end{bmatrix} + W_1 < 0 \tag{14}$$

where

$$H_1 = [A_{\Delta 1} + B_{\Delta 1}K \quad A_{\Delta 2} + B_{\Delta 2}K \quad A_{\Delta 1d} \quad A_{\Delta 2k}],$$
for all $\|F(i,j)\| \le 1$,

then there will exist a state feedback controller $u(i,j) = Kx(i,j)$ such that system (12) is quadratic stable with a guaranteed cost and for all admissible uncertainties the cost function satisfies the bound

$$J_0 < \sum_{i=1}^{r_2-1} x^T(i,0)(P - Q - Q_1 - Q_2)x(i,0) + \sum_{j=1}^{r_1-1}\sum_{i=-d}^{0} x^T(i,j)Q_1x(i,j)$$
$$+\sum_{j=1}^{r_1-1} x^T(0,j)Qx(0,j) + \sum_{i=1}^{r_2-1}\sum_{j=-k}^{0} x^T(i,j)Q_2x(i,j) \tag{15}$$

Remark 4 Lemma3 can be obtained directly by applying lemma 2 to closed-loop system (12).

Applying Theorem 1 to the closed-loop system (12) we have the following result:

Theorem 2 If there exist a positive scalar $\varepsilon > 0$ and positive definite symmetric matrices P, $P_1 = \varepsilon P^{-1}$, Y_1, Y_2, Y_3, and U such that the following LMI is feasible

$$
\begin{bmatrix}
-P_1 & \overline{A_1} & \overline{A_2} & A_{1d}P_1 & A_{2k}P_1 & L & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & \mathrm{T}_{22} & 0 & 0 & 0 & 0 & \overline{M_{11}} & P_1S_1^{1/2} & 0 & 0 & 0 & U^TR_1^{1/2} & 0 \\
* & * & -Y_1 & 0 & 0 & 0 & \overline{M_{12}} & 0 & P_1S_2^{1/2} & 0 & 0 & 0 & U^TR_2^{1/2} \\
* & * & * & -Y_2 & 0 & 0 & P_1M_{13}^T & 0 & 0 & P_1S_3^{1/2} & 0 & 0 & 0 \\
* & * & * & * & -Y_3 & 0 & P_1M_{14}^T & 0 & 0 & 0 & P_1S_4^{1/2} & 0 & 0 \\
* & * & * & * & * & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -I & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -\varepsilon I & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -\varepsilon I & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -\varepsilon I & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & -\varepsilon I & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & -\varepsilon I & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & -\varepsilon I
\end{bmatrix} < 0 \qquad (16)
$$

where

$\overline{A_1} = A_1P_1 + B_1U$, $\overline{M_{11}} = P_1M_{11}^T + U^TM_{21}^T$,

$\overline{A_2} = A_2P_1 + B_2U$, $\overline{M_{12}} = P_1M_{12}^T + U^TM_{22}^T$,

$\mathrm{T}_{22} = -P_1 + Y_1 + Y_2 + Y$, $K = UP_1^{-1}$

then, there exists a static-state feedback controller $u(i,j) = Kx(i,j)$ such that system (1) is quadratic stable with a guaranteed cost and for all admissible uncertainties the cost function satisfies the bound

$$
\begin{aligned}
J_0 < \varepsilon[&\sum_{i=1}^{r_2-1} x^T(i,0)(P_2 - P_2Y_3P_2 - P_2Y_1P_2 - P_2Y_2P_2)x(i,0) \\
&+ \sum_{j=1}^{r_1-1}\sum_{i=-k}^{0} x^T(i,j)P_2Y_3P_2x(i,j) + \sum_{j=1}^{r_1-1} x^T(0,j)P_2Y_1P_2x(0,j) \\
&+ \sum_{i=1}^{r_2-1}\sum_{j=-d}^{0} x^T(i,j)P_2Y_2P_2x(i,j)]
\end{aligned} \qquad (17)
$$

where $P_2 = P_1^{-1}$.

The proof of this theorem is similar to that of Theorem 1 and, hence is omitted here for simplicity.

Remark 5 In Theorem 2, we constructed a static-state feedback controller and guaranteed cost upper bound can be obtained. The bound is dependent on the cost controller and the initial conditions.

IV. Numerical Example

In this section, we shall illustrate the results via an example. All simulations were performed with LMI control toolbox. Consider the following 2-D system (1)

$$
A_1 = \begin{bmatrix} -0.2450 & 0.0307 \\ -0.1444 & 0.0008 \end{bmatrix}, A_2 = \begin{bmatrix} 0.2860 & 0.1800 \\ -0.1435 & -0.4601 \end{bmatrix},
$$

$$
A_{1d} = \begin{bmatrix} 0.1453 & 0.1489 \\ 0.0824 & 0.0536 \end{bmatrix}, A_{2k} = \begin{bmatrix} 0.0880 & 0.1367 \\ 0.1867 & 0.0425 \end{bmatrix},
$$

$$
B_1 = \begin{bmatrix} 0.8392 & 0.1338 \\ 0.6288 & 0.2071 \end{bmatrix}, B_2 = \begin{bmatrix} 1.0322 & 0.6298 \\ 1.0708 & 0.9778 \end{bmatrix}, L = \begin{bmatrix} 0.2257 \\ 0.0219 \end{bmatrix},
$$

and $d = 2, k = 3$, with uncertainty modeled represented by

$$
M_{11} = [0.0272 \quad 0.3127], M_{12} = [0.0129 \quad 0.3840],
$$

$$
M_{13} = [0.3043 \ 0.0082], M_{14} = [0.2935 \quad 0.1838],
$$

$$
M_{21} = [0.1366 \quad 0.0186], M_{22} = [0.0071 \quad 0.1225],
$$

$$
S_1 = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}, S_2 = \begin{bmatrix} 0.15 & 0 \\ 0 & 0.15 \end{bmatrix}, S_3 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}, S_4 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}
$$

The initial conditions are that there exist two positive integers $r_1 = 5$ and $r_2 = 6$ such that

$$
x(i,j) = \begin{bmatrix} \frac{\sin(i+j)}{10} & \frac{\sin(i+j)}{10} \end{bmatrix}^T, \quad j < r_1, i = -d, -d+1, \cdots 1, 0; \quad or \quad i < r_2, j = -k, -k+1, \cdots 1, 0
$$

In this case, LMI (16) is feasible and the matrices are

$$
P_1 = \begin{bmatrix} 0.6941 & -0.2400 \\ -0.2400 & 0.6920 \end{bmatrix}
$$

$$
Y_1 = \begin{bmatrix} 0.2681 & -0.1592 \\ -0.1592 & 0.3126 \end{bmatrix}, Y_2 = \begin{bmatrix} 0.2158 & -0.0559 \\ -0.0559 & 0.1235 \end{bmatrix},
$$

$$
Y_3 = \begin{bmatrix} 0.1427 & 0.0060 \\ 0.0060 & 0.0933 \end{bmatrix}, U = \begin{bmatrix} 0.2237 & -0.2701 \\ -0.2701 & 0.3810 \end{bmatrix}.
$$

And $\varepsilon = 35.3571$ which yields closed-loop matrix K and guaranteed cost upper bound equal to

$$
K = \begin{bmatrix} 0.2129 & -0.3164 \\ -0.2258 & 0.4723 \end{bmatrix}, J_0 < 8.1738 .
$$

The resulting system is quadratic stable with a guaranteed cost by Theorem 2.

V. Conclusion

In this paper, we have considered the problem of robust guaranteed cost control for 2-D discrete systems with shift-delays by static-state feedback. A desired robust guaranteed cost controller can be constructed by solving a given LMI. The approach can also be applied to obtain dynamic output feedback controller. Further, the results presented in this paper can also be extended for uncertain *m*-D (*m*>2) systems. Further development on robust control involving performance specification will be required to reduce conservative for 2-D systems to be stable and obtain more relaxed criteria for the existence of a guaranteed cost controller.

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