Centralized H∞ Fusion Filter Design in Multi-Sensor Data Fusion System

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***Abstract*—Some multi-sensor fusion systems are not asymptotic stable like target tracking system. When using classical H_∞ filtering theory to design the fusion filter for these systems, there are no feasible solutions to the problem. Applying H_∞ theory and LMI methods to design the H_∞ fusion filter for those kind of fusion systems, in which the process and measurement noise have unknown statistic characteristic but bounded power, a new approach is presented in this paper. Finally, an example is given to illustrate the effectiveness of our method.**

***Keywords*—H∞ filtering, multi-sensor data fusion, LMI**

I. INTRODUCTION

In recent years, the multi-sensor data fusion has received great attention due to extensive application backgrounds. The data fusion Kalman filter has been widely applied in many fields including guidance, defence, robotics, integrated navigation, target tracking, and GPS positioning, see [1-3] and the references therein. Generally speaking, there are two methods of fusion filtering estimation, i.e. state vector fusion estimation and measurement fusion estimation. When information of process and measurement are accurate, the latter will develop an optimal global estimation. However, in practice, the states in multi-sensor data fusion system have to be estimated from noisy measurement, and in some special cases, the characteristics of the noise were unknown. Therefore, classical Kalman filter has difficult in solving these problems.

The H∞ filtering technique has been widely studied for the benefit of different time and frequency domain properties to the H_2 filtering technique. In the H∞ setting, the exogenous input signal is assumed to be energy bounded rather than Gaussian. An H∞ filter is designed such that the H∞ norm of the system, which reflects the worst-case "gain" of the system, is minimized. It has been widely used to cope with disturbances of partially unknown statistics but with an upper bound of the signal power, see [4-9]). In particular, [4] gave a very useful lemma, which is called "stochastic bounded real lemma (SBRL)" on discrete-time system. [7] applied the SBRL to design robust filter for stochastic uncertainty system. In [9] and [7], linear stochastic H_∞ filtering was studied based on linear matrix inequality (LMI) technique. [8] presented a nonlinear stochastic H_∞ filtering design by means of Hamilton-Jacobi inequality (HJI). When we construct the H_∞ filter for different systems, all the works above are base on the premise that the systems we studied are asymptotic stable. Otherwise, there are no feasible solutions to the filter design problems, especially for some multi-sensor fusion systems like target tracking system. So, we should change the classical H_∞ filter design methods to satisfy for multi-sensor H_∞ fusion filtering.

This paper considers fusion filtering problems of the multi-sensor fusion system, and the process and measurement noise in the system have unknown statistic characteristic but bounded power. Applying H∞ theory and LMI methods, a new approach to design the filter is presented. And, in the fourth section of this paper, an example is given to illustrate the effectiveness of our theory.

For convenience, we adopt the following traditional notations: A' : transpose of the matrix A . $A \geq 0 (A > 0)$: A is a positive semidefinite (positive definite) matrix. S_n : the set of all real symmetric matrices. R^n : n-dimensional Euclidean Space. $I_{n\times n}$: $n\times n$ identity matrix.

II. PROBLEM FORMULATION

We consider the following stochastic discrete time fusion system

$$\begin{cases} x_{k+1} = Ax_k + (B + Gr_k)w_k \\ y_{1k} = C_1 x_k + D_1 v_{1k} \\ y_{2k} = C_2 x_k + D_2 v_{2k} \\ \quad .... \\ y_{pk} = C_p x_k + D_p v_{pk} \\ z_k = Lx_k \qquad k = 0,1,2,... \end{cases} \tag{1}$$

where $x_k \in R^n$ is the system state, $y_{1k}, y_{2k}, ... y_{pk} \in R^l$ are the measurement outputs , v_1, v_2, … , v_p are measurement noise which belongs to $l_2[0,\infty)$, $w_k \in R^m$ is stochastic process noise which belongs to $l_2[0,\infty)$, and $z_k \in R^m$ is the state combination to be estimated and $\{r_k\}$ is the standard random scalar sequences with zero mean that satisfy: $E\{r_k, r_j\} = \delta_{kj}, \ \forall k, j \geq 0$. A, B,

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$C_1, C_2, \ldots, C_p$, $D_1, D_2, \ldots, D_p$ and L are constant matrices of the appropriate dimensions.

For centralized fusion system, the measurement model can be denoted as the following augmented measurement equations:

$$y_k = Cx_k + Dv_k \tag{2}$$

where

$$\begin{aligned} y_k &= \left[y'_{1k}, y'_{2k}, \ldots, y'_{pk}\right]' \\ v_k &= \left[v'_{1k}, v'_{2k}, \ldots, v'_{pk}\right]' \\ C &= \left[C_1, C_2, \ldots, C_p\right]' \\ D &= Diag[D_1, D_2, \ldots, D_p] \end{aligned} \tag{2'}$$

So, system (1) can be simply expressed as:

$$\begin{cases} x_{k+1} = Ax_k + (B_1 + G_1 r_k)\varsigma_k \\ y_k = Cx_k + D_1\varsigma_k \\ z_k = Lx_k, \quad k = 0,1,2,\ldots \end{cases} \tag{3}$$

where

$$B_1 = [B \quad 0], D_1 = [0 \quad D],$$

$$G_1 = [G \quad 0] \; \varsigma_k = [w'_k \quad v'_k]'.$$

According to classical H∞ filtering theory (set G=0), the centralized fusion filter can be constructed as the following form:

$$\begin{cases} \hat{x}_{k+1} = A_f \hat{x}_k + B_f y_k \\ \hat{z}_k = L_f \hat{x}_k, \quad k = 0,1,2,\ldots \end{cases} \tag{4}$$

where the constant matrices $\{A_f, B_f, L_f\}$ are filter parameters to be determined to meet certain performance criteria, $\hat{x}_k$ and $\hat{z}_k$ denote the estimates of x_k and z_k, respectively. It follows from (1) and (2) that the extended state vector $\xi_k = \begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix}$ satisfies:

$$\begin{cases} \xi_{k+1} = \overline{A}\xi_k + \overline{B}_1\zeta_k \\ \tilde{z}_k = z_k - \hat{z}_k = \left[L - L_f \quad L_f\right]\zeta_k \end{cases} \tag{5}$$

where

$$\overline{A} = \begin{bmatrix} A & 0 \\ B_f C & A_f \end{bmatrix}, \overline{B}_1 = \begin{bmatrix} B_1 \\ B_f D_1 \end{bmatrix}.$$

For a given scalar $\gamma > 0$, we define the following performance index:

$$J = E\sum_{k=0}^{\infty}(\tilde{z}'_k\tilde{z}_k - \gamma^2\varsigma'_k\varsigma_k) \tag{6}$$

According to theorem 10.1.1 and 10.1.2 in [10] we can easily obtain the centralized H_∞ fusion filter where for all nonzero ς_k, the above performance index $J < 0$. However, a sufficient condition should be satisfied for above theorems, that is, the system we considered must be asymptotically stable. It means the classical H_∞ filtering deign methods are not fit for those unstable system, such as radar tracking fusion system.

In order to settle this problem, we re-investigate the design of a linear estimator for z_k of the following form

$$\begin{cases} \hat{x}_{k+1} = A\hat{x}_k + K(y_k - C\hat{x}_k) \\ \hat{z}_k = L\hat{x}_k \end{cases} \tag{7}$$

where $\hat{x}(k) \in R^n, \hat{z}(k) \in R^r$ are the estimated state and output, respectively, and K is well known as the Kalman gain, $\hat{x}_k$ and $\hat{z}_k$ denote the estimates of x_k and z_k, respectively. Let $\tilde{x}_k = x_k - \hat{x}_k$, denote the state error vector. It follows from (3) and (7) that satisfy

$$\begin{cases} \tilde{x}_{k+1} = \overline{A}\tilde{x}_k + (\overline{B} + G_1 r_k)\varsigma_k \\ \tilde{z}_k = z_k - \hat{z}_k = L\tilde{x}_k \end{cases} \tag{8}$$

where

$$\overline{A} = A - KC, \; \overline{B} = B_1 - KD_1 \tag{8'}$$

For centralized H_∞ fusion filter, we look for an estimator of the form (7) such that for all nonzero ς_k, the above performance index $J < 0$, and the system (8) is asymptotically stable when $\zeta_k = 0$.

We first put forward to the following lemma which is very useful for the proof of our main theorems.

Lemma 2.1 (Schur's complement):

For real matrices N, $M = M'$, $R = R' > 0$, the following two conditions are equivalent.

1) $M - NR^{-1}N' > 0$.

2) $\begin{bmatrix} M & N \\ N' & R \end{bmatrix} > 0.$

III. ASYMPTOTIC STABILITY

We firstly consider the internal stability of the system (8) in the absence of disturbance ζ_k, i.e. the asymptotic stability of

$$\xi_{k+1} = \overline{A}\,\xi_k \quad (9)$$

Assuming there exists $P>0$, we shall seek a Lyapunov functional of the following form:

$$V(\eta_k) = \xi_k' P \xi_k \quad (10)$$

Note that

$$\mathrm{E}V(\eta_{k+1}) - V(\eta_k) = \mathrm{E}[\xi_{k+1}' P \xi_{k+1}] - \xi_k' P \xi_k$$

$$= \mathrm{E}\left[\left(\overline{A}\,\xi_k\right)' P\left(\overline{A}\,\xi_k\right)\right] - \xi_k' P \xi_{kk}$$

$$= \xi_k' \overline{A}' P \overline{A}\,\xi_k - \xi_k' P \xi_k = \xi_k'(\overline{A}' P \overline{A} - P)\xi_k \quad (11)$$

We can see that if

$$\overline{A}' P \overline{A} - P < 0 \quad (12)$$

Then

$$\mathrm{E}V(\eta_{k+1}) - V(\eta_k) < 0$$

which implies the system (8) is asymptotically stable.

By Lemma 2.1, (12) is equivalent to

$$\begin{bmatrix} -P & \overline{A}'P \\ P\overline{A} & -P \end{bmatrix} < 0 \quad (13)$$

Now, we substitute $\overline{A}$ into (13), and it yields (14)

$$\begin{bmatrix} -P & A'P - C'K'P \\ AP - KCP & -P \end{bmatrix} < 0 \quad (14)$$

So, if there exist matrices $P>0, K$ to matrix inequality (14), the system (8) will be asymptotically stable.

IV. CENTRALIZED H_∞ FUSION FILTER DESIGN

We firstly bring the lemma that derived in [4] for the system (15)

$$\begin{cases} x_{k+1} = Ax_k + (B + Gr_k)w_k \\ z_k = Lx_k \end{cases} \quad (15)$$

where the exogenous disturbance $\{w_k\}$ is assumed to be of finite energy and may depend only on current and past values of the state-vector, $\{r_k\}$ is defined as in (1). Considering the performance index

$$\hat{J} = E\sum_{k=0}^{\infty}(z_k' z_k - \gamma^2 w_k' w_k) \quad (16)$$

and using the arguments of [4], the following holds:

Lemma 4.1 (The discrete-time bounded real lemma):

Consider the system of (15), Given $\gamma > 0$, a necessary and sufficient condition for $\hat{J}$ to be negative for all nonzero $\{w_k\}$ where $\{w_k\} \in l_2[0,\infty)$ is that there exists a solution $P = P' > 0$ to

$$-P + A'PA + A'PB\Theta^{-1}B'PA + L'L < 0 \quad (17)$$

Which satisfies $\Theta > 0$, where $\Theta = \gamma^2 I - B'PB - G'PG$.

Considering the system of (8) and applying Lemma 4.1, we arrive at the following result:

Theorem 4.1: If there exist feasible solutions $P = P' > 0, Q$ to LMIs (18) and (19), then the system (8) will be asymptotically stable and have H_∞ performance level of γ.

$$\begin{bmatrix} -P & 0 & A'P - C'Q & 0 & L' \\ 0 & -\gamma^2 I & B_1'P - D_1'Q & G_1'P & 0 \\ PA - Q'C & PB_1 - Q'D_1 & -P & 0 & 0 \\ 0 & PG_1 & 0 & -P & 0 \\ L & 0 & 0 & 0 & -I \end{bmatrix} < 0 \quad (18)$$

$$\begin{bmatrix} -P & A'P - C'Q \\ PA - Q'C & -P \end{bmatrix} < 0 \quad (19)$$

Proof. Applying Lemma 3.1 on system (8), and Lemma 2.1 on (17), then (17) is equivalent to (20)

$$\begin{bmatrix} -P + \overline{A}'P\overline{A} + L'L & \overline{A}'P\overline{B} \\ \overline{B}'P\overline{A} & -\gamma^2 I + \overline{B}'P\overline{B} + G_1'PG_1 \end{bmatrix} < 0 \quad (20)$$

And, (20) can be rewrote as

$$\begin{bmatrix} -P + L'L & 0 \\ 0 & -\gamma^2 I + G_1'PG_1 \end{bmatrix} - \begin{bmatrix} \overline{A}' \\ \overline{B}' \end{bmatrix}(-P)\begin{bmatrix} \overline{A} & \overline{B} \end{bmatrix} < 0 \quad (21)$$

Applying Lemma2.1, (21) can be readily put in the following matrix inequality form

$$\begin{bmatrix} -P + L'L & 0 & \overline{A}'P \\ 0 & -\gamma^2 I + G_1'PG_1 & \overline{B}'P \\ P\overline{A} & P\overline{B} & -P \end{bmatrix} < 0 \quad (22)$$

Then, take (8') into (20), define $Q = K'P$, and after series of computation, we can see (22) is equivalent to (18), (16) is equivalent to (19), respectively. So the proof is ended.

In Theorem 4.1, if we let $\gamma^2 = \bar{\gamma}$, and the matrix inequalities (18) and (19) can be wrote down as

$$\psi_1(P,Q,\bar{\gamma})<0 \quad (23a)$$

$$\psi_2(P,Q)<0 \quad (23b)$$

Then the inequalities (23) are linear respect to $\{P,Q,L,\bar{\gamma}\}$, and we can obtain the optional H_∞ fusion filter by solving the following optimization problem in using of Matlab software.

$$\min_{P>0,Q,\bar{\gamma}} \bar{\gamma} \quad \text{subject to LMIs(23)} \quad (24)$$

And the system (8) is asymptotically stable and have a least H_∞ performance level of $\sqrt{\bar{\gamma}}$, furthermore, the desired parameter K of the filter is given by

$$K=(QP^{-1})' \quad (25)$$

V. EXAMPLE

Consider certain tracking system with two sensors, the system model is as follows:

$$\begin{aligned} x_{k+1} &= Ax_k + [B+Gr_k]w_k \\ y_{1k} &= C_1x_k + v_{1k} \\ y_{2k} &= C_2x_k + v_{2k} \\ z_k &= Lx_k \qquad k=0,1,2,\dots \end{aligned} \quad (26)$$

where T is the sampling period. The state is $x_k=[s(k) \quad \dot{s}(k)]'$,where $s(k),\dot{s}(k)$ are the position and velocity of the target at time $k\times T$. $y_{ik}, i=1,2$ are the measurement signals, $v_{ik}, i=1,2$ are the measurement noises of the two sensors, with mean zero and variance are σ_{v1}^2 and σ_{v2}^2. w(t) is the system noises with mean zero and variance σ_w^2. Our aim is to find the optimal H_∞ fusion filter of form (7).

In fusion system (26), we set

$$A=\begin{bmatrix}1 & T\\ 0 & 1\end{bmatrix}, B=\begin{bmatrix}0.5T^2\\ T\end{bmatrix}, G=\begin{bmatrix}0.1T\\ 0.3T\end{bmatrix},$$

$$C=\begin{bmatrix}0.75 & 0.12\\ 0.18 & 0.8\end{bmatrix}, D=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, L=[1 \quad 1].$$

$$T=0.1, \sigma_w=30, \sigma_{v1}=90, \sigma_{v2}=120.$$

the initial velocity is 300m/s, initial position is 8000m.

According to Theorem 4.1, (24) and (25), we obtain the following results:

$$P=\begin{bmatrix}5.8628 & -9.4128\\ -9.4128 & 23.4678\end{bmatrix}, Q=\begin{bmatrix}1.8268 & 0.0748\\ 0.4684 & 1.7703\end{bmatrix},$$

$$K=\begin{bmatrix}0.8899 & 0.5649\\ 0.3602 & 0.3021\end{bmatrix}, \gamma=1.5553$$

The simulation outputs are showed in figure 1 and figure 2. The mean error square of H_∞ fusion filter outputs for $\hat{x}_1$ and $\hat{x}_2$ are 1.5007×10^5 m and 7.4381×10^4 m^2/s^2, respectively.

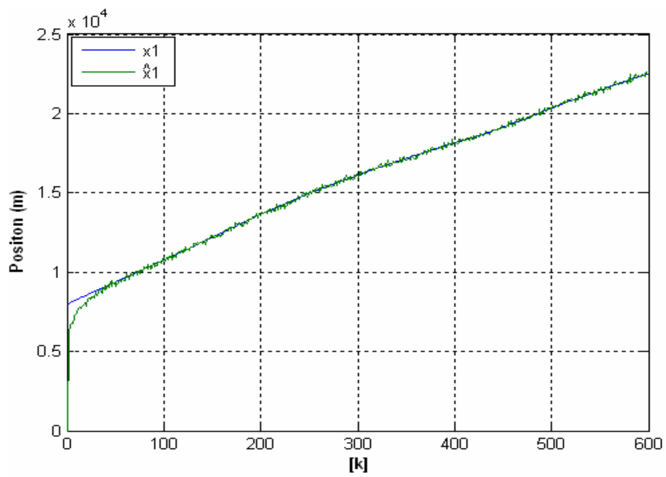

Figure 1. The trajectories of x_1 and $\hat{x}_1$.

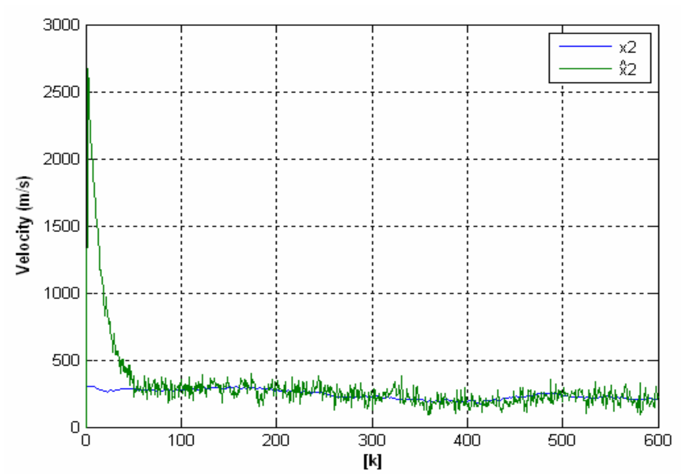

Figure.2. The trajectories of x_2 and $\hat{x}_2$.

Changing the parameters of fusion system as

$$A=\begin{bmatrix}1 & T\\ 0 & 1\end{bmatrix}, B=\begin{bmatrix}T^2/3\\ 2T\end{bmatrix}, G=\begin{bmatrix}0.4T\\ 0.1T\end{bmatrix},$$

$$C=\begin{bmatrix}7.6 & 1.2\\ 0.9 & 8.3\end{bmatrix}, T=0.5$$

the initial velocity as 500m/s, initial position as 8000m, the variance of measurement noise and system noise as $\sigma_w = 10, \sigma_{v1} = 190, \sigma_{v2} = 220$, we obtain

$$P=\begin{bmatrix}1.1077 & 0.9841\\ 0.9841 & 1.0030\end{bmatrix}, Q=\begin{bmatrix}0.1259 & 0.1097\\ 0.1671 & 0.1643\end{bmatrix},$$

$$K=\begin{bmatrix}0.1283 & 0.0415\\ -0.0165 & 0.1231\end{bmatrix}, \gamma = 1.1309$$

And the simulation results showed in figure 3 and figure 4. The mean error square $\hat{x}_1$ and $\hat{x}_2$ are 1.0695×10^5 m and 1.2566×10^4 m^2/s^2, respectively.

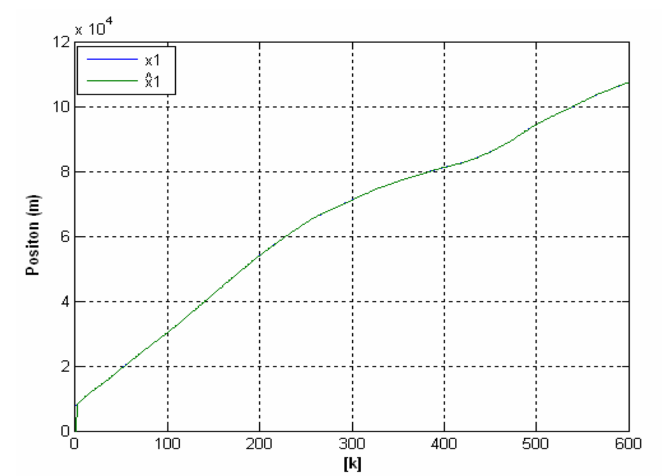

Figure 3. The trajectories of x_1 and $\hat{x}_1$ when the system parameters are changed.

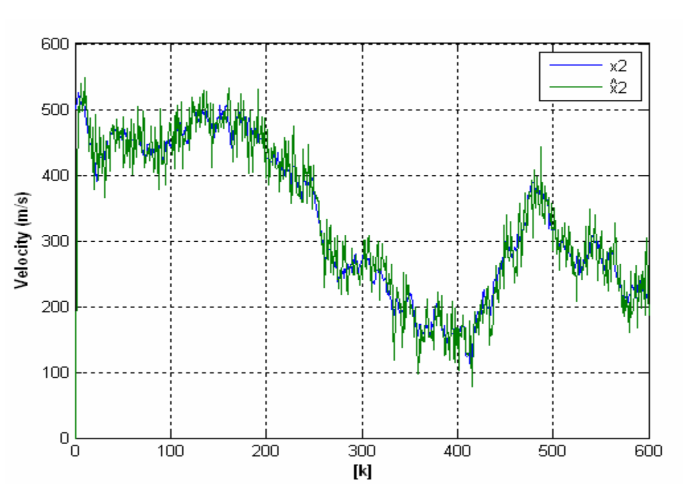

Figure 4. The trajectories of x_2 and $\hat{x}_2$ when the system parameters are changed.

From the simulation results, we can see that the tracking performance of the H∞ fusion filter for system (26) is in an acceptable precision, and the filter can still work well when the system parameters and the characteristics of measurement noise and system noise are changed.

VI. CONCLUSIONS

In above sections, we have discussed fusion filter design problem of the multi-sensor fusion system, which is not asymptotic stable. We changed the classical H_∞ filter's structure and presented a new approach to realize it by applying H_∞ theory and LMI methods. The H_∞ fusion filter is quite fit for coping with disturbances of partially unknown statistics but with an upper bound of the signal power in multi-sensor fusion systems. Obviously, it is very valuable in the study of this field, while many other problems merit further study.

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