Sliding Mode Control for Multi-robot Formation

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***Abstract*—This paper investigated the formation control of multi-mobile robots under the environment without obstacles, the feedback linearization is used for the robot, a nonlinear sliding mode controller is proposed in accordance with multi-robot system to coordinate a group of nonholonomic mobile robots so that a desired formation can be achieved. A prescribed trajectory is followed by using sliding mode control approaches, We prove theoretically that under certain reasonable assumptions the formation is asymptotically stable, that is, the proposed sliding mode controller can asymptotically stabilize the formation. The simulation results verify the effectiveness of the control laws.**

***Index Terms*—Formation Control, Feedback Linearization, Nonholonomic Mobile Robot, Sliding Mode Controller**

I. Introduction

With the development of computer technique, micro-electric technique and wireless communication technique, it becomes probable that multi-agent intelligent systems work in coordination with each other, and it is being used in many fields. Multi-agent intelligent systems work in coordination can achieve the task which a single system couldn't accomplish. In artificial intelligence and robot research, people focus on how to organize and coordinate multi-agent intelligent systems, so that they can achieve complex missions under unstructured environment effectively[1][2].

Nowadays, formation control is widely used in fields such as geologic detecting, search-rescue, mining, space exploration, and aeroplane formation without man. In the research about multi-robot formation problem, the methods for coordination control mainly contain leader-follower schemes[3][4], behavior-based methods[5][6], virtual structure technique[7], artificial potential function methods[8][9], kinetic energy shaping method[10], decentralization control-based method[11], and so on.

This paper studies the problem about coordination formation control of multi-mobile robots, that is, path tracking problem about a team of mobile robots. A sliding mode controller is proposed to realize the formation of multi-robot. The simulation result shows that the sliding mode control method is a feasible scheme, which overcomes some limitations of the feedback linearization method[12].

II. Dynamics and Kinematics Model of Mobile Robot

A nonholonomic constraint mobile robot in n dimension space can be described by Euler-Lagrange formulation[13]:

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = B(q)\tau + A^T(q)\lambda \tag{1}$$

where $q \in R^n$ is the coordinate vector, $\tau \in R^r$ is the torque control input vector, $\lambda \in R^m$ is the constraint force vector, $H(q) \in R^{n\times n}$ is the symmetric positive definite matrix. $G(q) \in R^n$ corresponds to gravity and $C(q,\ \dot{q}) \in R^{n\times n}$ to the centrifugal and Coriolis forces, $B(q) \in R^{n\times r}$ is the input transform matrix. $A(q) \in R^{m\times n}$ is the matrix corresponding to nonholonomic constraint. Let $r = n - m$ in the following text.

Nonholonomic dynamic constraint is described by the following equation:

$$A(q)\dot{q} = 0 \tag{2}$$

Set $q = (r_x,\ r_y,\ \theta)^T$ in (2), then the robot's nonholonomic constraint can be denoted as:

$$\dot{r}_x \sin\theta - \dot{r}_y \cos\theta = 0 \tag{3}$$

As shown in Fig. 1, the dynamics and kinematics model

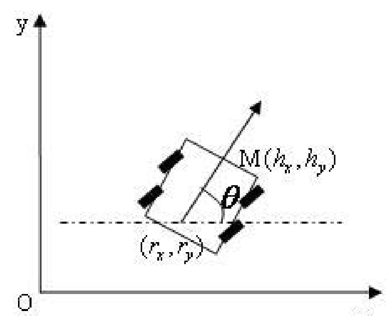

Fig. 1. Model of mobile robot

of mobile robot is given by:

$$\begin{bmatrix} \dot{r}_{xi} \\ \dot{r}_{yi} \\ \dot{\theta}_i \\ \dot{v}_i \\ \dot{\omega}_i \end{bmatrix} = \begin{bmatrix} v_i C_i \\ v_i S_i \\ \omega_i \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1/m_i & 0 \\ 0 & 1/J_i \end{bmatrix} \begin{bmatrix} F_i \\ M_i \end{bmatrix} \tag{4}$$

Where $r_i = (r_{xi}, r_{yi})^T$ are the coordinates of the center of the robot in inertia coordinates system, θ_i is the heading angle of the robot, v_i and ω_i are the linear and angular velocities of the robot, respectively. m_i is the mass of the robot, and J_i is the moment of inertia, F_i is control force, M_i is control torque, here, F_i and M_i are denoted as control inputs. Let $C_i = \cos\theta_i$, $S_i = \sin\theta_i$, $u_i = [F_i,\ M_i]^T$, the subscript i represents the i_{th} robot.

The heading position coordinates of the robot can be described as:

$$h_i = r_i + L_i \begin{bmatrix} C_i \\ S_i \end{bmatrix} \tag{5}$$

Where L_i is the distance from the heading position to the center of the robot.

III. Controller Design

Take

$$u_i = \begin{bmatrix} C_i/m_i & -L_iS_i/J_i \\ S_i/m_i & L_iC_i/J_i \end{bmatrix}^{-1} \cdot \left(\nu_i - \begin{bmatrix} -v_i\omega_iS_i - L_i\omega_i^2C_i \\ v_i\omega_iC_i - L_i\omega_i^2S_i \end{bmatrix}\right) \tag{6}$$

ν_i is to design. After the feedback linearization of the robot, then

$$\ddot{h}_i = \nu_i \tag{7}$$

The problem of formation control proposed in the paper [14] is comprised of two task:

1) *Geometric task*: Force the output h_i to converge to the desired path $\varepsilon_i(\beta)$

$$\lim_{t\to\infty} |h_i(t) - \varepsilon_i(\beta(t))| = 0 \tag{8}$$

for any continuous function $\beta(t)$.

2) *Dynamic task*: Satisfy one or more of the following assignments:

a) *Time assignment*: Force the path variable $\beta(t)$ to converge to a desired time signal $v_s(t)$,

$$\lim_{t\to\infty} |\beta(t) - v_s(t)| = 0 \tag{9}$$

b) *Speed assignment*: Force the path speed $\dot{\beta}(t)$ to converge to a desired speed $v_s(\beta, t)$,

$$\lim_{t\to\infty} |\dot{\beta}(t) - v_s(\beta, t)| = 0 \tag{10}$$

c) *Acceleration assignment*: Force the path acceleration $\ddot{\beta}(t)$ to converge to a desired acceleration $v_s(\dot{\beta}, \beta, t)$,

$$\lim_{t\to\infty} |\ddot{\beta}(t) - v_s(\dot{\beta}, \beta, t)| = 0 \tag{11}$$

Throughout this paper, the dynamic task is specified as a *speed assignment*.

Disassembling the system (7) as:

$$\dot{x}_{1i} = x_{2i} \tag{12}$$

$$\dot{x}_{2i} = \nu_i \tag{13}$$

$$h_i = x_{1i} \tag{14}$$

Set the error variations as

$$z_{1i} = x_{1i} - \varepsilon_i(\beta) \tag{15}$$

$$z_{2i} = x_{2i} - \alpha_{1i} \tag{16}$$

$$\omega_s = v_s(\beta, t) - \dot{\beta} \tag{17}$$

Where α_{1i} is virtual control variation, it will be given in the following text. For the i_{th} robot , its sliding plane is designed as

$$s = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} c_1z_1 \\ c_2z_2 \\ k_0\omega_s \end{bmatrix} \tag{18}$$

Where $s_1 = (s_{11},\ s_{12})^T$, $s_2 = (s_{21},\ s_{22})^T$, $c_1 = diag(c_{11},\ c_{12}) > 0$, $c_2 = diag(c_{21},\ c_{22}) > 0$, $k_0 > 0$.

First, it will be proved that the formation control is stable on the sliding plane.

Theorem 1: If it reaches the sliding plane within finite time, then z_1, z_2, ω_s converge to zero asymptotically.

Proof : On the sliding plane $s = 0$, since $s_1 = c_1z_1$, $s_2 = c_2z_2$, $s_3 = k_0\omega_s$, and $|c_1| > 0$, $|c_2| > 0$, $k_0 > 0$, it is easily to see that z_1, z_2, ω_s converge to zero asymptotically.

Next, it will be proved that it definitely reaches the sliding plane within finite time.

Theorem 2: For the i_{th} robot, given the control laws $\nu = \sigma_1 + \rho_2 v_s - (c_2^Tc_2)^{-1}c_1^Ts_1 - sgn(s_2)$, $\alpha_1 = z_1 + \dot{\varepsilon}(\beta)v_s$, and set $\dot{\omega}_s = -\frac{\tau}{k_0^2} - \lambda sgn(\omega_s)$, where $\sigma_1 = x_2 + \dot{\varepsilon}(\beta)\dot{v}_s$, $\rho_2 = -\dot{\varepsilon}(\beta) + \ddot{\varepsilon}(\beta)v_s$, $\tau = z_1^Tc_1^Tc_1\dot{\varepsilon}(\beta) + z_2^Tc_2^Tc_2\rho_2$, $\lambda > 0$, $sgn(\cdot)$ is sign function. Then the control laws satisfy the reachability condition.

Proof: Take the differential of the equations (15) and (16)

$$\dot{z}_1 = z_2 + \alpha_1 - \dot{\varepsilon}(\beta)\dot{\beta} \tag{19}$$

$$\begin{aligned} \dot{z}_2 &= \nu - (x_2 - \dot{\varepsilon}(\beta)\dot{\beta} \\ &\quad + \ddot{\varepsilon}(\beta)\dot{\beta}v_s + \dot{\varepsilon}(\beta)\dot{v}_s) \\ &= \nu - \sigma_1 - \rho_2\dot{\beta} \end{aligned} \tag{20}$$

Then

$$
\begin{aligned}
s^T\dot{s} &= (c_1z_1)^Tc_1\dot{z}_1 + (c_2z_2)^Tc_2\dot{z}_2 + k_0^2\omega_s\dot{\omega}_s \\
&= z_1^Tc_1^Tc_1[z_2 + \dot{\varepsilon}(\beta)\omega_s + (\alpha_1 - \dot{\varepsilon}(\beta)v_s)] \\
&\quad + z_2^Tc_2^Tc_2[\rho_2\omega_s + (\nu - \sigma_1 - \rho_2 v_s)] + k_0^2\omega_s\dot{\omega}_s \\
&= -z_1^Tc_1^Tc_1z_1 - z_2^Tc_2^Tc_2sgn(s_2) + (\tau + k_0^2\dot{\omega}_s)\omega_s \\
&= -z_1^Tc_1^Tc_1z_1 - c_{21}|s_{21}| - c_{22}|s_{22}| - \lambda k_0|s_3| \\
&= -s_1^Ts_1 - c_{21}|s_{21}| - c_{22}|s_{22}| - \lambda k_0|s_3| < 0
\end{aligned}
$$

So the reachability is satisfied.

It guarantees that when $t \to \infty$, $z_{1i} \to 0$, $z_{2i} \to 0$, $\omega_s \to 0$, thereby $h_i(t) \to \varepsilon_i(\beta(t))$, $\dot{\beta}(t) \to v_s(\beta(t), t)$, so the formation control is satisfied.

IV. Simulation

Three robots are chose for the experiment, the first robot is defined as the leader, its desired path is $\varepsilon_1(\beta) = (\beta,\ \sin(0.5\beta))^T$, where $\beta = t$, the desired path of the second robot is $\varepsilon_2(\beta) = (\beta,\ \sin(0.5\beta) + 0.8)^T$, and the third desired path is $\varepsilon_3(\beta) = (\beta,\ \sin(0.5\beta) - 0.8)^T$. In the experiment, the initial position of the three robots are $r_1 = (-0.1,\ 0.4)^T$, $r_2 = (-0.8,\ 1)^T$, $r_3 = (-0.6,\ -0.6)^T$, the initial heading angle are $\theta_1 = \pi/4$, $\theta_2 = \pi/2$, $\theta_3 = \pi/3$.

To avoid vibrating severely, function $g(\cdot)$ is used to replace the sign function $sgn(\cdot)$, function $g(\cdot)$ is defined as

$$g(\cdot) = \frac{2}{1 + e^{-5x}} - 1 \tag{21}$$

The results of simulation are shown in Fig. 2 to Fig. 7

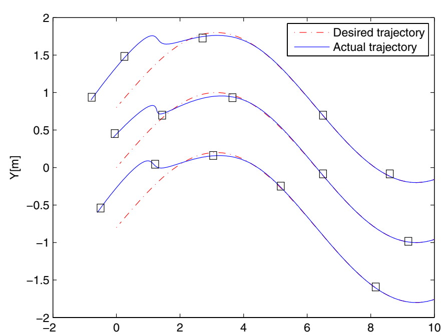

Fig. 2 Formation path of the robots

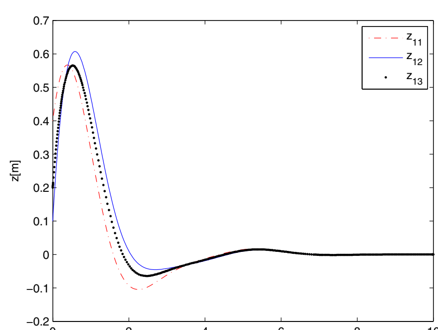

Fig. 3 Position error of the robots

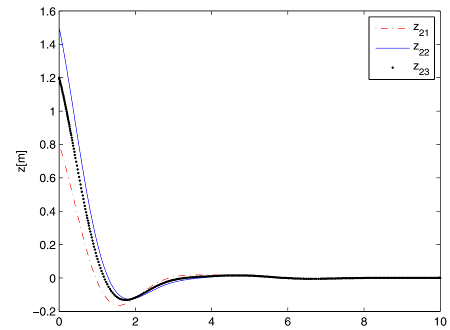

Fig. 4 Virtual error of the robots

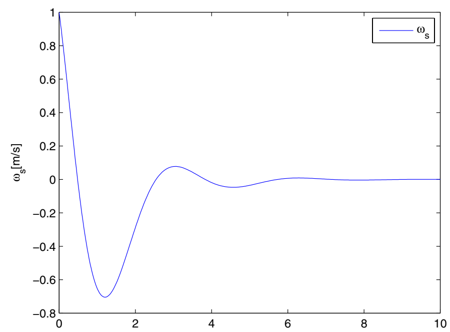

Fig. 5 Dynamic tracking error of the formation

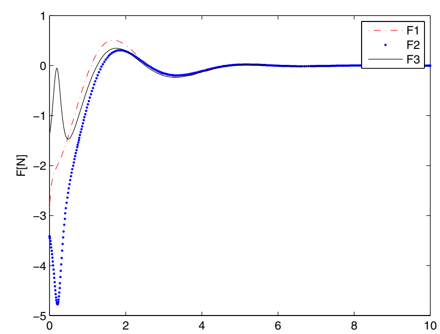

Fig. 6 Control force curve of the robots

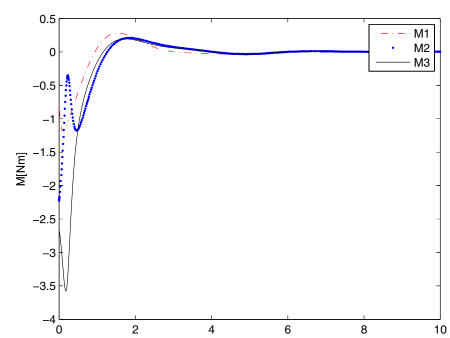

Fig. 7 Control torque curve of the robots

Fig. 2 shows the formation path of the robots, three robots are denoted with rectangles, from the top to the bottom are the paths of robots R_2, R_1, R_3, from the figures, it is easy to see three robots converging to the desired path soon. Fig. 3 shows the position error of them, after seven seconds, the errors almost converge to zero. Fig. 4 shows the indirect virtual errors of the three robots, they get stable rapidly. Fig. 5 shows the dynamic velocity error, it makes the three robots keep in regular formation. Fig. 6 and Fig. 7 show the control inputs curves of the three robots, making the formation going stably.

V. Conclusion

In this paper, a sliding control method is proposed to control the formation of mobile robots, which is an extension of tracking control. The results of simulation shows the robots can formate stably, the sliding model is simple but effective. Though the experiment in this paper is just for three robots, it is also available when it extends to n robots.

Acknowledgment

This work is supported partly by National Science Foundation(60474009), Shuguang Program of Shanghai(05SG48) and Development Foundation of Shanghai Education Committee(F07zz90).

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