Centralized H∞ Fusion Filter Design in Multi-sensor Nonlinear Fusion System

Qinghua Li, Hongjun Wang, Jun Du
School of Information Science and Engineering,
Shandong University,
Jinan 250100, P. R. China.
liqinghua1977@163.com

Qinghua Li, Xuezhen Liu, Fei Lin
School of Electronic Information and Control Engineering, Shandong Institute of Light Industry,
Jinan 250353, P. R. China.
xuezhen_liu@163.com

***Abstract*— Centralized nonlinear fusion filter design problem is to construct an asymptotically stable observer that leads to a stable estimation error process whose L_2 gain with respect to disturbance signal is less than a prescribed level for state estimation of the multi-sensor fusion system. This paper tries to resolve the problem in using of H_∞ filtering theory and LMI methods under certain conditions. Finally, a simulation example is given to illustrate the effectiveness of our methods.**

***Keywords*—nonlinear fusion system, H∞ filtering, LMI**

I. Introduction

In recent years, Kalman filter theory applied in linear fusion system became more and more sophisticated[1-4]. And at the same time, the state fusion estimation and parameter estimation of multi-sensor system also had great development. More study works were obtained refer to different sensor system, such as uncertain multi-sensor system, heterogeneity multi-sensor system, multi-resolution system, nonlinear system, etc[5-10]. For nonlinear fusion system, EKF was widely used as the approximate method to solve the non-Gaussian nonlinear problem which is put forward by Anderson and Moore since 1979. This method is base on system state evolution function and the first order Taylor expansion of measure function to obtain the MMSE estimation. It is actually approximate state evolution equation by local linearization of system predictive state. However, this local linearization may lead to filter's divergence. GSF combines a number of Gaussian models to approximate the posterior probability of system state. It equals to a group of EKFs which have the same drawbacks as EKF. Another method to solve the nonlinear non-Gaussian problems is base on stochastic sample filtering. Its main idea is using a series of weighted stochastic sample sets in state space to approximate the posterior pdf of system state, it is a statistical filtering base on simulation.

Recently, the H_∞ filtering becomes another important method in the estimation theory of stochastic system. If the measurement of definite system has known white noise or spectral density noise, the variance of estimation error can act as the filter's performance index, and the optimum filter can be designed by minimizing the performance index. When the statistical character of system disturbance is unknown, the disturbance can be regarded as random signal which has finite energy. Therefore, the filter performance index is the H_∞ norm of transfer function which is from disturbance input to evaluated error. We needn't know the definite statistical characteristic of disturbing signal, and only demand it should be finite energy signal in H_∞ filter, which is different from Kalman filter. The H_∞ filtering has very good robustness for uncertain system[11-14]. In [11], a bounded real lemma was presented for linear continuous-time stochastic uncertain systems, according to which full-order robust H_∞ filtering problems for stationary continuous-time linear stochastic uncertain systems were discussed by [13]. The above works are limited to the linear stationary stochastic systems, whereas [16] investigated the same problem for a class of special nonlinear stochastic systems. [14] presented a nonlinear stochastic H_∞ filtering design by means of Hamilton-Jacobi inequality (HJI).

In the present paper, we consider the centralized nonlinear fusion filter design problem in using of H_∞ filtering theory. Our goal is to construct an asymptotically stable (in some sense) observer that leads to a stable estimation error process whose L_2 gain with respect to disturbance signal is less than a prescribed level for state estimation of the centralized nonlinear fusion system. We also give a simulation example to show the effectiveness of our theory.

For convenience, we adopt the following traditional notations:

A' : transpose of the matrix A.
$A \geq 0 (A > 0)$: A is a positive semidefinite (positive definite) matrix.
I : the identity matrix.
$\|x\|$: Euclidean 2-norm of n-dimensional real vector x.
$L^2(R_+, R^l)$: the space of nonanticipative stochastic processes $y(t)$ with respect to filtration F_t satisfying $\|y(t)\|_{L_2}^2 := E\int_0^\infty \|y(t)\|^2 dt < \infty$.
$C^{2,1}(U,T)$: class of functions $V(x,t)$ twice continuously differentiable with respect to $x \in U$ and once continuously differentiable with respect to $t \in T$ except possibly at $x = 0$.

$V_t(x,t) := \dfrac{\partial V(x,t)}{\partial t}$; $V_x(x,t) := (\dfrac{\partial V(x,t)}{\partial x_i})_{n\times 1}$;

$V_{xx}(x,t) := (\dfrac{\partial V(x,t)}{\partial x_i \partial x_j})_{n\times n}$.

II. PROBLEM FORMULATION

Consider the following nonlinear stochastic fusion system

$$\begin{cases} dx(t) = (Ax(t) + Bv_0(t))dt + f(\mathrm{x(t)})dW \\ dy_i(t) = (C_i x(t) + g_i(x) + D_i v_i(t))dt \\ z(t) = Lx(t), i = 1,2,\ldots,p \end{cases} \tag{1}$$

Where $x(t) \in R^n$ is called the system state, $y(t) \in R^r$ is the measurement; $W(\cdot)$ is a standard one dimensional Wiener process defined on a complete filtered space $(\Omega, F, \{F_t\}_{t\in R_+}, P)$ with a filtration $\{F_t\}_{t\in R_+}$ satisfying usual conditions; $z(t) \in R^m$ is the state combination to be estimated, and $v_i \in L^2(R_+, R^{n_v}), i = 0,1,\ldots,p$ stands for the exogenous disturbance signal. $f(x(t)), g_i(x(t))$, $i = 1,2,\ldots,p$ are smooth functions with $f_i(0) = g(0) = 0$, A, B, C_i, D_i (i=1, 2,…,p) and L are constant matrices of the appropriate dimensions.

For centralized fusion system, the measurement model can be denoted as the following augmented measurement equations:

$$dy(t) = (Cx(t) + G(x) + Dv(t))dt \tag{2}$$

where

$$y(t) = [y_1'(t), y_2'(t), \ldots, y_p'(t)]'$$

$$v(t) = [v_1'(t), v_2'(t), \ldots, v_p'(t)]'$$

$$G(x(t)) = [g_1(x(t)), g_2(x(t)), \ldots, g_p(x(t))]' \tag{2'}$$

$$C = [C_1, C_2, \ldots, C_p]'$$

$$D = Diag[D_1, D_2, \ldots, D_p]$$

So, system (1) can be simply expressed as:

$$\begin{cases} dx(t) = (Ax(t) + B\tilde{v}(t))dt + f(x)dW \\ y(t) = (Cx(t) + G(x) + D\tilde{v}(t))dt \\ z(t) = Lx(t) \end{cases} \tag{3}$$

Where $\tilde{v}(t) = [v_0'(t), v'(t)]'$.

In what follows, we construct the following filtering equation for the estimation of $z(t)$:

$$\begin{cases} d\hat{x}(t) = A_f \hat{x}(t)dt + B_f dy(t) \\ \hat{z}(t) = L\hat{x}(t) \end{cases} \tag{4}$$

Set $\eta(t) = [x'(t) \quad \hat{x}'(t)]'$, and let

$$\tilde{z}(t) = z(t) - \hat{z}(t) = L(x(t) - \hat{x}(t)) \tag{5}$$

denote the estimation error, then we get the following augmented system(the time variable t is suppressed):

$$d\eta = \tilde{A}\eta dt + \tilde{F}(x)dW + \tilde{B}\tilde{v} + \tilde{G}(x)dt \tag{6}$$

Where

$$\tilde{A} = \begin{bmatrix} A & 0 \\ B_f C & A_f \end{bmatrix}, \tilde{B} = \begin{bmatrix} B \\ B_f D \end{bmatrix},$$
$$\tilde{G}(x) = \begin{bmatrix} 0 \\ B_f G(x) \end{bmatrix}, \tilde{F} = \begin{bmatrix} f(x) \\ 0 \end{bmatrix} \tag{6'}$$

For a given scalar $\gamma > 0$, we define the following performance index:

$$J_s = \|\tilde{z}(t)\|_{L_2}^2 - \gamma^2 \|\tilde{v}(t)\|_{L_2}^2 \tag{7}$$

In our paper, we are look for an H_∞ fusion estimator of the form (4) such that for all nonzero $\tilde{v}(t)$, the above performance index $J_s < 0$, and the system (5) is asymptotically internally stable when $\tilde{v}(t) = 0$.

We first put forward to the following lemma which is very useful for the proof of our main theorems.

Lemma 1.1[15]**:**

Consider the following nonlinear stochastic system:

$$\begin{cases} dx = (f(x) + g(x)v)dt + (h(x) + s(x)v)dW \\ x(0) = x_0 \in R^n, f(0) = h(0) = 0 \end{cases} \tag{8}$$

$f(x), h(x), g(x), s(x)$ are smooth functions. Assume there exists a positive Lyapunov function $V(x) \in C^2(R^n)$ satisfying

$$L_{v=0}V(x) = \frac{\partial V'}{\partial x}(f(x) + g(x)v) + \frac{1}{2}(h(x) + s(x)v)' \frac{\partial^2 V}{\partial x^2}(h(x) + s(x)v) < 0$$

for all nonzero $x \in R^n$, then, the equilibrium point $x \equiv 0$ of (8) is globally asymptotically stable in probability. $L_v V(x)$ is called the infinitesimal operator of (8).

Lemma 1.2 (Schur's complement):

For real matrices N, $M = M'$, $R = R' > 0$, the following two conditions are equivalent

1) $M - NR^{-1}N' > 0$.
2) $\begin{bmatrix} M & N \\ N' & R \end{bmatrix} > 0$.

Lemma 1.3 :

Let $x \in R^n, y \in R^n$ and $\varepsilon > 0$, then we have

$$2x'y \le \varepsilon x'x + \varepsilon^{-1} y'y$$

III. ASYMPTOTIC STABILITY

In order to guarantee the stability of system (6) when design the fusion filter, we firstly give the following theorem:

Theorem 3.1:

If there exist scalars $\mu > 0, \lambda > 0, \alpha > 0, \beta > 0$ and a positive definite matrix $P > 0$, $P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$, satisfy the following inequalities(9), then the system (6) will be asymptotically stable.

$$P \le \alpha I \tag{9a}$$

$$P\widetilde{A} + \widetilde{A}'P + P + 2(\lambda^2 + \mu^2)\alpha I - 2\lambda^2 \beta I < 0 \tag{9b}$$

$$B_f' P_2 B_f \le \beta I \tag{9c}$$

Proof: Take the Lyapunov candidate as $V(\eta) = \eta' P \eta$, where P>0, and Let $\hat{L}_{\widetilde{v}}$ be the infinitesimal operator of equation (6), then

$$\hat{L}_{\widetilde{v}=0} V(\eta) = \eta'(P\widetilde{A} + \widetilde{A}'P)\eta + \widetilde{F}'(x)P\widetilde{F}(x) + 2\widetilde{G}'(x)P\eta \tag{10}$$

Suppose there exists a scalar $\mu > 0, \lambda > 0, \alpha > 0,$ $\beta > 0$ and a positive definite matrix $P > 0$ and $P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$, such that

$$\left\|\widetilde{F}(x)\right\| \le \mu\|x\|,\ \|G(x)\| \le \lambda\|x\|, \forall x \in R^n \tag{11}$$

$$P \le \alpha I,\ B_f' P_2 B_f \le (\alpha - \beta) I \tag{12}$$

So according to Lemma 1.3, we obtain

$$\begin{aligned} 2\widetilde{G}'(x)P\eta &\le \widetilde{G}'(x)P\widetilde{G}(x) + \eta'P\eta \\ &\le G'(x)B_f'PB_f G(x) + \eta'P\eta \\ &\le 2(\alpha - \beta)\lambda^2 \|x\|^2 + \eta'P\eta \\ &\le 2\lambda^2(\alpha - \beta)\|\eta\|^2 + \eta'P\eta \end{aligned} \tag{13}$$

$$\widetilde{F}'(x)P\widetilde{F}(x) \le 2\mu^2 \alpha \|\eta\|^2 \tag{14}$$

Substituting (13)-(14) into (10), we have

$$\begin{aligned} \hat{L}_{\widetilde{v}=0} V(\eta) &= \eta'(P\widetilde{A} + \widetilde{A}'P)\eta + \widetilde{B}'P\widetilde{B} + 2\widetilde{F}'P\eta \\ &\le \eta'(P\widetilde{A} + \widetilde{A}'P + P + 2(\lambda^2 + \mu^2)\alpha I - 2\lambda^2\beta I)\eta \end{aligned} \tag{15}$$

If there exists k>0 such that

$$P\widetilde{A} + \widetilde{A}'P + P + 2(\lambda^2 + \mu^2)\alpha I - 2\lambda^2 \beta I < -kI \tag{16}$$

Therefore

$\hat{L}_{\widetilde{v}=0} V(\eta) \le -k\|\eta\|^2$ which yields (6) being asymptotically stable for $\widetilde{v} = 0$ according to lemma 1.1, and the proof of Theorem 3.1 is completed.

IV. H∞ FUSION FILTER DESIGN

In order to solve the filtering problem for centralized nonlinear fusion system, some linearization procedures have to be adopted in this section. By using stability Theorem 3.1 and LMI techniques, we give the following theorem:

Theorem 4.1:

Given scalars $\mu > 0, \lambda > 0, \alpha > 0, \beta > 0$, If there exist matrices $\{P_1 > 0, P_2 > 0, Z_1, Z_2\}$ solving the linear matrix inequalities (17)-(19), then the system (6) is asymptotically stable and have a robust H_∞ performance level of γ, furthermore, the desired parameters $\{A_f, B_f\}$ of estimator is give by (20).

$$\begin{bmatrix} P_1 A + AP_1 + P_1 + T & C'Z_2' & P_1 B & 0 \\ Z_2 C & Z_1 + Z_1' + P_2 + T & 0 & Z_2 D \\ B'P_1 & 0 & -\gamma^2 I & 0 \\ 0 & D'Z_2' & 0 & -\gamma^2 I \end{bmatrix} < 0 \tag{17}$$

Where $T = 2(\lambda^2 + \mu^2)\alpha I - 2\lambda^2 \beta I$

$$\begin{bmatrix} (\beta - \alpha) I & Z_2' \\ Z_2 & -P_2 \end{bmatrix} < 0 \tag{18}$$

$$\begin{bmatrix} P_1 - \alpha I & 0 \\ 0 & P_2 - \alpha I \end{bmatrix} < 0 \tag{19}$$

$$A_f = P_2^{-1} Z_1, \quad B_f = P_2^{-1} Z_2 \tag{20}$$

Proof: Note that for any T>0

$$\begin{aligned} J_s(T) &:= E\int_0^T (\|\widetilde{z}\|^2 - \gamma^2 \|\widetilde{v}\|^2) dt \\ &= E\int_0^T [(\eta'Q\eta - \gamma^2 \|\widetilde{v}\|^2) dt + d(\eta'P\eta) - d(\eta'P\eta)] \\ &= -E\eta'(T)P\eta(T) + E\int_0^T [(\eta'Q\eta - \gamma^2 \|\widetilde{v}\|^2) + \hat{L}_{\widetilde{v}} V(\eta)] dt \\ &\le E\int_0^T [(\eta'Q\eta - \gamma^2 \|\widetilde{v}\|^2) + \hat{L}_{\widetilde{v}=0} V(\eta) + \widetilde{v}'\widetilde{B}'P\eta + \eta'P\widetilde{B}\widetilde{v}] dt \\ &\le E\int_0^T [\eta'(P\widetilde{A} + \widetilde{A}'P + P + 2(\lambda^2 + \mu^2)\alpha I - 2\lambda^2 \beta I + Q)\eta - \gamma^2 \|\widetilde{v}\|^2] dt \\ &\quad + E\int_0^T (\widetilde{v}'\widetilde{B}'P\eta + \eta'P\widetilde{B}\widetilde{v}) dt \\ &\le E\int_0^T \begin{bmatrix} \eta \\ \widetilde{v} \end{bmatrix}' \begin{bmatrix} P\widetilde{A} + \widetilde{A}'P + P + 2(\lambda^2 + \mu^2)\alpha I - 2\lambda^2 \beta I + Q & P\widetilde{B} \\ \widetilde{B}'P & -\gamma^2 I \end{bmatrix} \begin{bmatrix} \eta \\ \widetilde{v} \end{bmatrix} dt \end{aligned}$$

$$(21)$$

Where $Q = \begin{bmatrix} L'L & -L'L \\ -L'L & L'L \end{bmatrix}$

Therefore, if

$$\begin{bmatrix} P\tilde{A}+\tilde{A}'P+P+2(\lambda^2+\mu^2)\alpha I-2\lambda^2\beta I+Q & P\tilde{B} \\ \tilde{B}'P & -\gamma^2 I \end{bmatrix}<0 \tag{22}$$

Then there exists $d>0, J_s(T)\le -d^2E\int_0^T \|\tilde{v}\|^2 dt<0$, for any nonzero $\tilde{v}\in L^2(R_+,R^{n_v})$, which yields $J_s \le -d^2 E\int_0^T \|\tilde{v}\|^2 dt<0$.

Substituting (6') into (20), setting $P=\begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}, P_2A_f=Z_1,$ $P_2B_f=Z_2, L=2(\lambda^2+\mu^2)\alpha I-2\lambda^2\beta I+L'L$, we obtain (17).

According to Theorem3.1, inequalities (9) should also be satisfied. Obviously, if (21) is satisfied, so is (9b). According to Lemma 1.2, inequality(9c) can be rewrote as

$$\begin{bmatrix} (\alpha-\beta)I & B_f'P_2 \\ P_2B_f & -P_2 \end{bmatrix} = \begin{bmatrix} (\alpha-\beta)I & Z_2' \\ Z_2 & -P_2 \end{bmatrix}<0 \tag{23}$$

Inequality (9a) can directly lead to (19), and the proof of Theorem 4.1 is completed.

V. SIMULATION EXAMPLE

Consider the following nonlinear multi-sensor fusion system with two sensors:

$$\begin{aligned}
dx(t) &= \begin{bmatrix} -20 & 2 \\ 11 & -21 \end{bmatrix} x dt + \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}\begin{bmatrix} u \\ v_{02} \end{bmatrix} dt + \begin{bmatrix} \sin(x_1) \\ \cos(x_2) \end{bmatrix} dW \\
dy_1(t) &= ([0.9 \quad 0.12]x + \cos(\sqrt{x_1^2+x_2^2}) + v_1(t))dt \\
dy_2(t) &= ([0.21 \quad 0.8]x + \frac{x_1+x_2}{\sqrt{2}+\sqrt{x_1^2+x_2^2}} + v_2(t))dt \\
z(t) &= [1 \quad 1]x(t)
\end{aligned} \tag{24}$$

where $x(t)=[x_1(t) \quad x_2(t)]'$, $y_i(t), i=1,2$ are the measurement signals, $v_{ik}, i=1,2$ are the measurement noises of the two sensors, with mean zero and variance are σ_{v1}^2 and σ_{v2}^2. v_{02} is the system noises with mean zero and variance $\sigma_{v_{02}}^2$. u is the set value. Our aim is to find the H_∞ fusion filter of form (6).

From the fusion system (24), we obtain

$$A=\begin{bmatrix} -20 & 2 \\ 11 & -21 \end{bmatrix}, B=\begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}, C_1=[0.9 \quad 0.12], f(x)=\begin{bmatrix} \sin(x_1) \\ \cos(x_2) \end{bmatrix},$$

$$C_2=[0.21 \quad 0.8], D_1=D_2=1, L=[1 \quad 1], G(x)=\begin{bmatrix} \cos(\sqrt{x_1^2+x_2^2}) \\ \dfrac{x_1+x_2}{\sqrt{2}+\sqrt{x_1^2+x_2^2}} \end{bmatrix}.$$

Set $u=1, \sigma_{v_{02}}=0.021, \sigma_{v1}=0.011, \sigma_{v2}=0.027$, the initial value of x_1 and x_2 are 0.

Obviously, $F(x)$ and $G(x)$ satisfy (11) with $\mu=1$ and $\lambda=\sqrt{2}$. According to Theorem 4.1 with $\gamma=0.71$, we solve the LMIs (17)-(19) together with the constraints $\alpha>0$ and $\beta=5$, we obtain the following results:

$$P_1=\begin{bmatrix} 2.0268 & 0.5869 \\ 0.5869 & 1.6708 \end{bmatrix}, P_2=\begin{bmatrix} 6.8837 & 0 \\ 0 & 6.8837 \end{bmatrix}, Z_1=\begin{bmatrix} -36.3954 & -0.5 \\ -0.5 & -36.3954 \end{bmatrix},$$

$$Z_2=\begin{bmatrix} 0.3267 & 0.3461 \\ 0.3267 & 0.3461 \end{bmatrix}, \alpha=12.8512.$$

Then

$$A_f=\begin{bmatrix} -5.2872 & -0.0726 \\ -0.0726 & -5.2872 \end{bmatrix}, B_f=\begin{bmatrix} 0.0475 & 0.0496 \\ 0.0475 & 0.0496 \end{bmatrix}.$$

The simulation outputs are showed in Fig 1 and Fig 2.

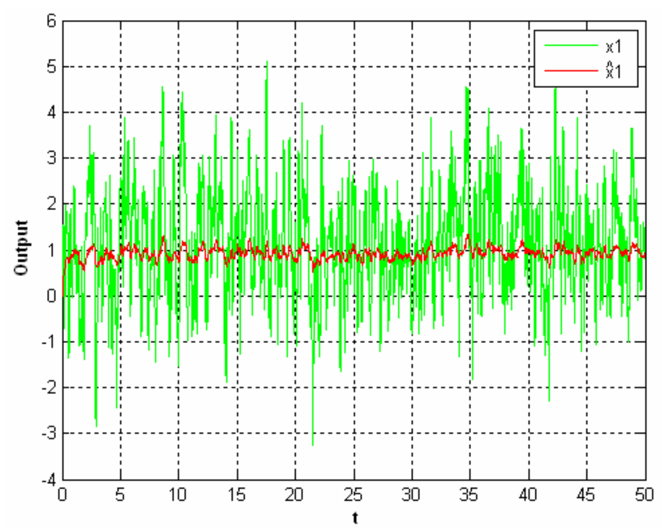

Fig. 1. The trajectories of x_1 and $\hat{x}_1$

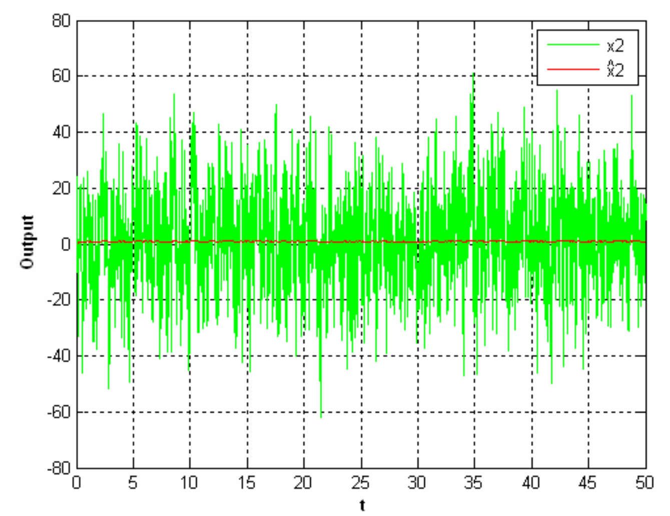

Fig. 2. The trajectories of x_2 and $\hat{x}_2$.

VI. CONCLUSIONS

In above sections, we have discussed fusion filtering problem of centralized multi-sensor nonlinear fusion system. By giving Theorem 3.1 and Theorem 4.1, we can present a new approach to design the asymptotically stable and effective H_∞ fusion filter. We also give a simulation example to illustrate the effectiveness of our theorems. It is very valuable in the study of stochastic H_∞ fusion filter for the nonlinear multi-sensor system, while many other problems merit further study.

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