LMI-based Criterion for the Robust Guaranteed Cost Control of 2-D Systems Described By the Roesser Model

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***Abstract*—This paper considers the problem of the guaranteed cost control for a class of two-dimensional (2-D) discrete systems described by the Roesser model with norm-bounded uncertainties. A linear matrix inequality (LMI)-based criterion for the existence of robust guaranteed cost controller is established .Such controller render the closed-loop system asymptotically stable for all admissible uncertainties and guarantee an adequate level of performance.**

***Keywords*—Guaranteed cost control; Linear matrix inequality; Lyapunov methods; Robust stability; 2-D discrete systems; uncertain systems.**

I. Introduction

Two-dimensional (2-D) discrete systems exist in many areas such as image data processing and transmission, seismographic data processing, thermal processes, gas absorption, water stream heating, etc[1-4].So far, many important results have been reported in the literature. For example, the stability analysis problem for 2-D systems has been investigated in [5, 6], H_∞ control and positive real control problems have been considered in [7, 8, 22].

On the other hand, the guaranteed cost control problem has recently drawn a great deal of research interests. The aim of the guaranteed cost control problem is to design a robust controller such that the associated closed-loop system satisfies the asymptotic stability and a specified level of the performance index for all the uncertainties. Based on this idea, many significant results have been obtained for the continuous-time case [9-11] and for the discrete-time case [12-14].Those results are only concerned with one-dimensional (1-D) systems. Recently, the guaranteed cost control problem for 2-D discrete uncertain systems in the FMSLSS model has been also considered [15-17]. Further, the stability properties of 2-D discrete systems described by the Roesser model have been investigated extensively [1], where the Roesser model [18-19] is well known to be important as well as the FMSLSS(Fornasini-Marchesini second local state-space) model. The Roesser model has wide applications and special structure. With the special structure, better results can be achieved.

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This paper, therefore, deals with the problem of robust guaranteed cost control for a class of 2-D discrete uncertain systems described by the Roesser model with norm-bounded uncertainties. A criterion for the existence of robust controller of the uncertain Roesser model is developed. The presented approach enables the formulation of the criterion based on the true LMI which is beneficial in terms of numerical complexity. The paper is organized as follows. The description of the system under consideration is given in Section 2. Section 3 presents robust guaranteed cost performance analysis of the uncertain 2-D systems described by the Roesser model, In Section 4, an LMI-based sufficient condition for the existence of static-state feedback controller is established. Finally, some concluding remarks are given in section 5.

II. System description

The paper deals with the problem of robust guaranteed cost control of a class of 2-D discrete uncertain system in the Roesser model. Specifically, the system under consideration is given by

$$\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = (A+\Delta A)\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + (B+\Delta B)u(i,j) \quad (1a)$$

Where $x^h(i,j)\in R^n$ and $x^v(i,j)\in R^m$ represent the horizontal and vertical states, respectively, $u(i,j)\in R^q$ is the control input. The matrices $A\in R^{(n+m)\times(n+m)}$ and $B\in R^{(n+m)\times q}$ are known constant matrices representing the nominal plant. The matrices ΔA and ΔB represent parameter uncertainties, which are assumed to be of the form

$$\begin{bmatrix}\Delta A & \Delta B\end{bmatrix} = LF(i,j)\begin{bmatrix}M_1 & M_2\end{bmatrix} \quad (1b)$$

In the above L, M_1, M_2 can be regarded as known structural matrices of uncertainty and $F(i,j)$ is an unknown matrix representing parameter uncertainty which satisfies

$$\| F(i,j) \| \le 1 \quad (1c)$$

Note that the uncertainty of (1b) satisfying (1c) has been widely adopted in robust control and filtering for uncertain systems.

It is assumed that the system (1a) has a finite set of initial condition. i.e., there exist two positive integer r_1 and r_2 such that

$$x^h(i,0)=0, i\geq r_1; x^v(0,j)=0, i\geq r_2 \tag{1d}$$

And the initial conditions are arbitrary, but belong to the set [15-17]:

$$S=\{x^h(i,0), x^v(0,j): x^h(i,0)=MN_1, \\ x^v(0,j)=MN_2, N_k^T N_k<1, k=1,2\} \tag{1e}$$

Equation (1) may be used to describe a class of uncertain 2-D discrete dynamical systems which include digital filters, digital control systems and so on..

Associated with the uncertain system (1a) is the cost function [16-17]:

$$J=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}u^T(i,j)Ru(i,j) \\ +\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}x^T(i,j)W_1x(i,j) \tag{2}$$

Where T denotes the transpose and

$R=R^T>0,\ W=W^T>0, x=\begin{bmatrix}x^h(i,j)\\ x^v(i,j)\end{bmatrix}$.

The main objective of this paper is to derive LMI-based sufficient condition for the existence of static-state feedback robust controller for system (1) with the cost function (2) such that the closed-loop system is asymptotically stable and the closed-loop cost function is not more than a specified upper bound. In the next section, we will first carry out robust guaranteed cost performance analysis for the uncertain 2-D free system.

III. Robust guaranteed cost performance analysis

Consider the 2-D free system (setting $u\equiv 0$):

$$\begin{bmatrix}x^h(i+1,j)\\ x^v(i,j+1)\end{bmatrix}=(A+\Delta A)\begin{bmatrix}x^h(i,j)\\ x^v(i,j)\end{bmatrix} \tag{3}$$

and the associated cost function

$$J_0=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}x^T(i,j)W_1x(i,j) \tag{4}$$

The uncertainties ΔA are said to be admissible if (1b)-(1c) hold true. Sufficient condition for the asymptotic stability of system (3) with ΔA =0 have been established in [20].It is obvious that system (3) is quadratically stable if it satisfies the condition of asymptotic stability for all admissible uncertainties. As an extension of the result for the asymptotic stability condition of 2-D discrete the Roesser model given in [1], one can easily arrive at the following definitions.

Definition 1. The uncertain system (3) is said to be quadratically stable if there exist a block-diagonal matrix $P=diag\{P_h,P_v\}>0$, where $P_h\in R^{n\times n}$ and $P_v\in R^{m\times m}$, such that

$$\Gamma=(A+\Delta A)^T P(A+\Delta A)-P<0 \tag{5}$$

Definition2. The uncertain system (3) with cost function (4) is said to be robustly stable with a quadratic guaranteed cost matrix (QGCM) $P=diag\{P_h,P_v\}>0$,if it satisfy:

$$\Omega=(A+\Delta A)^T P(A+\Delta A)-P+W_1\leq 0 \tag{6}$$

for all $\|F(i,j)\|\leq 1$

Where $W_1=W_1^T$ is a positive definite matrix.

In the following, we introduce an important Lemma1.

Lemma1[21].Let $A\in R^{n\times n}, H\in R^{n\times k}, E\in R^{l\times n}$ and $Q=Q^T\in R^{n\times n}$ be given matrices. Then there exist a positive definite matrix P such that

$$[A+HFE]^T P[A+HFE]-Q<0 \tag{7}$$

for all F satisfying $F^TF\leq I$,if and only if there exists a scalar $\varepsilon>0$ such that

$$\begin{bmatrix}-P^{-1}+\varepsilon HH^T & A\\ A^T & \varepsilon^{-1}E^TE-Q\end{bmatrix}<0 \tag{8}$$

Next, we aim to solve the connection between the existence of the QGCM and the quadratic stability of the system.

Lemma2. Suppose there exists a QGCM $P=diag\{P_h,P_v\}>0$ for system (3) with initial conditions (1d) and cost function (4) .Then (i) system (3) is quadratically stable and (ii) for all admissible uncertainties the cost function satisfies the bound

$$J_0<[(r_1-1)\lambda_{max}(M^TP_hM)+(r_2-1)\lambda_{max}(M^TP_vM)] \tag{9}$$

Where $\lambda_{max}(\bullet)$ denote the maximum eigenvalue.

Proof : Proof of (i) directly follows from definition 1 and 2

To prove (ii) .consider a quadratic 2-D Lyapunov function

$$V(x)=x^TPx$$

Let $\Delta V(x)$ be defined as

$$\Delta V(x)=V(x')-V(x) \tag{10}$$

$$x'=\begin{bmatrix}x^h(i+1,j)\\ x^v(i,j+1)\end{bmatrix}, x=\begin{bmatrix}x^h(i,j)\\ x^v(i,j)\end{bmatrix}$$

Equation (10) .in view of (3), take the form:

$$\Delta V(x) = x^T(i,j)\Gamma x(i,j) \quad (11)$$

Where Γ are defined in (5) respectively .Since P is a QGCM, it follows from definition 2 that

$$x^T(i,j)(\Gamma + W_1)x(i,j) < 0 \quad (12)$$

From (11) and (12), we obtain

$$\Delta V(i,j) + x^T(i,j)W_1 x(i,j) < 0 \quad (13)$$

Summing up (13) over $i, j = 0 \to \infty$ yields

$$J_0 < -\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\Delta V(i,j) \quad (14)$$

On the other hand, for any integers $P_1 > 0, P_2 > 0$, we have

$$\sum_{i=0}^{P_1}\sum_{j=0}^{P_2}\Delta V(i,j) =$$

$$\sum_{j=0}^{P_2}[(x^h)^T(P_1+1,j)P_h x^h(P_1+1,j) - (x^h)^T(0,j)P_h x^h(0,j)]$$

$$+\sum_{i=0}^{P_1}[(x^v)^T(i,P_2+1)P_v x^v(i,P_2+1) - (x^v)^T(i,0)P_v x^v(i,0)]$$

and

$$-\sum_{i=0}^{P_1}\sum_{j=0}^{P_2}\Delta V(i,j) =$$

$$-\sum_{j=0}^{P_2}[(x^h)^T(P_1+1,j)P_h x^h(P_1+1,j) - (x^h)^T(0,j)P_h x^h(0,j)]$$

$$-\sum_{i=0}^{P_1}[(x^v)^T(i,P_2+1)P_v x^v(i,P_2+1) - (x^v)^T(i,0)P_v x^v(i,0)]$$

$$< \sum_{j=0}^{P_2}(x^h)^T(0,j)P_h x^h(0,j) + \sum_{i=0}^{P_1}(x^v)^T(i,0)P_v x^v(i,0)$$

$$\leq \sum_{j=0}^{r_1-1}(x^h)^T(0,j)P_h x^h(0,j) + \sum_{i=0}^{r_2-1}(x^v)^T(i,0)P_v x^v(i,0)$$

$$< (r_1-1)\lambda_{max}(M^T P_h M) + (r_2-1)\lambda_{max}(M^T P_v M) \quad (15)$$

From (14) and (15) we have

$$J_0 < [(r_1-1)\lambda_{max}(M^T P_h M) + (r_2-1)\lambda_{max}(M^T P_v M)]$$

Where use has been made of (4), (1c) and (1d) and the relation $\lim_{i+j\to\infty} x(i,j) = 0$.This completes the proof .

Lemma3.(Schur complements) Let M(t),N(t) and P(t) are matrices with appropriate dimension, $M(t) = M(t)^T$, $N(t) = N(t)^T$, then for any $t \in R$ such that $\begin{bmatrix} M(t) & P(t) \\ P^T(t) & N(t) \end{bmatrix} < 0$ if and only if $N(t)<0$ and $M(t) - P(t)N^{-1}(t)P^T(t) < 0$.

The following theorem provides a sufficient condition for the QGCM.

Theorem1. A block-diagonal matrix $P = diag\{P_h, P_v\} > 0$ is a QGCM for system (3) with initial conditions (1d) and cost function (4) if there exist a scalar $\varepsilon > 0$ and a block-diagonal matrix $S = \varepsilon P^{-1} = diag\{S_h, S_v\} > 0$, such that the following LMI is feasible

$$\begin{bmatrix} -S & AS & L & 0 & 0 \\ SA^T & -S & 0 & SM_1^T & SW_1^{1/2} \\ L^T & 0 & -I & 0 & 0 \\ 0 & M_1 S & 0 & -I & 0 \\ 0 & W_1^{T/2} S & 0 & 0 & -\varepsilon I \end{bmatrix} < 0 \quad (16)$$

Where I is the identity matrix of appropriate dimension. Moreover, the cost function satisfies the bound

$$J_0 < \sum_{j=0}^{r_1-1}(x^h)^T(0,j)P_h x^h(0,j) + \sum_{i=0}^{r_2-1}(x^v)^T(i,0)P_v x^v(i,0) \quad (17)$$

Proof: Using (1b), lemma 1 and (6) can be rearranged as

$$\begin{bmatrix} -P^{-1} + \varepsilon^{-1}LL^T & A \\ A^T & \varepsilon M_1^T M_1 - P + W_1 \end{bmatrix} < 0 \quad (18)$$

Premultiplying and postmultiplying (18) by the matrix

$$\begin{bmatrix} \varepsilon^{1/2} I & 0 \\ 0 & \varepsilon^{1/2} P^{-1} \end{bmatrix}$$

One obtains

$$\begin{bmatrix} -S & AS \\ SA^T & -S \end{bmatrix} + \begin{bmatrix} LL^T & 0 \\ 0 & SM_1^T M_1 S + \varepsilon^{-1} SW_1 S \end{bmatrix} < 0 \quad (19)$$

Where $S = \varepsilon P^{-1}$ (20)

The equivalence of (19) and (16) follows trivially from the Schur complements.

Using (20), the bound of the cost function can be easily obtained from (9).

Remark1. It should be observed that the matrix inequality (16) is linear in the variables S and ε. Hence, it can be solved efficiently by employing the Matlab LMI Toolbox.

IV. Robust guaranteed cost control via static-state feedback

In this section, we are interested in finding a static-state feedback $u(i,j)=Kx(i,j)$ for system (1) and cost function (2) such that the closed-loop system is asymptotically stable and the closed-loop cost function is not more than s specified upper bound.

The closed-loop system (1a) with $u(i,j)=Kx(i,j)$ can be expressed as

$$\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = (A+\Delta A+KB+K\Delta B)\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} \tag{21}$$

and the cost function (2) reduces to

$$J_0=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}x^T(i,j)W_2x(i,j)$$

Where $W_2=W_1+K^TRK$ (22)

Definition 3.A state feedback controller $u(i,j)$ is said to define a quadratic guaranteed cost control associated with cost matrix $P=P^T>0$,for system (21) and cost function (22) if there exist a $n\times n$ positive-definite symmetric matrix W_2 given by (22) such that

$$[A+\Delta A+KB+K\Delta B]^TP[A+\Delta A+KB+K\Delta B]$$

$$-P+W_2<0 \qquad \text{for all } \|F(i,j)\|\le 1$$

Now, as an extension of the result presented in section 3 .the following theorem is easily arrived at

Theorem2. Consider system (20) with initial condition (1d) subject to (1e) and cost function (22) ,then there exist a static-state feedback controller $u(i,j)=Kx(i,j)$ that solves the addressed robust cost control problem .if there exist a positive scalar ε ,an $m\times n$ matrix U, a block-diagonal matrix $S=\varepsilon P^{-1}=\mathrm{diag}\{S_h,S_v\}>0$,such that

$$\begin{bmatrix} -S & \bar{A} & L & 0 & 0 & 0 \\ \bar{A}^T & -S & 0 & \bar{M}_1 & SW_1^{1/2} & U^TR^{1/2} \\ L^T & 0 & -I & 0 & 0 & 0 \\ 0 & \bar{M}_1^T & 0 & -I & 0 & 0 \\ 0 & W_1^{T/2}S^T & 0 & 0 & -\varepsilon I & 0 \\ 0 & R^{T/2}U & 0 & 0 & 0 & -\varepsilon I \end{bmatrix}<0 \tag{23}$$

Where

$\bar{A}=AS+BU$, $\bar{M}_1=SM_1^T+U^TM_2^T$.

In this situation, a suitable control law is given by $K=US^{-1}$. Moreover, cost function (22) satisfies the bound

$$J_0<\varepsilon[(r_1-1)\lambda_{max}(M^TS_h^{-1}M)+(r_2-1)\lambda_{max}(M^TS_v^{-1}M)] \tag{24}$$

Remark 2.Matrix inequality (23) is linear in the variables U,S and ε.Thus, the Matlab LMI Toolbox [18] can be applied to ascertain the existence of static-state feedback controller can be constructed and guaranteed cost upper bound can be obtained. It is clear that the upper bound on the closed-loop cost function is dependent on the choice of the guaranteed cost controllers.

V. Numerical example

In this section, we shall illustrate the robust guaranteed cost control problem via an example.

Consider the following 2-D systems described by the Roesser model

$$A=\begin{bmatrix} 0.8 & 1 \\ 0 & 0.1 \end{bmatrix}, B=\begin{bmatrix} -0.01 \\ 0 \end{bmatrix},$$
$$M_1=\begin{bmatrix} 0.0005 & -0.0005 \end{bmatrix}, M_2=0,$$
$$W_1=\begin{bmatrix} 0.0064 & 0 \\ 0 & 0.0064 \end{bmatrix}, R=0.25.$$

Using the Matlab LMI Control Toolbox, one can find that LMI (23) is feasible for this example, we obtain the solution as

$$S=\begin{bmatrix} 26.8356 & 0 \\ 0 & 5.6105 \end{bmatrix}, U=\begin{bmatrix} 0.4913 & 0.1329 \end{bmatrix},$$
$$\varepsilon=15.9774$$

and the guaranteed cost controller for this system is

$$u(i,j)=\begin{bmatrix} 0.0183 & 0.0237 \end{bmatrix}x(i,j)$$

VI. Conclusion

This paper has presented solutions for the robust guaranteed cost control for a class of 2-D discrete uncertain systems described by the Roesser model with norm-bounded uncertainties. A criterion for robust guaranteed cost control by static-state feedback is established. The criterion is LMI -based and can effectively be solved by using Matlab LMI Toolbox. The presented approach can also be applied to obtain sufficient condition for existence of dynamic output feedback controller. But, it provides only sufficient condition for the stability and is not necessary. Further work will be required to reduce conservative nature.

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