A New Image Thresholding Method based on Isoperimetric Ratios

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Abstract—**This paper presents a new thresholding method based on isoperimetric theory for image segmentation. The proposed method uses isoperimetric ratio as a criterion in selecting optimal segmenting threshold, that is, segmenting image with the gray value of minimal isoperimetric ratio as optimal threshold. Experiments show that this method outperforms normalized cut thresholding method in terms of segmentation quality, speed and noise immunity.**

Keywords—**Isoperimetric theory, isoperimetric ratio, image segmentation, threshold selection, graph theory**

I. INTRODUCTION

Image segmentation is a common and focusing issue of image processing. Thresholding method offers an efficient way, in terms of both the implementation simplicity and the processing time. However, it is still a very difficult task in this technology to determine automatically the optimal threshold. During the last few years some methods have been proposed aimed at alleviating the problem and M. Sezgin, B. Sankur [1] summarized the latest development results in this regard. In recent years, image segmentation techniques based on graph theory have become a new research hotspot. Unfortunately, because of their high complexity in computation and poor real-time performance, these techniques are rarely practical in many image segmentation problems.

This paper presents a new thresholding method based on isoperimetric theory, referred to as isoperimetric thresholding. It uses a measure of graph cut called isoperimetric cut or isoperimetric ratio as criterion in selecting optimal segmenting threshold. Similar to the existing techniques based on graph theory, the proposed method constructs a weighted undirected graph G with nonempty sets of vertices and edges. G corresponds to the image, the vertices to pixels, and the edges to adjacency relations between these pixels. The weight on the edge should reflect the likelihood that the two pixels belong to the same segment. The presented method is neither like that of J. Shi et al. [2], who developed general image segmenting approach by solving eigen system, nor does it be the same as the method proposed in [3] that segments an image by solving linear system. In 2007, Tao and Jin et al. [4] developed a new image thresholding method based on graph spectral theory, referred to as Ncut thresholding for describing convenience, which suffers a tedious computation of 256×256 symmetrical weight matrix based on gray levels. The author's purpose is to develop a simple and efficient thresholding approach based on isoperimetric theory, eliminating the need for computing 256×256 symmetrical weight matrix while obtaining more quickly the isoperimetric ratio values for every possible threshold t through faster matrix computation. Therefore the author's approach runs faster than the method [4]. In addition, the proposed method can select the segmenting threshold more accurately than others.

The remainder of this paper is organized as follows: Section 2 introduces isoperimetric theory and its derivation. Section 3 presents the proposed method based on isoperimetric theory. Section 4 presents the results of applying the approach to various images and compares these results with that of Ncut thresholding method [4]. Section 5 discusses some details of the proposed approach such as parameters and computational cost. Section 6 summarizes the main contributions of this work.

II. ISOPERIMETRIC THEORY

A. *Isoperimetric theory*

A point set in an arbitrary feature space can be represented as a weighted undirected graph $G=(V, E)$, where V is the set of vertices and E is the set of edges. G corresponds to the image, the vertices to pixels, and the edges to adjacency relations between these pixels. An edge, e, spanning two vertices, v_i and v_j, is denoted by e_{ij}. Let $n=|V|$ and $m=|E|$ where $|.|$ denotes cardinality. Each edge has a value called weight, which is typically nonnegative and real. The weight of edge e_{ij}, is denoted by w_{ij}. The degree of a vertex v_i, denoted d_i, is $d_i=\sum w_{ij}$ $\forall$ $e_{ij}\in E$.

For a graph, G, the isoperimetric constant [5], h_G is

$$h_G = \inf_S \frac{|\partial S|}{Vol s} \quad (1)$$

Where $S \subset V$ and $Vols \leq Volv/2$, where $Volv$ denotes volume of G. In graphs with a finite node set, the infimum in (1) is a minimum. Since the present context is that of finite graphs, we will henceforth use the minimum in place of the infimum. The boundary of a set, S, is defined as $\partial S=\{e_{ij} \mid v_i \in S, v_j \in \overline{S}\}$, where $\overline{S}$ denotes the set complement, and

$$|\partial S| = \sum_{e_{ij} \in \partial S} w_{ij} \qquad (2)$$

In order to determine a notion of volume for a graph, a metric must be defined. Different choices of a metric lead to different definitions of volume. Dodziuk presented [6][7] two different notions of combinatorial volume,

$$Vols = |S| \qquad (3)$$

And $$Vols = \sum_i d_i \quad \forall v_i \in S \qquad (4)$$

We use the latter metric as the definition of volume in our algorithm. The reason is that the second metric is more suited to image segmentation since regions of uniform intensity are given preference over regions that simply possess a large number of pixels.

For a subset of vertices, S, isoperimetric ratio, $h(S)$ is,

$$h(S) = \frac{|\partial S|}{Vols} \qquad (5)$$

Leo Grady et al. [3] pointed out that finding a region of an image that is both large (i.e., high volume) and that shares a small perimeter with its surroundings (i.e., small boundary) is intuitively appealing as a "good" image segment.

B. Derivation of Isoperimetric computation

Define an indicator vector, x, that takes a binary value at each node, x is defined as follows:

$$x_i = \begin{cases} 1 & if\ v_i \in S \\ 0 & if\ v_i \in \overline{S} \end{cases} \qquad (6)$$

Note that x may be viewed as a partition for a graph, the nodes with x_i of 1 belong to a segment, and the remainder belongs to another segment.

Given that the number of vertices in graph G is n. Defining the $n \times n$ matrix, L, of a graph as:

$$L_{v_i v_j} = L_{ij} = \begin{cases} d_i & if\ i = j \\ -w_{ij} & if\ e_{ij} \in E \\ 0 & otherwise \end{cases} \qquad (7)$$

This matrix is known as the Laplacian matrix [6] in the context of finite difference methods. The notation L_{ij} is used to indicate that the matrix L is being indexed by vertices v_i and v_j.

By definition of L, we can easily get the equation:

$$|\partial S| = x^T L x \qquad (8)$$

We can also derive that $Vols = x^T d$, where d is the vector of node degrees. If we use r indicating the vector of all ones, minimizing (8) subject to the constraint that the set, S, has fixed volume may be accomplished by asserting

$$Vols = x^T d = k \qquad (9)$$

Where $0 < k < r^T d/2$ is an arbitrary constant. Thus, the isoperimetric constant (1) of a graph, G, may be rewritten in terms of the indicator vector as,

$$h_G = \min_x \frac{x^T L x}{x^T d} \qquad (10)$$

For a given partition of graph, S, its isoperimetric constant, or so-called isoperimetric ratio, $h(s)$ can be rewritten as,

$$h(S) = h(x_s) = \min_{x_s} \frac{x_s^T L x_s}{x_s^T d} \qquad (11)$$

Where x_s denotes the indictor vector corresponding to the partition S.

III. THRESHOLDING BASED ON ISOPERIMETRIC RATIO

The proposed method uses isoperimetric ratio as a criterion in selecting optimal segmenting threshold, that is, segmenting an image with the gray value of minimal isoperimetric ratio as the optimal threshold.

A. Determination of graph and edge weight

For a given image, the set of pixels is $P=\{(i,j): i=0,1,\ldots,N_h-1; j=0,1,\ldots,N_w-1\}$, where N_h and N_w denotes the height and width of the image, respectively. Let $f(x,y)$ be the gray level value of the image at pixel (x,y). So, $f(x,y) \in T=\{0,1,\ldots,255\}$.

By taking each pixel as a node and connecting each pair of adjacent pixels by an edge, a weighted undirected graph $G=(V, E)$ can be constructed. Let's define the weight of the graph edge connecting two node v_i and v_j as:

$$w_{ij} = exp(-\beta \|I_i - I_j\|_2^2) \qquad (12)$$

Where β represents a control parameter and I_i is the intensity value at node v_i, $\|\cdot\|_2$ denotes vector norm. In our method, the topologic structure of a graph is of 4-connection character, that is, each pixel (excluding boundary pixels) should be connected to four adjacent pixels on four different directions by means of an edge.

B. Determination of optimal threshold

In order to determine the optimal segmenting threshold, isoperimetric ratios of all possible thresholds need to be calculated, and the threshold of minimal isoperimetric ratio should be chosen as the optimal threshold for segmenting. For a constructed graph G and calculated weight matrix W, the isoperimetric ratio corresponding to every threshold value, t, would be computed in the following sequence:

1) Acquire image gray value by arranging the image's all pixels in the order of column by rows, and followed by listing the associated gray level value for each pixel point, a $N \times 1$ column vector, $Vals$, is obtained, where N denotes the total number of pixels.

2) Compute node degree column vector d and Laplacian matrix L, where d_i is equal to the sum of all elements of ith row

in matrix W, $L=diag(d)–W$, where function, "*diag*", is used to diagonalize the column vector d and form a diagonal matrix.

3) Construct the primary indicator vector x by comparing each gray level value of column vector, *Vals*, with the threshod t. If $Vals(i) > t$, $x_i=0$, otherwise, $x_i=1$.

4) Compute isoperimetric ratio $h(x)$ associated with vector x, with reference to equation (11).

C. Algorithm flow

Let variable t be possible segmenting threshold value, $0 \le t \le 255$, with T denoting the optimal segmenting threshold, and variable, *minRatio*, denoting minimal isoperimetric ratio. The detailed flow of the proposed algorithm is as follows.

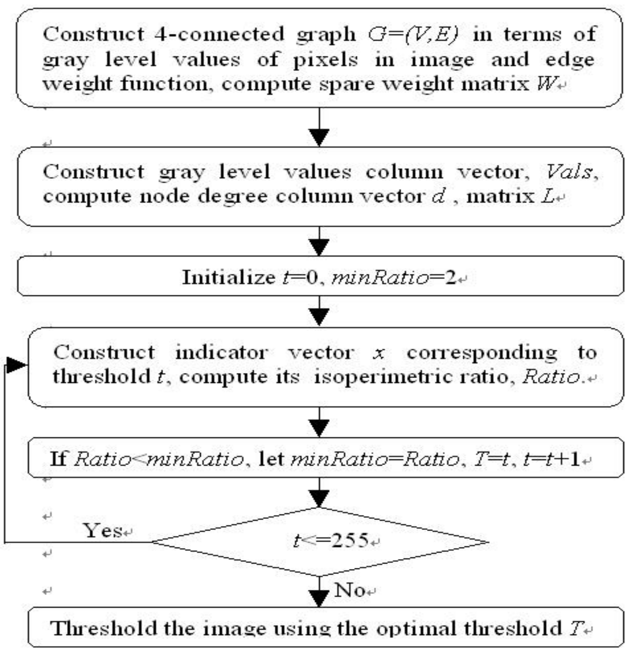


Fig.1 The flow chart of the proposed method

IV. PERFORMANCE EVALUATION AND COMPARISON

To verify the effectiveness of author's algorithm in image segmentation and evaluate the performance difference between various approaches of threshold segmentation, a set of real images was chosen as samples and organized in groups for experimental testing. The results are compared against that of Normalized cut threshold segmentation. To make evaluation and comparison of more practical significance, the chosen images are all real types; each containing distinct target and background, and the target can be exactly distinguished from the background by some suitable threshold. Since the performance of Ncut thresholding method is recognized as a better one than some of the commonly used algorithm. We only compare our results with that of Ncut thresholding. Among all the examples used in this section, the underlying graph topology was the 4-connected lattice for the proposed method, while an 8-connected lattice for Ncut thresholding method, the reason behind the differing connection being due to the failure to obtain satisfied quality results for the latter with 4-connected lattice. The parameters in equation (12) are set to $\beta=95$ for the proposed method and $\beta=35$ for Ncut thresholding method.

To make a comprehensive comparison in segmentation performance between the proposed method and Ncut thresholding method. Experiments are organized into three groups. The first group is to test the performance of segmenting ordinary gray images. The second group is for segmenting images with noise. The last group is for the runtime. All results are displayed in Fig2, Fig3, Fig4, respectively.

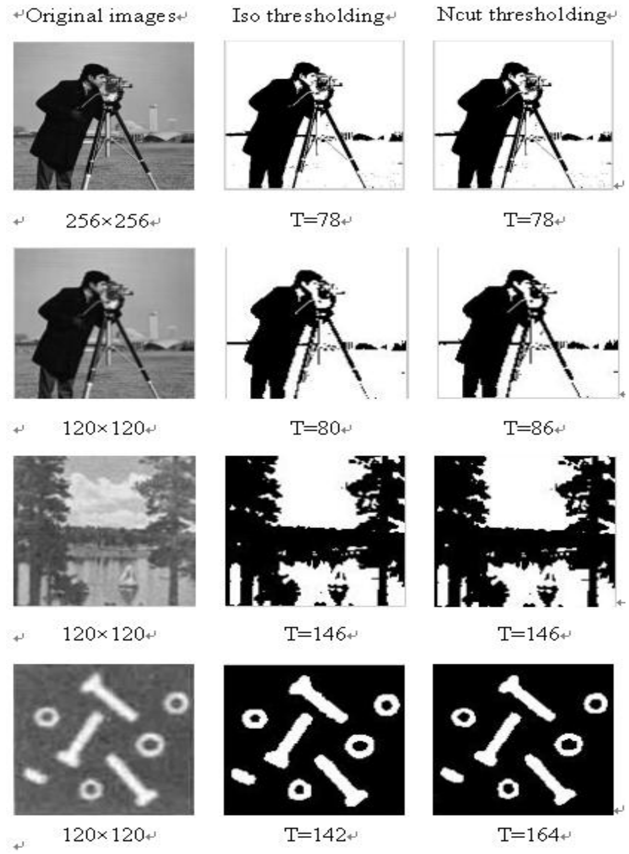
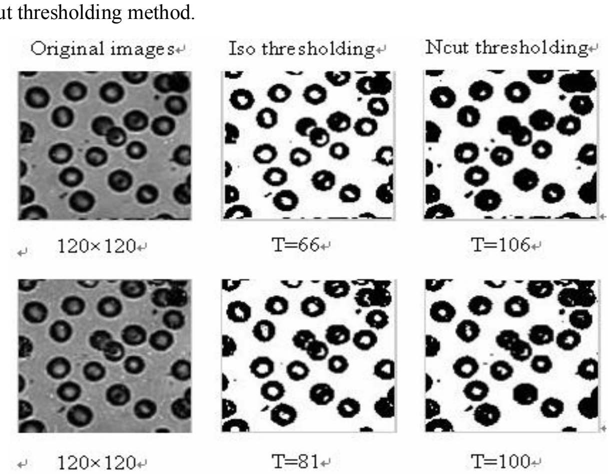


Fig.2 Comparison of segmentation quality between the proposed method (Isoperimetric thresholding) and Ncut thresholding method for ordinary gray level images. The first column corresponds to original images, the middle column to the results of the proposed method, the last column to the results of Ncut thresholding method.



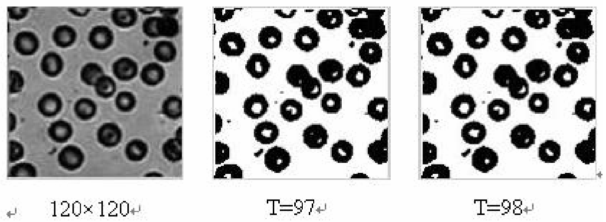


Fig.3 Stability analysis relative to shot, multiplicative noise. The first and second row represent an increasing amount of shot noise. The bottom row represents multiplicative noise. The first column corresponds to original blood images, the second column to the results of the proposed method, the last column to the results of Ncut thresholding method.

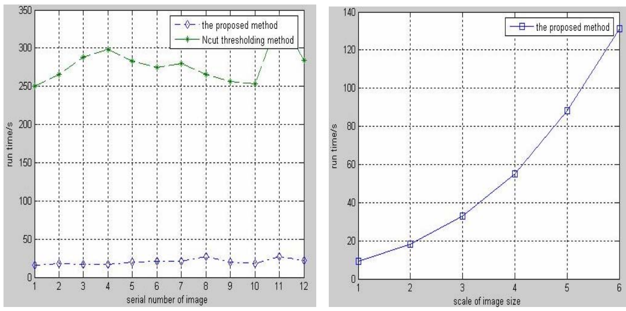


Fig. 4 Comparison of the runtime. Left subfigure corresponds to the runtime of the two methods on 12 different gray images of 120×120. Right subfigure to the runtime of the proposed method on images of different resolution and same content. Image resolutions are 100×100, 120×120, 140×140, 160×160, 180×180, 200×200 respectively.

The segmentation results of ordinary gray level images are shown in Fig 2. The results indicate that the segmentation quality of the proposed method is roughly the same or slightly better than that of Ncut thresholding method for images of different size or content. For images of increased resolution, however, the segmentation quality of image becomes better for the new algorithm. The explanation for it may attribute to the more detailed information contained in higher resolution images that would be in favor of segmenting process.

Fig 3 shows the results of stability analysis. It shows that the two approaches are both noise immune. For segmenting images with different amount of shot noise and multiplicative noise, the proposed method outperforms Ncut thresholding method in terms of segmentation quality.

Fig 4 depicts the runtime comparison. The remarkable shorter runtime for the newly proposed method (10 fold faster) comes from the fact that it eliminates the need for calculation of a 256×256 gray level weight matrix, which would otherwise take a lot of time. As image size is increased, the benefit in efficiency for the proposed approach would be even more appreciated.

V. THE PARAMETER AND COMPUTATION COST

In the proposed method, only one parameter, β, exists to control the effect of gray scale difference of the two nodes to the weight. Proper selection of this parameter is all you need to segment an image. In most case, the typical value of β ranges between 20 and 150. Furthermore, it was found that the proposed method is highly insensitive to the graph topology, that is, 4-connected and 8-connected graph topology have little effect on segmentation results.

The proposed method is in essence a thresholding method that explains the computation efficiency. On the other hand, no calculation need for gray level weight matrix dramatically reduces the segmentation time. The experimental results show that the segmentation time is less than 30s for the images of 120×120. All experiments are carried out on a 3.2G Pentium PC with 512M RAM.

VI. CONCLUSION

A new thresholding method based on the isoperimetric ratio measure has been developed. It uses isoperimetric ratio as criterion to select the optimal segmenting threshold, in which, the threshold of minimal isoperimetric ratio will be taken as the optimal threshold to segment an image. The proposed method enjoys the advantages of being more efficient and using more advanced criterion to select optimal threshold. In addition, there is no need to compute gray level weight matrix, thus reducing the segmenting time. Experimental results have also shown that the proposed method outperforms Ncut thresholding method in terms of segmentation quality, speed and noise immunity.

ACKNOWLEDGMENT

This research was supported by the National Natural Science Foundation of China (No. 60632050), the National 863 Item (No. 2006AA04Z238, 2006AA01Z119), the technology item of the education department of Fujian province (No. JB07170) and the technology startup item of Minjiang university (No. YKQ07001).

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