An Improved Result for Observer-based H_∞ Fuzzy Control

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Abstract

This paper deals with the observer-based H_∞ control problem for T-S fuzzy systems. By Lyapunov stability and the three index combination technique, a new nonlinear matrix inequality condition for the existence of an observer-based H_∞ controller is derived. To solve controllers with LMI algorithms, a set of LMI conditions which are necessary and sufficient to the derived nonlinear condition are developed as well. It can be shown that the present result is more relaxed than the existing ones and includes them as special cases.

Keywords T-S fuzzy systems, linear matrix inequality (LMI), H_∞ control.

I. Introduction

Fuzzy sets and systems have gone through substantial development since the introduction of fuzzy set theory by Zadeh since 1965 [18]. There have been a great variety of successful applications in the area of image processing, industrial applications, medicine, finance, control engineering and so on in the literature [1,2,6,10,17,19]. In particular, the T-S fuzzy model, also called the Type-III fuzzy model by Sugeno [11,12], is recognized as a powerful tool. For example, it offers an alternative approach to describing nonlinear systems [20]. By using the fuzzy model, lots of nonlinear control problems can be easily solved [13,15,16]. Therefore, considerable attention has been paid to the analysis and synthesis of T-S fuzzy systems and various techniques have been developed during the decade [4,14].

The H_∞ control problem of T-S fuzzy systems has been investigated by many authors recently. For example, the papers [3,7,8,9] and the references therein solved the H_∞ control problem via fuzzy observer-based feedback control. Among which, the result of [8] is quite interesting. It introduced a new type of observer and developed a nonlinear condition for the existence of controllers. For obtaining controllers, a sufficient LMI condition was proposed and solved by a two-step procedure. Later on, the reference [7] proposed an improved result, in which an LMI condition equivalent to the nonlinear condition was developed. By the LMI conditions, the controller and observer can be obtained simultaneously. Thus remove the drawback of two-step procedure in [8]. In our paper, a new nonlinear condition which is more relaxed than that of [8] is proposed. An LMI condition which is equivalent to our nonlinear condition is also given. The present LMI condition is more relaxed than those of [7] and [8]. Such improvement on relaxization makes possible for finding a controller achieving a better H_∞ performance and allowing a larger size of uncertainties for robust control.

II. Preliminaries

Consider a nonlinear system that is represented by the following T-S fuzzy model：

Plant Rule i :If $\theta_1(t)$ is M_{i1} and … and $\theta_s(t)$ is M_{is}

Then $\begin{cases} \dot{x}(t) = A_i x(t) + B_{wi} w(t) + B_{ui} u(t) \\ z(t) = C_{zi} x(t) + D_{zui} u(t) \\ y(t) = C_{yi} x(t) + D_{ywi} w(t) \end{cases}$ (1)

In (1), M_{ij} (i = 1,2,…,r, j = 1,2,…,s) is the fuzzy set and r is the number of If-Then rules. $\theta_i(t)$, i=1,2,…,s are the premise variables which are measurable, $x \in \mathbb{R}^n$ is the state vector, $w \in \mathbb{R}^{m_w}$ is the exogenous disturbance, $z \in \mathbb{R}^p$ is the controlled output, $y \in \mathbb{R}^q$ is the output vector, and $u \in \mathbb{R}^{m_u}$ is the control input vector. Assume $A_i \in \mathbb{R}^{n\times n}$, $B_{ui} \in \mathbb{R}^{n\times m_u}$, $B_{wi} \in \mathbb{R}^{n\times m_w}$, $C_{zi} \in \mathbb{R}^{p\times n}$, $D_{zui} \in \mathbb{R}^{p\times m_u}$, $C_{yi} \in \mathbb{R}^{q\times n}$, and $D_{ywi} \in \mathbb{R}^{q\times m_w}$. Given a pair of ($x(t)$, $u(t)$), the final output of the fuzzy system is inferred as follows:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} h_i\left(\theta(t)\right)\left(A_i x(t) + B_{wi} w(t) + B_{ui} u(t)\right) \\ z(t) &= \sum_{i=1}^{r} h_i\left(\theta(t)\right)\left(C_{zi} x(t) + D_{zui} u(t)\right) \\ y(t) &= \sum_{i=1}^{r} h_i\left(\theta(t)\right)\left(C_{yi} x(t) + D_{ywi} w(t)\right) \end{aligned} \tag{2}$$

where $h_i\left(\theta(t)\right) = \dfrac{\beta_i\left(\theta(t)\right)}{\sum_{i=1}^{r} \beta_i\left(\theta(t)\right)}$ $\beta_i\left(\theta(t)\right) = \prod_{j=1}^{s} M_{ij}\left(\theta_j(t)\right)$.

For simplicity, $h_i\left(\theta(t)\right)$ is replaced by h_i in the sequel.

Definitions: If $w(t)$ = 0, the system (2) is termed to be "disturbance-free". A disturbance-free fuzzy system is said to be *quadratically stable* if there exists a $P > 0$ such that $\dot{V}\left(x(t)\right) < 0$, where $V\left(x(t)\right) = x^T(t)Px(t)$. If $u(t)$ = 0, the system (2) is termed to be "unforced". Given a prescribed scalar $\gamma > 0$, if for any $w(t) \in L^2\left(0, \infty, \mathbb{R}^{m_w}\right)$ (the set of square integrable functions), the response $z(t)$ of the unforced fuzzy system (2), under zero initial condition, satisfies

$$\int_0^\infty z^T(t)z(t)dt < \gamma^2 \int_0^\infty w^T(t)w(t)dt. \tag{3}$$

then the fuzzy system (2) is said to be stable with γ-disturbance attenuation.

Consider the observer model same as in [7,8]

$$\begin{aligned} \dot{\hat{x}}(t) &= \sum_{i=1}^{r} h_i\left(A_i \hat{x}(t) + B_{wi} w(t) + B_{ui} u(t) - L_i\left(y(t) - \hat{y}(t)\right)\right) \\ \hat{y}(t) &= \sum_{i=1}^{r} h_i\left(C_{yi}\hat{x}(t) + D_{ywi} w(t)\right) \end{aligned} \tag{4}$$

Assume the fuzzy controller is

$$u(t) = \sum_{i=1}^{r} h_i K_i \hat{x}(t) \tag{5}$$

where K_i and L_i are the controller gains and observer gains to be determined. Denote the estimation errors as $e(t) \equiv x(t) - \hat{x}(t)$ and construct an augmented system

$$\begin{aligned} \begin{bmatrix} \dot{x}(t) \\ \dot{e}(t) \end{bmatrix} &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \left(\begin{bmatrix} A_i + B_{ui}K_j & -B_{ui}K_j \\ 0 & A_i + L_i C_{yj} \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} B_{wi} \\ 0 \end{bmatrix} w(t) \right) \\ z(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \begin{bmatrix} C_{zi} + D_{ui}K_j & -D_{ui}K_j \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}. \end{aligned} \tag{6}$$

The objectives of the observer-based H_∞ control are

(i) The disturbance-free fuzzy system (6) is *quadratically stable*.

(ii) The unforced fuzzy system (6) is *stable with* γ *-disturbance attenuation*.

The following is the main results of [8]. It is stated here for the ease of relaxization comparison with our result from a theoretic viewpoint in next section.

Lemma 1 [8] : For a given real number $\gamma > 0$, the control objectives can be achieved if there exist matrices $X > 0$, $G > 0$, K_i, L_i, Z_{ii}, $i = 1,2,...,r$, Z_{ij}, $Z_{ji} = Z_{ij}^T$, $i = 1,2,...,r-1$, $j = i+1,...,r$ such that the following matrix inequalities hold

$$\begin{bmatrix} V_{ii}^T X + XV_{ii} + \gamma^{-2} X B_{wi} B_{wi}^T X & -XB_{ui}K_i \\ -K_i^T B_{ui}^T X & \Gamma_{ii}^T G + G\Gamma_{ii} \end{bmatrix} < Z_{ii}, \quad i = 1,\ 2, ...,\ r, \tag{7}$$

$$\begin{bmatrix} V_{ij}^T X + XV_{ij} + \gamma^{-2} X\left(B_{wi}B_{wj}^T + B_{wj}B_{wi}^T\right)X & -X\left(B_{ui}K_j + B_{uj}K_i\right) \\ -\left(K_i^T B_{uj}^T + K_j^T B_{ui}^T\right)X & \Gamma_{ij}^T G + G\Gamma_{ij} \end{bmatrix} \le Z_{ij} + Z_{ij}^T, \quad j > i, \tag{8}$$

$$\begin{bmatrix} Z_{11} & \cdots & Z_{1r} & U_{ik}^T \\ \vdots & \ddots & \vdots & \vdots \\ Z_{1r}^T & \cdots & Z_{rr} & U_{rk}^T \\ U_{1k}^T & \cdots & U_{rk} & -I \end{bmatrix} < 0, \quad k = 1,\ 2, \ldots,\ r, \qquad (9)$$

where

$V_{ii} = A_i + B_{ui} K_i$, $V_{ij} = A_i + A_j + B_{ui} K_j + B_{uj} K_i$,

$\Gamma_{ii} = A_i + L_i C_{yi}$, $\Gamma_{ij} = A_i + A_j + L_i C_{yj} + L_j C_{yi}$,

$U_{ik} = \begin{bmatrix} C_{zi} + D_{ui} K_k & -D_{ui} K_k \end{bmatrix}$

Note that the inequalities (7)-(9) are actually nonlinear with respect to K_i and L_i. In order to solve K_i and L_i with LMI algorithms, following the idea of [3,9], the paper [8] presented another sufficient condition and proposed a two-step procedure to solve the condition. The drawback of two-step procedure was overcome latterly by [**7**], in which a set of LMI conditions were developed and was proved to be equivalent to the nonlinear matrix conditions of [**7**]. In next section, a new nonlinear condition that is more relaxed than Lemma 1 is proposed. A new LMI condition necessary and sufficient to our developed nonlinear condition is also derived for obtaining the gains in one step. Actually, we will show the present result is more relaxed than those of [8] and [**7**] and includes them as special cases.

III. Main results

Lemma 2 : For a given positive number $\gamma > 0$, the control objectives can be achieved if there exist matrices $X > 0$, $G > 0$, K_i, L_i, Y_{iii}, $i = 1, 2, \ldots, r$; $Y_{jii} = Y_{iij}^T$, Y_{iji}, $i = 1, 2, \ldots, r$, $j \neq i$, $j = 1, 2, \ldots, r$, $Y_{\ell ji} = Y_{ij\ell}^T$, $Y_{j\ell i} = Y_{i\ell j}^T$, $Y_{\ell ij} = Y_{ji\ell}^T$, $i = 1, 2, \ldots, r-2$, $j = i+1, \ldots, r-1$, $\ell = j+1, \ldots, r$ such that the following matrix inequalities hold

$$\Lambda_{ii} < Y_{iii}, \quad i = 1, 2, \ldots, r, \qquad (10)$$

$$\Lambda_{ii} + \Lambda_{ij} + \Lambda_{ji} \le Y_{iij} + Y_{iji} + Y_{iij}^T, \quad i = 1, 2, \ldots, r, \quad j \neq i, \quad j = 1, 2, \ldots, r, \qquad (11)$$

$$\Lambda_{ij} + \Lambda_{i\ell} + \Lambda_{ji} + \Lambda_{j\ell} + \Lambda_{\ell i} + \Lambda_{\ell j} \le Y_{ij\ell} + Y_{i\ell j} + Y_{ji\ell} + Y_{ij\ell}^T + Y_{i\ell j}^T + Y_{ji\ell}^T$$
$$i = 1, 2, \ldots, r-2,\ j = i+1, 2, \ldots, r-1,\ \ell = j+1, 2, \ldots, r, \qquad (12)$$

$$\begin{bmatrix} Y_{1j1} & \cdots & Y_{1jr} & U_{1j}^T \\ \vdots & \ddots & \vdots & \vdots \\ Y_{rj1} & \cdots & Y_{rjr} & U_{rj}^T \\ U_{1j} & \cdots & U_{rj} & -I \end{bmatrix} < 0 \quad j = 1, 2, \ldots, r, \qquad (13)$$

where

$$\Lambda_{ij} = \begin{bmatrix} A_i^T X + X A_i + K_j^T B_{ui}^T X \\ + X B_{ui} K_j + \gamma^{-2} X B_{wi} B_{wj}^T X & -X B_{ui} K_j \\ -K_j^T B_{ui}^T X & A_i^T G + G A_i + \\ & C_{yj}^T L_i^T G + G L_i C_{yj} \end{bmatrix}$$

$$U_{ij} = \begin{bmatrix} C_{1i} + D_{12i} K_j & -D_{12i} K_j \end{bmatrix}$$

In what follows, we are going to show that Lemma 2 is more relaxed than Lemma 1 and includes it as a special case.

Theorem 1: The set of solutions to (7)-(9) in Lemma 1 is a subset of solutions to (10)-(13) in Lemma 2.

Remark 1: Since Lemma 2 is more relaxed than Lemma 1, the controller set obtained by Lemma 2 is lager than that of Lemma 1. This improvement makes possible for finding a controller achieving a better H_∞ control performance and allowing a bigger size of uncertainties for robust control. This will be demonstrated in the numerical example section.

Theorem 2 : For a given positive number $\gamma > 0$, there exist matrices $X > 0$, $G > 0$, K_i, L_i, Y_{iii}, $i = 1, 2, \ldots, r$, $Y_{jii} = Y_{iij}^T$, Y_{iji}, $i = 1, 2, \ldots, r$, $j \neq i$, $j = 1, 2, \ldots, r$, $Y_{ij\ell}$, $Y_{\ell ji} = Y_{ij\ell}^T$, $Y_{j\ell i} = Y_{i\ell j}^T$, $Y_{\ell ij} = Y_{ji\ell}^T$, $i = 1, 2, \ldots, r-2$, $j = i+1, \ldots, r-1$, $\ell = j+1, \ldots, r$, such that the matrix inequalities (10)-(13) hold if and only if there exist matrices $\bar{X} > 0$, $\bar{Y} > 0$, M_i, J_i, P_{iii}, Q_{iii}, $i = 1, 2, \ldots, r$, $P_{jii} = P_{iij}^T, Q_{jii} = Q_{iij}^T$, P_{iji}, Q_{iji}, $i = 1, 2, \ldots, r$, $j \neq i$, $j = 1, 2, \ldots, r$, $P_{\ell ji} = P_{ij\ell}^T$,

$Q_{\ell ji}=Q_{ij\ell}^T$, $P_{\ell ji}=P_{ij\ell}^T$, $Q_{j\ell i}=Q_{i\ell j}^T$, $P_{\ell ij}=P_{ji\ell}^T$, $Q_{\ell ij}=Q_{ji\ell}^T$, $i=1,2,\ldots,r-2$, $j=i+1,\ldots,r-1$, $\ell=j+1,\ldots,r$ such that the following matrix inequalities hold

$$\bar{X}A_i^T+A_i\bar{X}+M_i^TB_{ui}^T+B_{ui}M_i+\gamma^{-2}B_{wi}B_{wi}^T<P_{iii},\quad i=1,2\ldots r, \tag{14}$$

$$A_i^T\bar{Y}+\bar{Y}A_i+C_{yi}^TJ_i^T+J_iC_{yi}<Q_{iii},\quad i=1,2\ldots r, \tag{15}$$

$$\begin{aligned}&\bar{X}\left(2A_i^T+A_j^T\right)+\left(2A_i+A_j\right)\bar{X}+M_i^T\left(B_{ui}^T+B_{uj}^T\right)\\&+M_j^TB_{ui}^T+\left(B_{ui}+B_{uj}\right)M_i+B_{ui}M_j\\&+\gamma^{-2}\left(B_{wi}B_{wi}^T+B_{wi}B_{wj}^T+B_{wj}B_{wi}^T\right)\le P_{iij}+P_{iji}+P_{iij}^T,\\&i=1,2\ldots r,\quad i\neq j,\quad j=1,2\ldots r\end{aligned} \tag{16}$$

$$\begin{aligned}&\left(2A_i^T+A_j^T\right)\bar{Y}+\bar{Y}\left(2A_i+A_j\right)+C_{yi}^T\left(J_i^T+J_j^T\right)\\&+C_{yj}^TJ_i^T+J_iC_{yj}+\left(J_i+J_j\right)C_{yi}\\&\le Q_{iij}+Q_{iji}+Q_{iij}^T,i=1,2\ldots r,i\neq j,j=1,2\ldots r\end{aligned} \tag{17}$$

$$\begin{aligned}&2\bar{X}\left(A_i^T+A_j^T+A_\ell^T\right)+2\left(A_i+A_j+A_\ell\right)\bar{X}+M_j^TB_{ui}^T\\&+M_\ell^TB_{uj}^T+M_i^TB_{uj}^T+M_\ell^TB_{uj}^T+M_i^TB_{u\ell}^T+M_j^TB_{u\ell}^T\\&+B_{ui}M_j+B_{ui}M_\ell+B_{uj}M_i+B_{uj}M_\ell+B_{u\ell}M_i+B_{u\ell}M_j\\&+\gamma^{-2}\left(B_{wi}B_{wj}^T+B_{wi}B_{w\ell}^T+B_{wj}B_{wi}^T+B_{wj}B_{w\ell}^T\right.\\&\qquad\left.+B_{w\ell}B_{wi}^T+B_{w\ell}B_{wj}^T\right)\\&\le P_{ij\ell}+P_{i\ell j}+P_{ji\ell}+P_{ij\ell}^T+P_{i\ell j}^T+P_{ji\ell}^T\\&i=1,2\ldots r-2,\quad j=i+1\ldots r-1,\quad \ell=j+1\ldots r\end{aligned} \tag{18}$$

$$\begin{aligned}&2\left(A_i^T+A_j^T+A_\ell^T\right)\bar{Y}+2\bar{Y}\left(A_i+A_j+A_\ell\right)+C_{yj}^TJ_i^T\\&+C_{y\ell}^TJ_i^T+C_{yi}^TJ_j^T+C_{y\ell}^TJ_j^T+C_{yi}^TJ_\ell^T+C_{yj}^TJ_\ell^T+J_iC_{yj}\\&+J_iC_{y\ell}+J_jC_{yi}+J_\ell C_{yi}+J_\ell C_{yj}\\&\le Q_{ij\ell}+Q_{i\ell j}+Q_{ji\ell}+Q_{ij\ell}^T+Q_{i\ell j}^T+Q_{ji\ell}^T\\&i=1,2\ldots r-2,\quad j=i+1\ldots r-1,\quad \ell=j+1\ldots r\end{aligned} \tag{19}$$

$$\begin{bmatrix}P_{1k1}&\cdots&P_{1kr}&\bar{X}C_{11}^T+M_k^TD_{121}^T\\\vdots&\ddots&\vdots&\vdots\\P_{rk1}&\cdots&P_{rkr}&\bar{X}C_{1r}^T+M_k^TD_{12r}^T\\C_{11}\bar{X}+D_{121}M_k&\cdots&C_{1r}\bar{X}+D_{12r}M_k&-I\end{bmatrix}<0,\quad k=1,2,\ldots r \tag{20}$$

$$\begin{bmatrix}Q_{1k1}&\cdots&Q_{1kr}\\\vdots&\ddots&\vdots\\Q_{rk1}&\cdots&Q_{rkr}\end{bmatrix}<0,\quad k=1,2,\ldots r \tag{21}$$

In this case, the controller gains and the observer gains can be chosen as

$$K_i=M_i\bar{X}^{-1}\text{ and }L_i=\bar{Y}^{-1}J_i\ i=1,2,\ldots,r. \tag{22}$$

IV. A numerical example

Consider a nonlinear system [13]

$$\begin{aligned}&\dot{x}_1(t)=x_1(t)+\sin x_2(t)-0.1x_3(t)+\left(x_1^2(t)+1\right)u(t)+w(t)\\&\dot{x}_2(t)=2x_1(t)-3x_2(t)+w(t)\\&\dot{x}_3(t)=\sin x_2(t)-x_3(t)+w(t)\\&z(t)=x_1(t)+2.3u(t)\\&y_1(t)=\left(x_1^2(t)+1\right)x_2(t)+0.5w(t)\\&y_2(t)=x_1(t)+0.5w(t).\end{aligned}$$

Assume $x_1(t)\in[-a\quad a]$, $x_2(t)\in[-b\quad b]$, where a and b are positive numbers. The two parameters will be also used later to compare the conservativeness of conditions. The premise membership functions and the consequent matrices are

$$M_{11}=\frac{x_1^2}{a^2}=M_{21},\qquad M_{31}=1-M_{11}=M_{41},$$

$$M_{12}=M_{32}=\begin{cases}\dfrac{b\sin x_2-x_2\sin b}{x_2(b-\sin b)}&x_2\neq0\\1&x_2=0\end{cases},$$

$$M_{22}=1-M_{12}=M_{42},$$

$$A_1=\begin{bmatrix}1&1&-0.1\\2&-3&0\\0&1&-1\end{bmatrix},\ B_{u1}=\begin{bmatrix}1+a^2\\0\\0\end{bmatrix},$$

$$C_{y1}=\begin{bmatrix}0&1+a^2&0\\1&0&0\end{bmatrix},$$

$$A_2=\begin{bmatrix}1&\sin(b)/b&-0.1\\2&-3&0\\0&\sin(b)/b&-1\end{bmatrix},\ B_{u2}=\begin{bmatrix}1+a^2\\0\\0\end{bmatrix},$$

$$C_{y2}=\begin{bmatrix}0&1+a^2&0\\1&0&0\end{bmatrix},$$

$$A_3=\begin{bmatrix}1&1&-0.1\\2&-3&0\\0&1&-1\end{bmatrix},\ B_{u3}=\begin{bmatrix}1\\0\\0\end{bmatrix},\ C_{y3}=\begin{bmatrix}0&1&0\\1&0&0\end{bmatrix},$$

$$A_4=\begin{bmatrix}1&\sin(b)/b&-0.1\\2&-3&0\\0&\sin(b)/b&-1\end{bmatrix},\ B_{u4}=\begin{bmatrix}1\\0\\0\end{bmatrix},$$

$$C_{y4}=\begin{bmatrix}0&1&0\\1&0&0\end{bmatrix},$$

$B_{w1}=B_{w2}=B_{w3}=B_{w4}=\begin{bmatrix}1&1&1\end{bmatrix}^T$,

$C_{z1}=C_{z2}=C_{z3}=C_{z4}=\begin{bmatrix}2.5&3&0\end{bmatrix}$, $D_{zu1}=D_{zu2}=D_{zu3}=D_{zu4}=0.001$, $D_{yw1}=D_{yw2}=D_{yw3}=D_{yw4}=0.005$. In this simulation, assume $a=1.4$ and $b=0.7$. By Theorem 2, choosing $\gamma=0.2$, we obtain

$$K_1 = [\text{-}6.6894 \quad \text{-}0.4333 \quad 0.0397],$$
$$K_2 = [\text{-}6.7732 \quad \text{-}0.3923 \quad 0.0390],$$
$$K_3 = [\text{-}10.8100 \quad \text{-}1.1630 \quad 0.1041],$$
$$K_4 = [\text{-}10.1789 \quad \text{-}1.0774 \quad 0.0990],$$

$$L_1 = \begin{bmatrix} \text{-}1.5494 & \text{-}16.7745 \\ \text{-}3.8066 & \text{-}0.6041 \\ \text{-}0.3339 & 0.6601 \end{bmatrix}, \; L_2 = \begin{bmatrix} \text{-}1.5705 & \text{-}16.6232 \\ \text{-}3.9595 & \text{-}0.5219 \\ \text{-}0.2907 & 0.6559 \end{bmatrix},$$
$$L_3 = \begin{bmatrix} \text{-}1.4381 & \text{-}16.5338 \\ \text{-}4.4155 & \text{-}2.6641 \\ \text{-}0.8670 & 0.6159 \end{bmatrix}, \; L_4 = \begin{bmatrix} \text{-}1.4346 & \text{-}16.2066 \\ \text{-}3.9927 & \text{-}2.5747 \\ \text{-}0.7884 & 0.6053 \end{bmatrix}.$$

Finally, we compare the conservativeness of Theorem 2 with [7] and [8]. For the above numerical example, the maximal interval of the positive parameter a such that the conditions are feasible is given Table 1. Our result allows the largest interval for three different cases.

Table 1. the maximal allowable interval of a by using different conditions

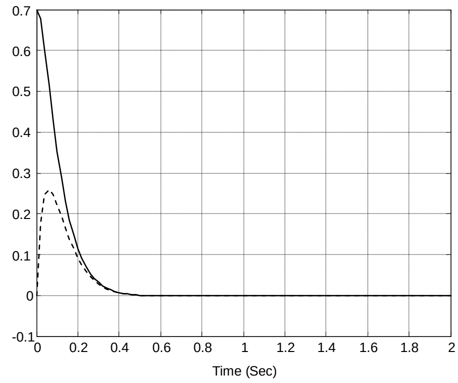
cases	Theorem 2	[7]	[8]
$\gamma = 0.2$, $b = 0.01$	$0 < a \le 7132042$	$0 < a \le 1931000$	$0 < a \le 1421000$
$\gamma = 0.2$, $b = \pi/2$	$0 < a \le 1527433$	$0 < a \le 1126768$	$0 < a \le 935466$
$\gamma = 0.1$, $b = 2$	$0 < a \le 2200000$	$0 < a \le 430414$	$0 < a \le 226590$

Since our condition is the most relaxed, the designed controller may achieve a better control H_∞ performance which is demonstrated as follows.

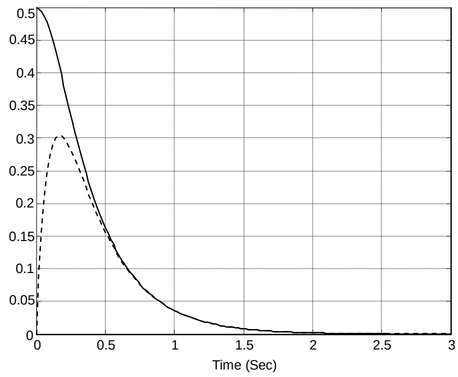
Table 2 the comparison of H_∞ control performance by using different design approaches

	Theorem 2	[7]	[8]
a=4358774, b=0.01	γ =0.198	γ =0.4	γ =1.5
a=1527433, $b = \pi/2$	γ =0.2	γ =1.87	γ =3.2
a=610558, b=2	γ =0.06	γ =0.2	γ =1.2

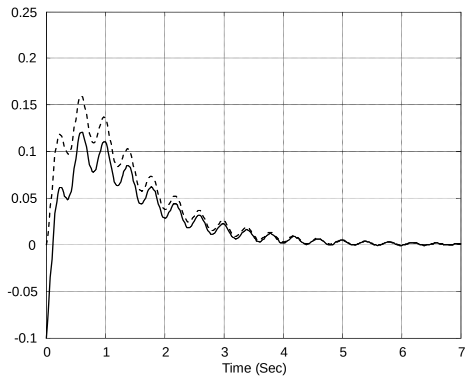
Fig. 2 show the state response of the closed-loop fuzzy system when the initial condition is $[0.7 \quad 0.5 \quad -0.1]^T$ and $w(t)$ is a disturbance given by $w(t) = 0.5e^{-0.5t}\sin(5\pi t)$. The solid line denotes the state variable, the dotted line denotes the observer state.



(a) $\hat{x}_1$ and x_1



(b) $\hat{x}_2$ and x_2



(c) $\hat{x}_3$ and x_3

Fig. 2 Responses of the state and its estimation

V. Conclusions

In this paper, a new nonlinear matrix inequality condition for the existence observer-based H_∞ control of T-S fuzzy systems is developed. The condition is shown to be more relaxed than [8] and include it as a special case. For solving the observer and the controller via LMI algorithms by one step, a set of LMI conditions necessary and sufficient to the new developed nonlinear condition is also derived. Our LMI condition is more relaxed than that of [7] since the result of [7] is only equivalent to the nonlinear condition of [8].

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