Fuzzy-GA and Multi-Objective Transportation Optimization

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Abstract—the spanning tree-based Genetic Algorithm (ST-GA) is an effective method to solve the multi-objective transportation optimization problem. The performance of this algorithm is definitely superior to the typical matrix-based GA. A new fuzzy-GA is proposed in this paper. At first, it adopts the Pareto method, so that the problem that the non-convex solutions are difficult to be found through common aggregation function method could be avoided; Moreover, the theory of fuzzy rules which can easily express explicit knowledge is introduced and merged in the ST-GA. Better Pareto front and Pareto optimal solutions are found in applications. Therefore, it displays that the new algorithm not only has better practicability but also has stronger intelligence. From the example, it can be found that the new algorithm is better then the ST-GA in performance.

Keywords—Pruefer number, Pareto optimal solution, fuzzy rules, spanning tree-based GA, multi-objective optimization.

I. Preface

In order to achieve the best optimization of overall objective in solving the multi-objective optimization problem, ordinarily, it is quite necessary to comprehensively consider all conflicitive sub-objectives, which is the comprehensive tradeoff of each sub-objective. In the multi-objective space, the spatial algebraic structure could only fulfill the partial order, but no longer have the good property in entire order of the spatial algebraic structure could only fulfill the partial tradeoff of each sub-objective. In the multi-objective space, all conflictive sub-objectives, which is the comprehensive ordinarily, it is quite necessary to comprehensively consider objective in solving the multi-objective optimization problem, but the greedy algorithm is difficult to find satisfactory solutions on optimization problem, but the Fuzzy-GA could find better Pareto front and Pareto optimal solutions in applications, therefore, it displays that the new algorithm not only has better practicability but also has stronger intelligence.

II. Symbol and Algorithm

According to the ST-GA [3, 4] mode, symbols and descriptions are given below:

\[ S = \{1,2,3,\ldots,m\} \] is the number set of m factories;
\[ D = \{m+1,m+2,\ldots,m+n\} \] is the number set of n warehouses;
\[ a = (a_1,a_2,a_3,\ldots,a_m) \] is the availability set of the product of the NO. i factory; \( b = (b_1,b_2,b_3,\ldots,b_n) \), \( b_i \) is the demand for a product of No. \( m+i \) warehouse; \( C_{\text{eff}} = (C_{\text{eff}})_{mn} \), \( q = 1,2,3,\ldots,Q \), \( C_y \) is the effectiveness units of the transportation load which is from No. i factory to No. m + j warehouse, such as transportation cost, profit, size, weight, and so on. \( a_1, b_1, C_y \) can be positive integers, and also can be positive real numbers.

Multi-objective optimization problem can be described as follows:

\[ \min Z_q = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij}, \quad q = 1,2,\ldots,Q \]
\[ \text{S.T.} \quad \sum_{j=1}^{n} x_{ij} \leq a_i, \quad i = 1,2,\ldots,m \]
The algorithm of constructing Spanning Tree $T$ according to Pruefer and Fuzzy rules:

**Pruefer Algorithm:**

1. **1st step:** Set $T = \{\emptyset\}$, $p = 0$, objective function $Z = (Z_1, Z_2, \ldots, Z_q) = (0,0,0,0,\ldots,0)$.

2. **2nd step:** Defining Pruefer remainder CP according to Pruefer number $P$.

3. **3rd step:** If $p \leq m + n - 2$, repeat (3.1) to (3.6).

(3.1) Select the rightmost number $j$ of $P$, and select the minimal number $i$ from CP.

(3.2) If $i,j \in S$ or $i,j \in D$, select next number from $P$ and marked as $j$, until $i,j$ are not all in $S$ or $D$, then $T = T \cup \{(i,j)\}$.

(3.3) Deleting $j$ from $P$, delete $i$ from $CP$. If $j$ does not reappear in left part of $P$, then add $j$ into $CP$.

(3.4) Appointing the transportation load $x_{ij}$ of border $(i,j)$ by Fuzzy rules, and updating the objective function $Z$ of border $(i,j)$.

(3.5) Updating availability and demand: $a_i = a_i - x_{ij}$, $b_j = b_j - x_{ij}$.

(3.6) Set $p = p + 1$.

4. **4th step:** If there is not any number existed in $P$, and then there must be 2 nodes $r$ and $s$ existed in CP. Adding $T = T \cup \{(r,s)\}$, and forming a tree of $m + n - 1$ borders. Likewise, appointing the transportation load of border $x_{rs}$ by Fuzzy rules, updating $Z$ and demand, $a_i = a_i - x_{rs}$, $b_j = b_j - x_{rs}$.

5. **5th step:** Repeat (5.1) and (5.2) until satisfying all demands.

(5.1) $\forall x_j > 0$, if $a_j > 0$, then add border $T = T \cup \{(i,j)\}$, appoint $x_{ij}$ by Fuzzy rules, update $Z$, availability and demand, $a_i = a_i - x_{ij}$, $b_j = b_j - x_{ij}$.

6. **6th step:** If there are borders of 0 transportation load existed in $T$, delete these borders.

From the above $P \rightarrow T$ algorithm, transportation tree set $\tilde{T} = \{T\}$ and objective function set $\tilde{Z} = \{Z\}$ can be constructed from the stocks $\tilde{P} = \{P_{i=1,\ldots,N}\}$. The set function method in literature [3,4] has difficulty in solving non-convex solution. This paper uses the following arena’s principle (AP) as the choice evaluating operator. This algorithm is also the fast algorithm [5] to construct Pareto solution or non-dominant solution.

**AP algorithm:**

1. **1st step:** Set $E = \emptyset$.

2. **2nd step:** According to $P \rightarrow T$ algorithm, transportation set $\tilde{T}$ and objective function set $\tilde{Z} = \{Z_{1} = (Z_1(T), Z_2(T), \ldots, Z_q(T))\}$ can be constructed by $\tilde{P}$.

3. **3rd step:** If $\tilde{Z} \neq \emptyset$, select $Z_i$ from $\tilde{Z}$ as the master of arena, repeat (3.1) and (3.2).

(3.1) Select another $Z_j$ from $\tilde{Z}$ to do partial comparison with $Z_j$, if $Z_j$ is superior to $Z_i$, delete $Z_i$ from $\tilde{Z}$, and put $Z_j$ as the new master of arena.

(3.2) If the master of arena have compared to other elements of $\tilde{Z}$ all over the $\tilde{Z}$, add the master of arena into $E$, and delete the master of arena from $\tilde{Z}$, then return to the 3rd step.

4. **4th step:** $PSet = \{P\} \subset \tilde{P}$ could be gained according to $E = \{Z\} \subset \tilde{Z}$.
When using AP algorithm as the matrix’s operator, and if PSet is too big, the entropy can be used to define density, and deleting the solution of larger density from PSet.

When using AP algorithm as the offspring’s operator, reducing the size of PSet could use the entropy density method. If the PSet size needs to be augmented, just randomly select new P to be added into the PSet.

This method called size adjustment of PSet

The crossover operator and mutation operator all adopt the method defined in the literature [3, 4].

Set P (t) as the chromosome stocks of current era t, C (t) as the chromosome of current era t, PSet (t) is the Pareto solution set of current era t. The integrated algorithm is as follows:

**Fuzzy-GA:**

Begin

$t \leftarrow 0$;

Randomly initialize P (t);

According to $P \rightarrow T$ and AP algorithm, and gain PSet (t) from P (t);

While termination conditions are not satisfied do

Recompose P (t), and gain C (t);

According to $P \rightarrow T$ and AP algorithm, and update PSet (t) from P (t) and C (t);

Select P (t+1) from P (t) and C (t);

$t \leftarrow t+1$;

End

The last corresponding $E=[Z]$ of PSet (t) gained Fuzzy-GA is Pareto front.

**III. Calculation Examples**

Realizing Fuzzy-GA method by C# language. The calculation examples of solving dual-objective transportation optimization problem are from the reference [4].

The first example is 3 factories and 4 warehouses, supply and demand are:

$a = (8,9,17)$  
$b = (11,3,14,16)$;

Effectiveness unit matrix:

$$C_{3\times 4} = \begin{pmatrix} 1 & 2 & 7 & 7 \\ 1 & 9 & 3 & 4 \\ 8 & 9 & 4 & 6 \end{pmatrix}, \quad C_{3\times 4}^2 = \begin{pmatrix} 4 & 4 & 3 & 4 \\ 5 & 8 & 9 & 0 \\ 6 & 2 & 5 & 1 \end{pmatrix}$$

Parameter setting of Fuzzy-GA: pop_size=30, max_gen=1000, run=20 times.

In order to build Fuzzy rules, firstly calculate:

$$C = C_{3\times 4}^1 + C_{3\times 4}^2 = \begin{pmatrix} 5 & 6 & 10 & 11 \\ 6 & 17 & 12 & 14 \\ 14 & 11 & 9 & 7 \end{pmatrix}$$

Construct fuzzy sets in the section [5,17], and the subject trigonometric function accord to the definition in literature [6]:

$$\text{triangle}(x, x_1, x_2, x_3) = \max(\min(\frac{x-x_1}{x_2-x_1}, \frac{x_3-x}{x_3-x_2}), 0)$$

That is:

"Low" = $\frac{11-x}{6}, 5 \leq x \leq 11$;

"Medium" = $\max(\min(\frac{x-11}{6}, \frac{17-x}{6}), 0), 5 \leq x \leq 17$;

"High" = $\frac{x-11}{6}, 11 \leq x \leq 17$.

First of all, distributing traffic load according to the following Fuzzy rules:

if $C_{ij} \in \text{"Low"}$ then $x_{ij} = \min\{a_i, b_j\}$ 
if $C_{ij} \in \text{"Medium"}$ then $x_{ij} = \frac{1}{2} \min\{a_i, b_j\}$ 
if $C_{ij} \in \text{"High"}$ then $x_{ij} = \frac{1}{6} \min\{a_i, b_j\}$

Then: $a_i = a_i - x_{ij}, b_j = b_j - x_{ij}$, Traversing unfinished transportation load according to the sequence—first row then line, then distributing it according to $\min\{a_i, b_j\}$. Obviously, the transportation load distributing method in literature [4] is above Fuzzy rule special case.

The Pareto front point sets educed by the ST-GA in literature [4] are $\{(143,265),(156,200),(176,175),(202,173)\}$, but the Fuzzy-GA of this paper educes not only the Pareto front point based on ST-GA, but also more Pareto front point sets, $\{(147,245),(160,195),(164,190),(172,185),(186,171)\}$, especially, the optimum solution (186,171) educed by Fuzzy-GA is obviously better then the (202,173) educed by ST-GA. Therefore, Fuzzy-GA is better than ST-GA.

The following chart shows Pareto front which is given birth by Pareto optimal solution:
The point "box" in this chart is the point set on Pareto front is given by ST-GA; the point "lap" is the point set on Pareto front is given by Fuzzy-GA; "Triangle" is the ideal point. It is obviously that Fuzzy-GA method is superior to ST-GA in solving Pareto front.

The second example is 8 factories and 9 warehouses, supply and demand are:

\[a = (10, 8, 12, 16, 21, 15, 7, 9)\]
\[b = (9, 7, 15, 10, 13, 16, 7, 10, 11)\]

Effectiveness unit matrix:

\[
\begin{bmatrix}
1 & 2 & 7 & 7 & 8 & 10 & 9 & 2 & 5 \\
8 & 9 & 4 & 6 & 4 & 1 & 6 & 9 & 2 \\
2 & 4 & 5 & 5 & 3 & 2 & 3 & 2 & 9 \\
5 & 4 & 5 & 1 & 9 & 9 & 1 & 6 & 1 \\
8 & 3 & 3 & 2 & 2 & 3 & 6 & 7 & 6 \\
1 & 2 & 6 & 4 & 5 & 9 & 3 & 5 & 2 \\
13 & 3 & 3 & 5 & 1 & 5 & 6 & 3 & 2 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
4 & 4 & 3 & 4 & 5 & 8 & 9 & 10 & 2 \\
6 & 2 & 5 & 1 & 7 & 14 & 12 & 4 & 4 \\
2 & 9 & 1 & 8 & 9 & 1 & 4 & 0 & 1 \\
3 & 5 & 3 & 2 & 8 & 3 & 3 & 2 \\
1 & 4 & 12 & 2 & 1 & 5 & 4 & 9 & 1 \\
2 & 23 & 4 & 4 & 6 & 2 & 4 & 6 & 7 \\
1 & 2 & 1 & 9 & 0 & 13 & 2 & 3 & 2 \\
14 & 3 & 4 & 2 & 1 & 8 & 5 & 3 & 1 \\
\end{bmatrix}
\]

Parameter setting of Fuzzy-GA: pop_size=100, max_gen=500, run=20. The following chart illustrates the Pareto front which is given birth by Pareto optimal solution:

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### IV. Conclusion

Fuzzy-GA algorithm improves the ST-GA method from Pareto optimal set and Fuzzy rules. From the results, the last result has distinct advantage, especially for the large-scale transportation optimization problem. For example, the situation of 8x9 in second example that shows the characteristics of the new algorithm that it finds not only better Pareto front, but also better Pareto solution. Fuzzy rules also reflect the unique advantages in resolving the multi-objective optimization problems. Explicit knowledge can be very easily expressed by Fuzzy rules and merged into the GA, so that the GA search has a knowledge orientation which greatly reduced the blindness of all searching by random search. How to more deeply discuss the intelligent effect of Fuzzy rules in the transportation multi-objective optimization problem and how to improve the efficiency of GA are the follow-up work.

### REFERENCES


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