Fuzzy-GA and Multi-Objective Transportation Optimization

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Abstract—**the spanning tree-based Genetic Algorithm (ST-GA) is an effective method to solve the multi-objective transportation optimization problem. The performance of this algorithm is definitely superior to the typical matrix-based GA. A new fuzzy-GA is proposed in this paper. At first, it adopts the Pareto method, so that the problem that the non-convex solutions are difficult to be found through common aggregation function method could be avoided; Moreover, the theory of fuzzy rules which can easily express explicit knowledge is introduced and merged in the ST-GA. Better Pareto front and Pareto optimal solutions are found in applications. Therefore, it displays that the new algorithm not only has better practicability but also has stronger intelligence. From the example, It can be found that the new algorithm is better then the ST-GA in performance.**

***Keywords*—Pruefer number, Pareto optimal solution, fuzzy rules, spanning tree-based GA, multi-objective optimization.**

I. Preface

In order to achieve the best optimization of overall objective in solving the multi-objective optimization problem, ordinarily, it is quite necessary to comprehensively consider all conflictive sub-objectives, which is the comprehensive tradeoff of each sub-objective. In the multi-objective space, the spatial algebraic structure could only fulfill the partial order, but no longer have the good property in entire order of single-objective optimization; as a result, this partial order makes the multi-objective optimization problem difficult to be solved. Michalewicz [1, 2] firstly discussed solving the linear and nonlinear transportation multi-objective optimization problems by using genetic algorithm, and then used matrices to build chromosome, and designed the composition and mutation operator based on matrix, so that the transportation problem was preferably solved. Gen, Li and Cheng [3, 4] discussed expressing the genetic algorithm of solving transportation problems by using Spanning Tree, and used the tree coding Pruefer number as a viable standard of designing chromosomes. The advantage of using spanning tree coding is that the required storage space for designing chromosomes has been greatly reduced. With regard to the m factories and n warehouses problem, for each chromosome, the matrix-based method will need $m \times n$ storage units in the evolutionary process, but the Pruefer number method only need $m+n-2$ storage units. Therefore, the performance of Pruefer number-based GA is superior to matrix-based GA in solving the multi-objective transportation problem.

This paper proposes a new Fuzzy-GA which improves the ST-GA from two aspects. At first, it adopts the Pareto method, but not the aggregation function method which was used in the paper [3, 4], so that the problem that the non-convex solutions are difficult to be found could be avoided; Moreover, the theory of fuzzy rules which can easily express explicit knowledge is introduced in the ST-GA to improve the transportation load reasonability on the edge of designated tree of ST-GA. Because the algorithm in the paper [3, 4] actually used the greedy algorithm to specify transportation load, and the greedy algorithm is difficult to find satisfactory solutions on optimization problem, but the Fuzzy-GA could find better Pareto front and Pareto optimal solutions in applications, therefore, it displays that the new algorithm not only has better practicability but also has stronger intelligence.

II. Symbol and Algorithm

According to the ST-GA [3, 4] mode, symbols and descriptions are given below:

$S=\{1,2,3,\cdots,m\}$ is the number set of m factories; $D=\{m+1,m+2,\cdots,m+n\}$ is the number set of n warehouses; $a=(a_1,a_2,a_3,\cdots a_m)$, a_i is the providable capacity for a product of the NO. i factory; $b=(b_1,b_2,\cdots,b_n)$, b_i is the demand for a product of No. m + i warehouse; $C^q_{n\times m}=(C^q_{ij})_{n\times m}$, $q=1,2,3\cdots,Q$, C^q_{ij} is the effectiveness units of the transportation load which is from No. i factory to No. m + j warehouse, such as transportation cost, profit, size, weight, and so on. a_i, b_j, C^q_{ij} can be positive integers, and also can be positive real numbers.

Multi-objective optimization problem can be described as follows:

$$\min Z_q = \sum_{i=1}^{m} \sum_{j=1}^{n} C^q_{ij} x_{ij}, \quad q=1,2,\cdots,Q$$

$$\text{S.T.} \quad \sum_{j=1}^{n} x_{ij} \le a_i, \quad i=1,2,\cdots,m$$

$$\sum_{i=1}^{m} x_{ij} \ge b_j, \quad j=1,2,\cdots,n \quad (1)$$

$$x_{ij} \ge 0, \quad \forall i,j$$

Z_q means the No. q objective function.

Let $\Omega = \{x=(x_{ij}) \in R^{m\times n} \mid \sum_{j=1}^{n} x_{ij} \le a_i, \sum_{i=1}^{m} x_{ij} \ge b_j,$ $i=1,2,\cdots,m,\ j=1,2,\cdots n\}$

Pruefer number: $P=[n_1,n_2,\cdots,n_k]$, $1 \le n_i \le n+m$, $k=n+m-2$;

Pruefer remainder: $CP=[m_1,m_2,\cdots,m_s]$, $1 \le m_i \le n+m$, $2 \le s \le n+m-1$, $m_i < m_{i+1}$, And if regard the P and CP as sets, $P \cup CP = S \cup D$, and $\forall i,j \quad m_i \ne n_j$;

Pareto optimal solutions set:

$$Pset = \{x \in \Omega \mid \neg \exists x' \in \Omega, x' \ne x, \ni Z_q(x') \le Z_q(x), \\ q=1,2\cdots Q\}$$

If restricting the Ω of PSet in finite set, PSet is called non-dominate solution [5].

Pareto front:

$$PF = \{Z(x)=(Z_1,Z_2,\cdots,Z_q) \in R^q \mid x \in PSet\}.$$

Fuzzy rule: IF C_{ij}^1 is A_{ij}^{1r} and C_{ij}^2 is A_{ij}^{2r} ··· THEN $x_{ij} = C_0^r$, here A_{ij}^{qr} is a fuzzy set,

Membership functions which figure the high, medium and low significances of effectiveness unit in (1), middle and low significance are triangular function. This is 0 rank's TSK rule [6].

The algorithm of constructing Spanning Tree T according to Pruefer and Fuzzy rules:

$P \to T$ Algorithm:

1st step: Set $T=\{\varnothing\}$, $p=0$, objective function $Z=(Z_1,Z_2,\cdots Z_q)=(0,0,0\cdots,0)$.

2nd step: Defining Pruefer remainder CP according to Pruefer number P.

3rd step: If $p \le m+n-2$, repeat（3.1）to（3.6）

(3.1) Select the rightmost number j of P，and select the minimal number i from CP.

(3.2)If $i,j \in S$ or $i,j \in D$, select next number from P and marked as j, until i, j are not all in S or D, then $T = T \cup \{(i,j)\}$.

(3.3)Deleting j from P, delete i from CP. If j does not reappear in left part of P, then add j into CP.

(3.4)Appointing the transportation load x_{ij} of border（i，j）by Fuzzy rules, and updating the objective function Z of border (i，j).

(3.5)Updating availability and demand: $a_i = a_i - x_{ij}$, $b_j = b_j - x_{ij}$.

(3.6)Set $p=p+1$.

4th step: If there is not any number existed in P, and then there must be 2 nodes r and s existed in CP. Adding $T = T \cup \{(r,s)\}$, and forming a tree of $m+n-1$ borders. Likewise, appointing the transportation load of border x_{rs} by Fuzzy rules, updating Z and demand, $a_r = a_r - x_{rs}, b_s = b_s - x_{rs}$.

5th step: Repeat（5.1）和（5.2）until satisfying all demands.

(5.1)$\forall b_j > 0$, if $a_i > 0$, then add border $T = T \cup \{(i,j)\}$, appoint x_{ij} by Fuzzy rules，update Z, availability and demand, $a_i = a_i - x_{ij}$, $b_j = b_j - x_{ij}$.

6th step: If there are borders of 0 transportation load existed in T, delete these borders.

From the above $P \to T$ algorithm, transportation tree set $\tilde{T}=\{T_i\}$ and objective function set $\tilde{Z}=[Z_i]$ can be constructed from the stocks $\tilde{P}=\{P_i \mid i=1,\cdots,N\}$. The set function method in literature [3,4] has difficulty in solving non-convex solution. This paper uses the following arena's principle (AP) as the choice evaluating operator. This algorithm is also the fast algorithm [5] to construct Pareto solution or non-dominant solution.

AP algorithm:

1st step: Set $E=\varnothing$

2nd step: According to $P \to T$ algorithm, transportation set $\tilde{T}$ and objective function set $\tilde{Z}=[Z_i=(Z_1^i(T),Z_2^i(T),\cdots,Z_q^i(T))]$ can be constructed by $\tilde{P}$

3rd step: If $\tilde{Z} \ne \varnothing$, select Z_i from $\tilde{Z}$ as the master of arena，repeat（3.1）and（3.2）

(3.1)Select another Z_j from $\tilde{Z}$ to do partial comparison with Z_i, if Z_j is superior to Z_i, delete Z_i from $\tilde{Z}$, and put Z_j as the new master of arena.

(3.2)if the master of arena have compared to other elements of $\tilde{Z}$ all over the $\tilde{Z}$, add the master of arena into E, and delete the master of arena from $\tilde{Z}$, then return to the 3rd step.

4th step: $PSet=\{P\} \subset \tilde{P}$ could be gained according to $E=[Z] \subset \tilde{Z}$.

When using AP algorithm as the matrix's operator, and if PSet is too big, the entropy can be used to define density, and deleting the solution of larger density from PSet.

When using AP algorithm as the offspring's operator, reducing the size of PSet could use the entropy density method. If the PSet size needs to be augmented, just randomly select new P to be added into the PSet.

This method called size adjustment of PSet

The crossover operator and mutation operator all adopt the method defined in the literature [3, 4].

Set P (t) as the chromosome stocks of current era t, C (t) as the chromosome of current era t, PSet (t) is the Pareto solution set of current era t. The integrated algorithm is as follows:

Fuzzy-GA:
Begin
  $t \leftarrow 0$;
  Randomly initialize P（t）;
  According to $P \rightarrow T$ and AP algorithm, and gain PSet（t）from P（t）;
  While termination conditions are not satisfied do
  Begin
    Recompose P（t）, and gain C（t）;
    According to $P \rightarrow T$ and AP algorithm, and update PSet（t）from P（t）and C（t）;
    Select P（t+1）from P（t）and C（t）;
    $t \leftarrow t+1$;
  End
End

The last corresponding $E=[Z]$ of PSet（t）gained Fuzzy-GA is Pareto front.

III. Calculation Examples

Realizing Fuzzy-GA method by C# language. The calculation examples of solving dual-objective transportation optimization problem are from the reference [4].

The first example is 3 factories and 4 warehouses，supply and demand are:

$a=(8,9,17)$ $\quad b=(11,3,14,16)$，

Effectiveness unit matrix:

$$C^1_{3\times4}=\begin{pmatrix}1&2&7&7\\1&9&3&4\\8&9&4&6\end{pmatrix},\qquad C^2_{3\times4}=\begin{pmatrix}4&4&3&4\\5&8&9&0\\6&2&5&1\end{pmatrix}$$

Parameter setting of Fuzzy-GA: pop_size=30, max_gen=1000, run=20 times.

In order to build Fuzzy rules, firstly calculate:

$$C=C^1_{3\times4}+C_{3\times4}=\begin{pmatrix}5&6&10&11\\6&17&12&14\\14&11&9&7\end{pmatrix}$$

Construct fuzzy sets in the section [5,17], and the subject trigonometric function accord to the definition in literature [6]:

$$triangle(x,x_1,x_2,x_3)=\max(\min(\frac{x-x_1}{x_2-x_1},\frac{x_3-x}{x_3-x_2}),0)$$

That is:

"Low" $=\frac{11-x}{6}$, $5\le x\le 11$;

"Medium" $=\max(\min(\frac{x-11}{6},\frac{17-x}{6}),0)$, $5\le x\le 17$;

"High" $=\frac{x-11}{6}$, $11\le x\le 17$.

First of all, distributing traffic load according to the following Fuzzy rules:

$$if\quad C_{ij}\in "Low"\qquad then\quad x_{ij}=\min\{a_i,b_j\}$$

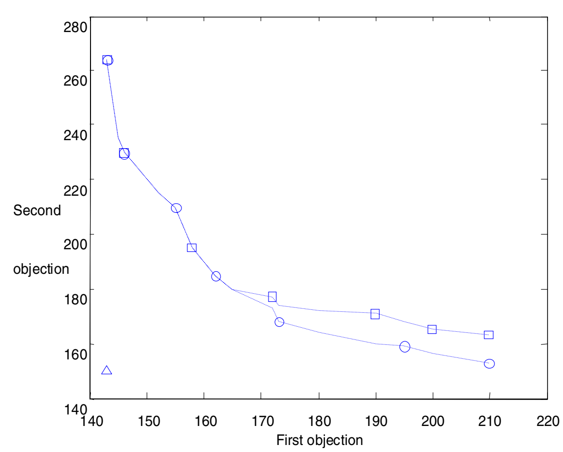
$$if\quad C_{ij}\in "Medium"\qquad then\quad x_{ij}=\frac{1}{2}\min\{a_i,b_j\}$$

$$if\quad C_{ij}\in "High"\qquad then\quad x_{ij}=\frac{1}{6}\min\{a_i,b_j\}$$

Then: $a_i=a_i-x_{ij}$, $b_j=b_j-x_{ij}$, Traversing unfinished transportation load according to the sequence—first row then line, then distributing it according to $\min\{a,b_j\}$. Obviously, the transportation load distributing method in literature [4] is above Fuzzy rule special case.

The Pareto front point sets educed by the ST-GA in literature [4] are $\{(143,265),(156,200),(176,175),(202,173)\}$, but the Fuzzy-GA of this paper educes not only the Pareto front point based on ST-GA, but also more Pareto front point sets, $\{(147,245),(160,195),(164,190),(172,185),(186,171)\}$, especially, the optimum solution $(186,171)$ educed by Fuzzy-GA is obviously better then the $(202,173)$ educed by ST-GA. Therefore, Fuzzy-GA is better than ST-GA.

The following chart shows Pareto front which is given birth by Pareto optimal solution:



The point "box" in this chart is the point set on Pareto front is given by ST-GA; the point "lap" is the point set on Pareto front is given by Fuzzy-GA; "Triangle" is the ideal point. It is obviously that Fuzzy-GA method is superior to ST-GA in solving Pareto front.

The second example is 8 factories and 9 warehouses, supply and demand are:

$a=(10 \quad 8 \quad 12 \quad 16 \quad 21 \quad 15 \quad 7 \quad 9)$

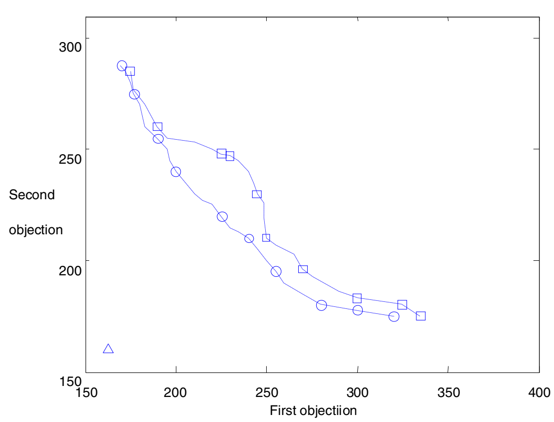
$b=(9 \quad 7 \quad 15 \quad 10 \quad 13 \quad 16 \quad 7 \quad 10 \quad 11)$,

Effectiveness unit matrix:

$$C^1_{8\times 9}=\begin{pmatrix} 1 & 2 & 7 & 7 & 8 & 10 & 9 & 2 & 5 \\ 1 & 9 & 3 & 4 & 3 & 5 & 7 & 1 & 1 \\ 8 & 9 & 4 & 6 & 4 & 1 & 6 & 9 & 2 \\ 2 & 4 & 5 & 5 & 3 & 2 & 3 & 2 & 9 \\ 5 & 4 & 5 & 1 & 9 & 9 & 1 & 6 & 1 \\ 8 & 3 & 3 & 2 & 2 & 3 & 6 & 7 & 6 \\ 1 & 2 & 6 & 4 & 5 & 9 & 3 & 5 & 2 \\ 13 & 3 & 3 & 5 & 1 & 5 & 6 & 3 & 2 \end{pmatrix}$$

$$C^2_{8\times 9}=\begin{pmatrix} 4 & 4 & 3 & 4 & 5 & 8 & 9 & 10 & 2 \\ 6 & 2 & 5 & 1 & 7 & 14 & 12 & 4 & 4 \\ 2 & 9 & 1 & 8 & 9 & 1 & 4 & 0 & 1 \\ 3 & 5 & 5 & 3 & 2 & 8 & 3 & 3 & 2 \\ 1 & 4 & 12 & 2 & 1 & 5 & 4 & 9 & 1 \\ 2 & 23 & 4 & 4 & 6 & 2 & 4 & 6 & 7 \\ 1 & 2 & 1 & 9 & 0 & 13 & 2 & 3 & 2 \\ 14 & 3 & 4 & 2 & 1 & 8 & 5 & 3 & 1 \end{pmatrix}$$

Parameter setting of Fuzzy-GA: pop_size=100, max_gen=500, run=20. The following chart illustrates the Pareto front which is given birth by Pareto optimal solution:



The point "box" in this chart is the point set on Pareto front is given by ST-GA; the point "lap" is the point set on Pareto front is given by Fuzzy-GA; "Triangle" is the ideal point. It is obviously that Fuzzy-GA method is superior to ST-GA in solving Pareto front.

IV. Conclusion

Fuzzy-GA algorithm improves the ST-GA method from Pareto optimal set and Fuzzy rules. From the results, the last result has distinct advantage, especially for the large-scale transportation optimization problem. For example, the situation of 8×9 in second example that shows the characteristics of the new algorithm that it finds not only better Pareto front, but also better Pareto solution. Fuzzy rules also reflect the unique advantages in resolving the multi-objective optimization problems. Explicit knowledge can be very easily expressed by Fuzzy rules and merged into the GA, so that the GA search has a knowledge orientation which greatly reduced the blindness of all searching by random search. How to more deeply discuss the intelligent effect of Fuzzy rules in the transportation multi-objective optimization problem and how to improve the efficiency of GA are the follow-up work.

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