Active Control of Space Flexible-Joint/Flexible-Link Manipulator

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***Abstract*—In this paper, active control of space manipulator with flexible-link and flexible-joint was investigated based on the singular perturbation method. Owing to the combined effects of the link and joint flexibilities, the dynamics model of this type of manipulator became more complex and led to a series of unsolved control system. To simplify the design of control system, singular perturbation method was exploited to obtain the two-time-scale simpler subsystem and composite control methods were designed to realize precise trajectory tracking and vibration suppression simultaneously of one-link and one-joint flexible manipulator with payload. In the slow subsystem, a fuzzy sliding-mode controller was designed to decrease the influences of external disturbance and parameters uncertainties, the system stability and asymptotic trajectory tracking performance were guaranteed by Lyapunov function; while a linear-quadratic controller was designed to suppress the vibration in the fast subsystem. The performances of proposed controller were demonstrated by the simulation results.**

***Keywords*—Space Flexible Manipulator, Flexible-joint/Flexible-link, singular perturbation, sliding-mode control, fuzzy system**.

I. INTRODUCTION

Over the past decades, the modeling and control of space flexible manipulators have been the challenging research topics. As the development of space technology, the research of space flexible manipulator has made a significant progress[1]. However, the characters of large-scale and lightweight of space manipulator link and the needs for a high speed, high payload to weight ratio for these flexible manipulators pose control system lots of the challenging problems. Furthermore, space manipulator actuators are usually driven by harmonic reducer which brings joint flexibility and lead to a series of nonlinear problems such as friction, hysteresis, backlash and resonances[2]. Thus, realizing precise trajectory tracking and vibration suppression of link and joint simultaneously by joint actuators brings great challenge for the design of control system.

There have been quite a number of studies dealing with stabilization and tracking control of flexible manipulators. In Ref[3], a dynamic state feedback controller is used to achieve robust regulation of the rigid modes as well as suppression of elastic vibrations. However, unknown disturbances included high-frequency modes generated by the nonlinearities of the plant and large parameter uncertainties arising from tip mass changes have greatly degraded the performances of control. In Ref.[4], an adaptive controller for the vibration suppression is addressed to guarantee the stability of the system in the presence of model uncertainty. In Ref.[3], a nonlinear adaptive and robust controllers were designed for two-link flexible arm. In Ref.[5], the H_∞ controller was designed for the purpose of incorporating robustness and also for attenuating the disturbances, and the experimental results suggested that the control scheme was more robust to uncertainties. In Ref.[6], a composite controller is designed based on singular perturbation model for one-link flexible manipulators, where a new adaptive sliding mode controller with robust tracking performance is designed for the slow subsystem, the adaptive algorithm is used to estimate the unknown perturbation part of system parameters, the proposed controller not only can attenuate effectively the effect of system uncertainties on tracking error, but also can suppress tip vibration. At present, intelligent control methods are used in the control of flexible manipulator more and more. In Ref.[7], a fuzzy composed controller with a nonlinear compensatory and a PD controller is proposed based on the dynamic model of a planar two-link flexible manipulator and the control method is verified by computer simulation. In Ref.[8], a fuzzy logic controller in the feedback configuration is proposed, and an efficient dynamic recurrent neural network in the feedforward configuration is developed, ang get remarkable performance for the control of two-link flexible manipulator system.

However, most research in these literatures have only concentrated on the control of flexible link and omitted the flexibility of joint. The flexibility of joint brings additory degree of freedom and makes the dynamic model more complicated. In Ref.[9], a dynamic modeling of an N-flexible-link and N-flexible-joint robot was reported, each flexible joint is modeled as a linearly elastic torsional spring and the method of assumed modes was adopted to describe the deformation of the link. Reference [10] take harmonic as a linearly rotational spring model and set up the dynamics model of flexible-joint/flexible-link, then a decoupled output feedback sliding mode control is used to servo the flexible manipulator and suppress residual vibrations. In harmonic driven actuator, the nonlinearities problems including friction, backlash and hysteresis always exist in practice. In Ref.[11], three-layer neural network is used to approximate the unknown plant function, then design a backstepping and variable structure controller to provide robustness to all the uncertainties.

In this paper, the singular perturbation technique is used to reduce the full order dynamic model of the flexible-link and flexible-joint manipulator into two reduced order systems including one slow rigid subsystem and one fast subsystem. The joint angular was taken as the slow state variables and the generalized flexible coordinates and the joint flexibility were viewed as the fast subsystem variables. Then, composite control method was adopted to realize precise trajectory tracking and vibration suppression simultaneously for this two-time-scale subsystem. A fuzzy sliding mode controller is adopted to decrease the influences of external disturbance and parameters uncertainties in the slow subsystem for the existence of high frequency mode and the model uncertainty, the system stability and asymptotic convergence are guaranteed by Lyapunov function; while a linear-quadratic controller is designed to suppress the vibration in the fast subsystem. The performances of proposed controller are demonstrated by the simulation results.

This paper is organized as follow: The dynamics of flexible-joint/flexible-link manipulator is built in Section Ⅱ. Then, singular perturbation method is realized in Section Ⅲ. Fuzzy sliding-mode control for slow subsystem and LQR control for fast subsystem are realized in Section Ⅳ. Simulation results and analysis are presented in Section Ⅴ. Finally, conclusions are driven in Section VI.

II. DYNAMICS OF FLEXIBLE MANIPULATOR

In this paper, an experimental flexible manipulator setup was fabricated and its schematic diagram was shown as Fig.1, The flexible joint is driven by Yaskawa SGMAH-02A AC servo motor and a harmonic reduction, an aluminum beam clamped to the output shaft of harmonic. Strain gauge sensors embedded on the boot of beam and an acceleration meter attached on tip of beam were used to measure vibration of beam. A shaft encoder of motor is used to measure the angular speed of rotation of motor and another MicroE rotary encoder is used to measure the rotate angle of joint output shaft. Trajectory tracking and vibration suppression of flexible manipulator are realized simultaneously by joint actuator. The dimensions and the mechanical properties of beam are given in Table 1.

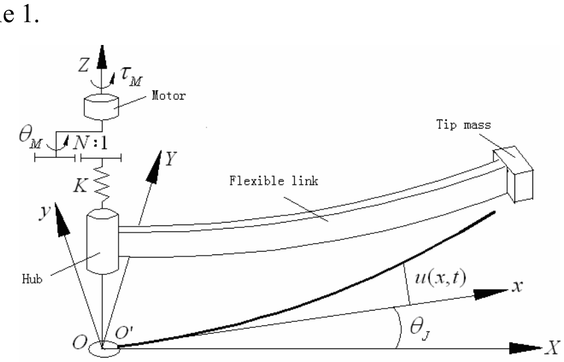


Figure 1. Schematic diagram of Flexible Manipulator

TABLE I. MECHANICAL PROPERTIES OF MANIPULATOR

Properties	Values
Length of beam, L	0.7 m
Thickness, h	0.004 m
Width, b	0.03 m
Linear density, ρ	2.7×10^{3} Kg/m^{3}
Modulus of elasticity, EI	11.36 N • m^{2}
Tip mass, m_p	0.2 Kg
Motor inertia, J_M	0.106×10^{-4}Kg • m^{2}
Joint inertia, J_J	0.42×10^{-4}Kg • m^{2}
Stiffness of harmonic, K	7.5×10^{3} Nm/rad
Reduction ratio of harmonic, N	100:1

The flexible manipulator is assumed to operate on the horizontal plane so that the effect of gravity is ignored, and the flexible beam is modeled as a spring-pinned beam attached to the rotating hub driven by harmonic. The beam deflection satisfies the Euler-Bernoulli beam theory and the assumed modes method can be used. The deflection of a point located at a distance x along the beam can be expressed as

$$u(x,t)=\sum_{i=1}^{n}\phi_i(x)\bullet q_i(t) \tag{1}$$

Where $u(x,t)$ is the deflection, $\phi_i(x)$ is the mode shape function, $q_i(t)$ is the time varying modal coordinate, n is the number of finite modes.

In Fig.1, r denotes the position vector of a point x on the manipulator with respect to the Cartesian coordinate and can be written as

$$r(x,t)=\{x\cos\theta_J-u\sin\theta_J,x\sin\theta_J+u\cos\theta_J\} \tag{2}$$

Where θ_J is the output angle of joint and we can get

$$\dot{r}(x,t)=\{-x\sin\theta_J-u\cos\theta_J,x\cos\theta_J-u\sin\theta_J\} \tag{3}$$

The total kinetic energy of the flexible manipulator system with the motion of motor, joint and link associated tip mass can be written as

$$T=\frac{1}{2}J_M\dot{\theta}_M^2+\frac{1}{2}J_J\dot{\theta}_J^2+\frac{1}{2}\rho A\int_0^{L_1}\dot{r}^2(x,t)dx+\frac{1}{2}m_p\dot{r}^2(L,t) \tag{4}$$

The potential energy due to the elastic deflection of link and the joint spring can be written as

$$V=\frac{1}{2}K(\theta_J-\frac{\theta_M}{N})^2+\frac{1}{2}\int_0^{L}EI[\frac{\partial^2u(x,t)}{\partial x^2}]^2dx \tag{5}$$

The boundary conditions of spring-pinned constraint and clamped-mass are used to get the mode shapes. Then, the dynamic equations of the system can be derived using Lagrange equation. The dynamic equation with joint flexibility and link flexibility can be expressed as[12]

$$\begin{cases} J_M\ddot{\theta}_M+K\cdot\delta/N=\tau_M \\ \mathbf{M(\theta,q)}\begin{bmatrix}\ddot{\theta}_J\\ \ddot{\mathbf{q}}\end{bmatrix}+\begin{bmatrix}\mathbf{g_1(\theta,\dot{\theta},q,\dot{q})}\\ \mathbf{g_2(\theta,\dot{\theta},q,\dot{q})}\end{bmatrix}+\begin{bmatrix}-\dfrac{K\cdot\delta}{N}\\ \mathbf{K_w}\cdot\mathbf{q}\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}\end{cases} \tag{6}$$

Where the inertia matrix $\mathbf{M(\theta,q)}$ is

$$\mathbf{M}=\begin{bmatrix} I+\rho A\int_0^L \phi_i^2(x)dxq_i^2+m(L^2+\phi_i(L)\phi_j(L)q_i) & \rho A\int_0^L x\phi_1(x)dx & \cdot & \rho A\int_0^L x\phi_n(x)dx+m_pL\phi_n(L) \\ \rho A\int_0^L x\phi_1(x)dx+m_pL\phi_1(L) & \rho A+m_p\phi_1^2(L) & \cdot & m_p\phi_1(L)\phi_n(L) \\ \cdot & \cdot & \cdot & \cdot \\ \rho A\int_0^L x\phi_n(x)dx+m_pL\phi_n(L) & m_p\phi_n(L)\phi_1(L) & \cdot & m_p\phi_n^2(L) \end{bmatrix}$$

Where $I = J_J + \rho A L^3/3$, $\delta = \frac{\theta_M}{N} - \theta_J$, the stiffness matrix $\mathbf{K_w}$ is written as $\mathbf{K_w} = diag(k_1, k_2, \cdots k_n)$,and $k_i = \omega_i^2$.

$$g_1 = 2\rho A\int_0^L \phi_i^2(x)dxq_i\dot{q}_i\dot{\theta}_J + m_p\phi_i(L)\phi_j(L)(\dot{q}_iq_i + q_i\dot{q}_i)\dot{\theta}_J$$

$$g_2 = \begin{bmatrix} -\dot{\theta}^2\rho A\int_0^L \phi_1^2 dx - m_p\dot{\theta}^2(\phi_1^2(L)q_1 + \phi_1(L)\phi_2(L)q_2 + \cdots + \phi_1(L)\phi_n(L)q_n) \\ -\dot{\theta}^2\rho A\int_0^L \phi_2^2 dx - m_p\dot{\theta}^2(\phi_1(L)\phi_2(L)q_1 + \phi_2^2(L)q_2 + \cdots + \phi_2(L)\phi_n(L)q_n) \\ \cdot \\ -\dot{\theta}^2\rho A\int_0^L \phi_n^2 dx - m_p\dot{\theta}^2(\phi_n(L)\phi_1(L)q_1 + \phi_n(L)\phi_2(L)q_2 + \cdots + \phi_n^2(L)q_n) \end{bmatrix}$$

In order to explain in brief, the dynamic equation can be rewritten as

$$\begin{cases} J_M\ddot{\theta}_M + K\cdot\delta/N = \tau_M \\ \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}\begin{bmatrix} \ddot{\theta}_J \\ \ddot{q} \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} + \begin{bmatrix} -\frac{K\cdot\delta}{N} \\ K_w\cdot q \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{cases} \tag{7}$$

III. SINGULAR PERTURBATION MODEL

The dynamic model can be transformed into a two-time-scale singular perturbation model as described below. Define a matrix $\mathbf{H}$.

$$\mathbf{H}(\theta_J, q) = \mathbf{M}^{-1}(\theta_J, q) = \begin{bmatrix} H_{11}(\theta_J, q) & H_{12}(\theta_J, q) \\ H_{21}(\theta_J, q) & H_{22}(\theta_J, q) \end{bmatrix} \tag{8}$$

Form the equation (6), we can get

$$\ddot{\theta}_J = -H_{11}(\theta_J, q)g_1(\theta_J, \dot{\theta}_J, q, \dot{q}) - H_{12}(\theta_J, q)g_2(\theta_J, \dot{\theta}_J, q, \dot{q}) + H_{11}(\theta_J, q)K\cdot\delta - H_{12}(\theta_J, q)K_w\cdot q \tag{9}$$

$$\ddot{q} = -H_{21}(\theta_J, q)g_1(\theta_J, \dot{\theta}_J, q, \dot{q}) - H_{22}(\theta_J, q)g_2(\theta_J, \dot{\theta}_J, q, \dot{q}) + H_{21}(\theta_J, q)K\cdot\delta - H_{22}(\theta_J, q)K_w\cdot q \tag{10}$$

$$\ddot{\delta} = \ddot{\theta}_M - \ddot{\theta}_J = \frac{1}{N}\cdot J_M^{-1}(\tau_M - \frac{K\cdot\delta}{N}) - \ddot{\theta}_J \tag{11}$$

Define a common scale factor k_c as the minimum of all the stiffness, $k_c = \min(K, k_1, k_2, \cdots, k_n)$ [13], define $\mu = 1/k_c$,the joint torsional spring stiffness K and flexible beam stiffness K_w can be scaled by k_c , so that $\tilde{K} = \mu\cdot K$, $\tilde{K}_w = \mu\cdot K_w$, and define $\tau_q = k_c\cdot q$, $\tau_\delta = k_c\cdot\delta$, and substitute $q = \mu\cdot\tau_q$, $\delta = \mu\cdot\tau_\delta$ into the equation (9),(10), we can get

$$\ddot{\theta} = -H_{11}(\theta_J, \mu\tau_q)g_1(\theta_J, \dot{\theta}_J, \mu\tau_q, \mu\dot{\tau}_q) - H_{12}(\theta_J, \mu\tau_q)g_2(\theta_J, \dot{\theta}_J, \mu\tau_q, \mu\dot{\tau}_q) - H_{12}(\theta_J, \mu\tau_q)\tilde{K}_w\tau_q + H_{11}(\theta_J, \mu\tau_q)\tilde{K}\tau_\delta \tag{12}$$

$$\mu\ddot{\tau}_q = -H_{21}(\theta_J, \mu\tau_q)g_1(\theta_J, \dot{\theta}_J, \mu\tau_q, \mu\dot{\tau}_q) - H_{22}(\theta_J, \mu\tau_q)g_2(\theta_J, \dot{\theta}_J, \mu\tau_q, \mu\dot{\tau}_q) - H_{22}(\theta_J, \mu\tau_q)\tilde{K}_w\tau_q + H_{21}(\theta_J, \mu\tau_q)\tilde{K}\tau_\delta \tag{13}$$

$$\mu\ddot{\tau}_\delta = \frac{1}{N}\cdot J_M^{-1}\cdot\tau_M - \frac{1}{N^2}\cdot J_M^{-1}\cdot\tilde{K}\cdot\tau_\delta - \ddot{\theta}_J \tag{14}$$

On setting $\mu \to 0$, we can get the rigid motion equation form (13), (14), when $\mu = 0$, we can get

$$\bar{\tau}_\delta = N\tilde{K}^{-1}\bar{\tau}_M - N^2\tilde{K}^{-1}J_M\ddot{\bar{\theta}}_J \tag{15}$$

$$\bar{\tau}_q = \tilde{K}_w^{-1}H_{22}^{-1}(\theta_J, 0)H_{21}(\theta_J, 0)\cdot(N\bar{\tau}_M - N^2J_M\ddot{\bar{\theta}}_J) \tag{16}$$

And note that:

$$M_{11}^{-1}(\bar{\theta}_J, 0) = H_{11}(\bar{\theta}_J, 0) - H_{12}(\bar{\theta}_J, 0)\cdot H_{22}^{-1}(\bar{\theta}_J, 0)H_{21}(\bar{\theta}_J, 0)$$

The slow subsystem in the state-space form can be written as

$$\ddot{\bar{\theta}}_J = M_{11}^{-1}(\theta_J, 0)\cdot(N\tau_M - N^2J_M\ddot{\bar{\theta}}_J) \tag{17}$$

This can be reduced as

$$\ddot{\bar{\theta}}_J = (M_{11} + N^2J_M)^{-1}\cdot N\tau_M \tag{18}$$

Where superscript "¯" indicate the value of the variable at $\mu = 0$. In order to get the state-space equation, the following state variables are defined[14]:

$x_1 = \theta_J$, $x_2 = \dot{\theta}_J$, $z_1 = \tau_q$, $z_2 = \varepsilon\dot{\tau}_q$, $y_1 = \tau_\delta$, $y_2 = \varepsilon\dot{\tau}_\delta$, $\varepsilon = \sqrt{\mu}$.

Based on the above state variables, the state-space equation of singular perturbation modal can be expressed as:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -H_{11}(x_1, \varepsilon^2 z_1)g_1(x_1, x_2, \varepsilon^2 z_1, \varepsilon z_2) - H_{12}(x_1, \varepsilon^2 z_1)g_2(x_1, x_2, \varepsilon^2 z_1, \varepsilon z_2) \\ &\quad - H_{12}(x_1, \varepsilon^2 z_1)\tilde{K}_w z_1 + H_{11}(x_1, \varepsilon^2 z_1)\tilde{K}y_1 \end{aligned} \tag{19}$$

$$\begin{aligned} \varepsilon\dot{z}_1 &= z_2 \\ \varepsilon\dot{z}_2 &= -H_{21}(x_1, \varepsilon^2 z_1)g_1(x_1, x_2, \varepsilon^2 z_1, \varepsilon z_2) - H_{22}(x_1, \varepsilon^2 z_1)g_2(x_1, x_2, \varepsilon^2 z_1, \varepsilon z_2) \\ &\quad - H_{22}(x_1, \varepsilon^2 z_1)\tilde{K}_w z_1 + H_{21}(x_1, \varepsilon^2 z_1)\tilde{K}y_1 \end{aligned} \tag{20}$$

$$\varepsilon \dot{y}_1 = y_2$$

$$\varepsilon \dot{y}_2 = -\frac{1}{N^2} J_M^{-1} \tilde{K} y_1 + \frac{1}{N} J_M^{-1} \tau_M - \dot{x}_2 \quad (21)$$

On setting $\varepsilon = 0$ in the equation (18), we can get the state-space equation of the slow subsystem

$$\begin{cases} \dot{\bar{x}}_1 = \bar{x}_2 \\ \dot{\bar{x}}_2 = (M_{11} + N^2 J_M)^{-1} \cdot N \bar{\tau}_M \end{cases} \quad (22)$$

In order to get the fast subsystem, a fast time scale $t_f = t/\varepsilon$ is introduced. At the boundary layer $\varepsilon = 0$, $dx_1/dt_f = 0$, $dx_2/dt_f = 0$, $g_1(x_1, x_2, 0, 0) = 0$, $g_2(x_1, x_2, 0, 0) = 0$, So the slow variables can be viewed as constant, Hence in the fast time scale, define new fast variables as[15]

$$\eta_{q1} = z_1 - \bar{\tau}_q,\ \eta_{q2} = z_2,\ \eta_{\delta 1} = y_1 - \bar{\tau}_\delta,\ \eta_{\delta 2} = y_2$$

Based on the above fast time scale variables, the fast subsystem state-space equation of singular perturbation modal can be determined as

$$\begin{cases} \eta_{q1}/dt_f = \eta_{q2} \\ \eta_{q2}/dt_f = -H_{22}(\bar{x}_1, 0)\tilde{K}_w \eta_{q1} + H_{21}(\bar{x}_1, 0)\tilde{K}\eta_{\delta 1} \\ \eta_{\delta 1}/dt_f = \eta_{\delta 2} \\ \eta_{\delta 2}/dt_f = -\frac{1}{N^2} J_M^{-1}\tilde{K}\eta_{\delta 1} + \frac{1}{N} J_M^{-1}(\tau_M - \bar{\tau}_M) \end{cases} \quad (23)$$

On setting $\tau_f = \tau_M - \bar{\tau}_M$ as the fast input torque variable, equation (21) can be rewritten in brief

$$\mathbf{\dot{x}_f} = \mathbf{A_f x_f} + \mathbf{B_f \tau_f} \quad (24)$$

Where: $\mathbf{A_f} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -H_{22}\tilde{K}_w & 0 & H_{21}\tilde{K} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -J_M^{-1}\tilde{K}/N^2 & 0 \end{bmatrix}$, $\mathbf{B_f} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{N} J_M^{-1} \end{bmatrix}$

IV. CONTROL SYSTEM DESIGN FOR MANIPULATOR

Based on two time-scale decomposition, the design of controller of the flexible manipulator can be decomposed into the control of the angle tracking of the slow subsystem and the vibration suppressing of fast subsystem.

Because of the unmodelled high frequency modes and the model uncertainty in the slow subsystem, the good performances can not always be guaranteed. The variable structure control strategy using the sliding-mode has very strong robustness to extern disturbances and parameter perturbations, and in order to make certain the upper bound of uncertainties in sliding mode control system, a fuzzy sliding-mode control schemes was designed to estimate the uncertainties upper bound by judging the sliding surface.

A. Fuzzy Sliding-mode Control for Slow Subsystem

In order to achieve excellent performance of position trajectory tracking with the unknown parameters and external disturbances, fuzzy sliding mode controller is designed. Define $e_J = \theta_J - \theta_J^*$ and $\dot{e}_J = \dot{\theta}_J - \dot{\theta}_J^*$, θ_J^* and $\dot{\theta}_J^*$ are desired joint angle and joint velocity, and assume that θ_J^*, $\dot{\theta}_J^*$ are continuous and bounded, the sliding-mode surface is defined as

$$s = \dot{e}_J + \lambda e_J \quad (25)$$

Where λ is positive constant and $\lambda > 0$. The controller is designed to drive dynamic variables to the manifold $s = 0$ and constrain them there, and the following sliding-mode control law is introduced

$$\bar{\tau}_M = \frac{1}{N}\left(M_{11} + N^2 J_M\right)\left(\ddot{\theta}_J^* - \lambda(\dot{\theta}_J - \dot{\theta}_J^*) - \hat{\rho}\,\mathrm{sgn}(s)\right) \quad (26)$$

Where the estimated gain $\hat{\rho} = |E| + \xi$, $|E|$ is the assumed uncertainties upper bound, and ξ is a positive constant. Define Lyapunov function as

$$V = \frac{1}{2} s^2 \quad (27)$$

Then: $\dot{V} = s\dot{s} = s(\ddot{e}_J + \lambda \dot{e}_J)$

$$= s((M_{11} + N^2 J_M)^{-1} \cdot N\bar{\tau}_M - \ddot{\theta}_J^* + \lambda \dot{e}_J + |E|) \quad (28)$$

Substitute control law (26) into (28), we can get

$$\dot{V} = s\left(-\hat{\rho}\,\mathrm{sgn}(s) + E(t)\right) \le -\xi |s| \quad (29)$$

In order to suppress the effect of chattering in practical system, a fuzzy control system is designed to estimate the upper bound of the uncertainties in sliding mode control. Proposed fuzzy system takes s and $\dot{s}$ as input variable and $\Delta\rho$ is the output variable, which is described in Fig.2. Define $\hat{\rho}$ as the fuzzy estimation value of upper bound of uncertainties, then, $\hat{\rho} = \rho + \Delta\rho$.

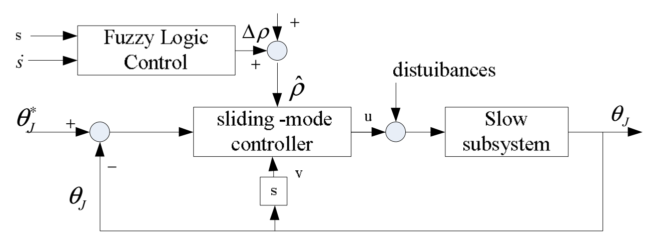


Figure 2. fuzzy sliding-mode control system

The linguistic input variable and output variable are defined as follow

$$s = \{P, Z, N\};\ \dot{s} = \{P, Z, N\};$$
$$\Delta\rho = \{NB, NM, NS, Z, PS, PM, PB\};$$

Where:

N=negative, Z=zero, P=positive
NB=negative big, NM=negative medium, NS=negative small
PS=positive small, PM=positive medium, PB=positive big

The universe of discourse of s is assigned as [-2, 2], $\dot{s}$ is [-4, 4], output $\Delta\rho$ is [-2, 2]. The triangular type input and output membership functions are used and described in Fig.3.

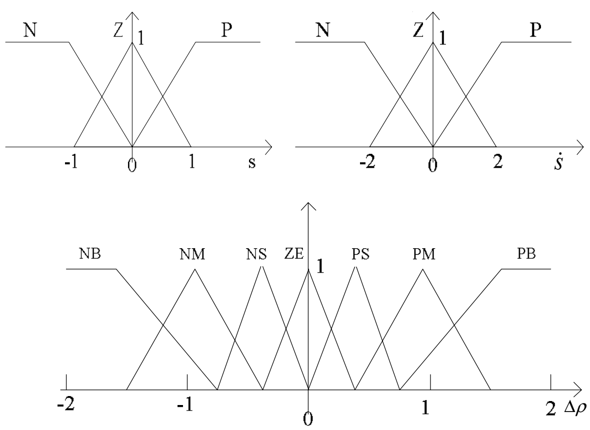


Figure 3. Membership functions of s, $\dot{s}$ and $\Delta\rho$

In sliding-mode control system, the reaching condition of sliding surface is $s\dot{s} < 0$, if $s\dot{s}$ is positive, ρ should be enlarged to overcome the effectiveness of uncertainties and drive the system states to the surface as soon as possible; If $s\dot{s}$ is negative, the reaching condition is satisfied, in order to reduce the magnitude of chattering besides of sliding surface, ρ can be decreased. Therefore, the switch gain can be described as follows:

R1: IF s is P and $\dot{s}$ is P THEN $\Delta\rho$ is NB;

R2: IF s is P and $\dot{s}$ is Z THEN $\Delta\rho$ is NM;

R3: IF s is P and $\dot{s}$ is N THEN $\Delta\rho$ is NS;

R4: IF s is Z and $\dot{s}$ is P THEN $\Delta\rho$ is NS;

R5: IF s is Z and $\dot{s}$ is Z THEN $\Delta\rho$ is ZE;

R6: IF s is Z and $\dot{s}$ is N THEN $\Delta\rho$ is PS;

R7: IF s is N and $\dot{s}$ is P THEN $\Delta\rho$ is PS;

R8: IF s is N and $\dot{s}$ is Z THEN $\Delta\rho$ is PM;

R9: IF s is N and $\dot{s}$ is N THEN $\Delta\rho$ is PB;

The fuzzy output $\Delta\rho$ can be calculated by the centre of area defuzzy method, described as follow

$$\Delta\rho = \sum_{i=1}^{9} x_i \mu(i) / \sum_{i=1}^{9} \mu(i) \tag{25}$$

Where: $\Delta\rho$ is output of fuzzy system, $\mu(i)$ is the fuzzy membership function, x_i is the centre of the membership function.

B. LQR Control for Fast Subsystem

In the fast subsystem, (A_f, B_f) is found to be completely state controllable, therefore, a fast state feedback control can be designed[6].

$$\tau_f = -K_{opt} x_f(t) = -R_f^{-1} B_f^T P x_f(t) \tag{26}$$

Where the feedback gains K_{opt} can be obtained through an LQR approach, the cost function given below

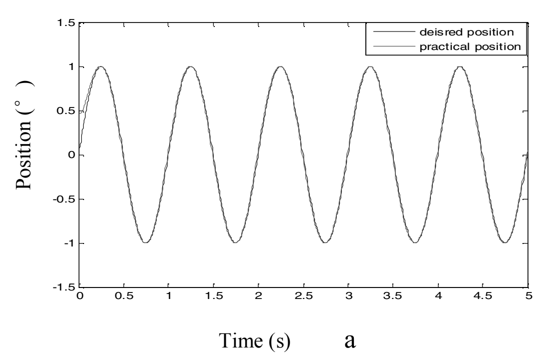
$$J_f = \frac{1}{2}\int_0^{\infty} \left[x_f^T(t) Q x_f(t) + u_f^T(t) R u_f(t) \right] dt \tag{27}$$

V. SIMULATION RESULTS AND ANALYSIS

The proposed controller was used to a manipulator with one flexible-joint and one flexible-link. Only two assumed modes is considered in the vibration suppress of flexible-link, and the mechanical properties of manipulator are described in table 1.

Presume the desired position trajectory $\theta_J^* = 2\sin(2\pi F t)$, $F = 1.0Hz$, and $\lambda = 70$, $\rho = 2$, unknown uncertainties $E(t) = 20\sin(t)$. The simulations are realized by sliding-mode method and fuzzy sliding-mode method, results of period sinusoidal trajectory tracking and tracking errors are depicted in Fig.4.a, b, c. From the simulated results, we can find that the sliding-mode method can track sinusoidal trajectory precisely with maximum 0.05° position error at the influence of uncertainties. Therefore, sliding-mode control system can track period sinusoidal precisely since the bound of lumped uncertainties is large enough to get rid of all kinds of influences of uncertainties. The result of fuzzy sliding-mode for trajectory tracking show that it degenerate the maximum position error to 0.015°, and the chattering of control output torque is reduced, which is described in Fig.5.

In the fast subsystem, selecting LQR matrix $Q = (100,10,10,10)$ and $R_f = 100$, then the gains $K_{opt} = (-0.7089, -0.418, 6.3735, 0.3369)$. The first assumed modes are described in Fig.6.



a

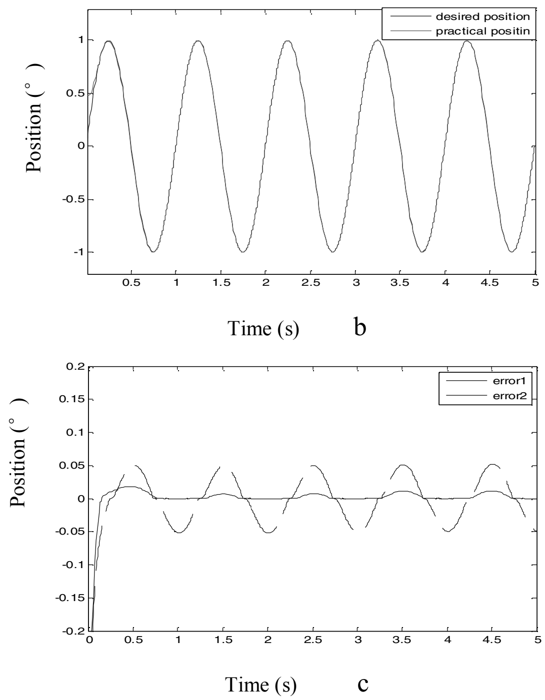


Figure 4. simulated response of sinusoidal tracking

a. tracking response of trajectory of sliding-mode controller;
b. tracking trajectory of fuzzy sliding-mode controller;
c. position errors for two controller.

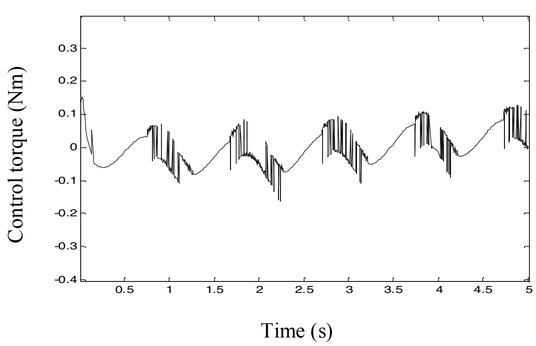


Figure 5. Control torque of fuzzy sliding-mode system

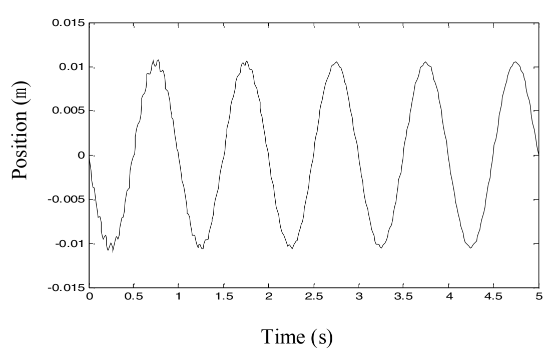


Figure 6. The first assumed modes

V. VI. CONCLUSION

The dynamic equation of space manipulator with one flexible-joint and one flexible-link has been derived by Euler-Lagrange principle and the assumed modes method. A composite controller is used to transfer complex one flexible-joint and one flexible-link manipulator into a slow-subsystem and a fast subsystem by singular perturbation approach. To achieve excellent performance of position trajectory tracking with the unknown parameters and external disturbances, a kind of sliding mode controller is designed and fuzzy system is used to suppress the affection of chattering; while a linear-quadratic controller was designed to suppress the vibration of flexible-link in the fast subsystem. The simulation results demonstrated that the proposed composite controller can track position trajectory precisely and suppress vibration of flexible-link simultaneously using flexible-joint actuators.

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