Fuzzy Control of Inverter Pendulum Robot via Perturbed Time-Delay Affine T-S Fuzzy Models

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***Abstract*—This paper presents a robust fuzzy control method, combining affine Takagi–Sugeno (T–S) fuzzy models and H_∞ performance constraints, for stability analysis and controller design of an inverted pendulum robot to simulate human stance. The inverted pendulum robot proposes that it is beneficial for the stance leg to behave like a pendulum. As mentioned above, in this paper, the T–S fuzzy model is used to describe a nonlinear inverted pendulum robot system with perturbations and time delays. Moreover, some sufficient conditions are derived on robust H_∞ disturbance attenuation in which both robust stability and a prescribed performance are achieved, simultaneously. Finally, a numerical simulation for the nonlinear inverted pendulum robot system is given to show the applications of the present design approach.**

***Keywords*—Takagi–Sugeno fuzzy models, time delay, inverted pendulum robot.**

I. INTRODUCTION

Human being stance has been investigated in detail for a long time [1]. In recent years, the researchers wish to simulate human stance on the machine. In this paper, the model is constructed based on purely inverted pendulum dynamics and on a movable supportive base. This work was based on the assumption that the act of maintaining an erect posture could be viewed. However, the problems often are a complicated nonlinear system. In general, the methods of linear control and those of local linearization and moving linearization are not well suited for the control problem of inverted pendulums. This is due to the fact that inverted pendulums constantly move among widely separated regions of their workspace such that no linearization valid for all regions can be found. In many practical systems, the system plants contain severe nonlinear properties.

Recently, some authors proposed several control methods to control the nonlinear systems by using Takagi–Sugeno (T–S) fuzzy system models [2–9]. The affine T–S fuzzy system means the fuzzy system of which consequent part is affined and which has a constant bias term. It is well known that such models can describe or approximate a wide class of nonlinear systems. Hence, it is important to study their stability and the design of stabilizing controllers. Besides, robust stability also has been considered in literatures which have presented robust stability analysis and methods for designing robust fuzzy controllers to stabilize a class of uncertain fuzzy systems. In general, stability analysis and synthesis can be extended to the time-delay systems. Time delays often appear in industrial systems and information networks. Thus, it is also important to analyze time-delay effects for the affine T–S fuzzy systems.

In this paper, for inverted pendulum robot, we consider robust stability and stabilization of uncertain affine T–S fuzzy systems with state delays where uncertainties come into the state and input matrices. Moreover, the H_∞ control scheme [9] is used in this paper to attack the problem of robust performance design problems for the perturbed affine T–S fuzzy models. It can provide the guaranteed H_∞ performance for the attenuation γ, which can cope with the worst case effect of disturbance on system states. The majority of T-S fuzzy controller design was developed by using the concept of Parallel Distributed Compensation (PDC) [2–9] and the Lyapunov stability criterion. Based on the Linear Matrix Inequalities (LMI) technique [2], one can find a suitable common positive definite matrix for the stability conditions, and then to obtain a stable fuzzy controller for the closed-loop T–S fuzzy models. However, the fuzzy controller design of the affine T–S fuzzy models is a challenging problem for the designers because the closed-loop stability conditions are not LMI formulations but Bilinear Matrix Inequalities (BMI) ones. The BMI conditions cannot be solved via a convex optimization algorithm. In this paper, an Iterative LMI (ILMI) algorithm [7, 9] is employed to solve the BMI formulations for the proposed control problem.

II. THE DYNAMIC MODEL OF SIMPLE HUMAN STANCE SYSTEM

In this section, the mathematical model of the simple inverted pendulum robot system is introduced. Referring to Fig. 1, a simplified dynamic model for describing inverted pendulum robot system to simulate human stance is proposed as follows [10].

$$x_1(k+1) = x_1(k) + T x_2(k) + e\,v(k)$$
$$x_2(k+1) = x_2(k) + \frac{T}{M + m(\sin x_3(k))^2}$$
$$\times\left(u(k) + ml\,x_4^2(k)\sin x_3(k) - b x_2 - mg\cos x_3(k)\sin x_3(k)\right)$$
$$x_3(k+1) = x_3(k) + T x_4(k)$$
$$x_4(k+1) = x_4(k) + \frac{T}{l\left(M + m(\sin x_3(k))^2\right)}$$
$$\times\left((M+m)g\sin x_3(k) - u(k)\cos x_3(k) + b x_2(k)\cos x_3(k)\right.$$
$$\left. - ml\,x_4^2(k)\sin x_3(k)\cos x_3(k)\right) \quad (1)$$

where

- m is the mass of the black on the pendulum.
- l is length of the pendulum.
- g is acceleration due to gravity.
- b is coefficient of viscous friction for motion of the cart.
- u is applied force.
- $v(t)$ is the denotes the disturbances.

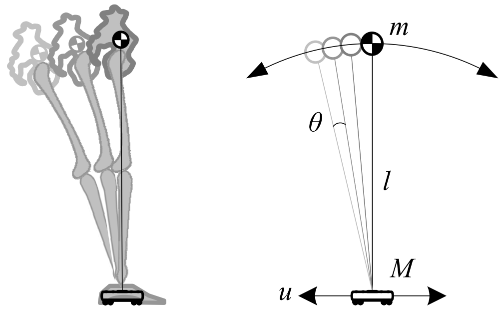

Fig. 1 Inverted pendulum robot system to simulate human stance

The four state variables stand for $x_1 = x$, $x_2 = \dot{x}$, $x_2 = \theta$, $x_4 = \dot{\theta}$ with x the position of the cart, and θ the angle the pendulum makes with vertical. This model is obtained by discretizing the continuous time model via Euler's method with T is 0.1s, $b = 12.98\,\mathrm{kg}/\mathrm{s}$, $M = 1.378$ kg, $l = 0.325$ m, $g = 9.8\,\mathrm{m}/\mathrm{s}^2$, $m = 0.051$ kg.

Considering premise nominal parameter uncertainties, the modified dynamic model for the inverted pendulum robot system can be described as follows:

$$x_1(k+1) = (0.1\cos(t) + 1)\sigma(k) + 0.1 x_2(k) + 0.2 v(k)$$
$$x_2(k+1) = x_2(k) + \frac{0.1}{1.378 + 0.051(\sin x_3(k))^2}$$
$$\times\left(u(k) - 12.98 x_2 + 0.0166 x_4^2(k)\sin x_3(k)\right.$$
$$\left. - 0.4998\cos x_3(k)\sin x_3(k)\right)$$
$$x_3(k+1) = x_3(k) + 0.1 x_4(k)$$
$$x_4(k+1) = x_4(k)$$
$$+ \frac{0.1}{0.4478\left(0.0166(\sin x_3(k))^2\right)} \times\left(13.4162\sin x_3(k)\right.$$
$$- u(k)\cos x_3(k) + 12.98 x_2(k)\cos x_3(k)$$
$$\left. - 0.0166 x_4^2(k)\sin x_3(k)\cos x_3(k)\right) \quad (2)$$

where

$$\sigma(k) = \rho x_1(k) + (1-\rho)x_1(k-\tau)$$

The $\sigma(k)$ is a time-delay function.

III. The Affine T-S Fuzzy Models

The affine T–S fuzzy model of the inverted pendulum robot system can be obtained by applying Jaccobian linearization technique [11]. Given a pair of $(x(t),\ u(t))$, the final outputs of the perturbed time-delay affine T–S fuzzy model of inverted pendulum robot system (2) are inferred as follows:

$$x(k+1) = \sum_{i=1}^{r} h_i(z(k))\left\{(\mathbf{A}_i + \Delta\mathbf{A}_i)x(k)\right.$$
$$+ (\mathbf{A}_{id} + \Delta\mathbf{A}_{id})x(k-\tau) + (\mathbf{B}_i + \Delta\mathbf{B}_i)u(k)$$
$$\left. + (\mathbf{a}_i + \Delta\mathbf{a}_i)\right\} + \mathbf{E}\,v(k) \quad (3)$$

where

$$h_i(z(k)) \geq 0 \text{ and } \sum_{i=1}^{r} h_i(z(k)) = 1 \quad (4)$$

The quantities $\mathbf{A}_i$, $\mathbf{A}_{id}$, $\mathbf{B}_i$, $\mathbf{a}_i$ and $\mathbf{E}$ are constant matrices. Besides, $\Delta\mathbf{A}_i$, $\Delta\mathbf{A}_{id}$, $\Delta\mathbf{B}_i$ and $\Delta\mathbf{a}_i$ are time-varying matrices with appropriate dimensions and they are structured in the following norm-bounded form:

$$\begin{bmatrix} \Delta\mathbf{A}_i & \Delta\mathbf{A}_{id} \\ \Delta\mathbf{B}_i & \Delta\mathbf{a}_i \end{bmatrix} = \mathbf{D}_i\Delta_i(t)\begin{bmatrix} \mathbf{Q}_{1i} & \mathbf{Q}_{2i} \\ \mathbf{Q}_{3i} & \mathbf{Q}_{4i} \end{bmatrix} \quad (5)$$

where $\mathbf{D}_i$, $\mathbf{Q}_{1i}$, $\mathbf{Q}_{2i}$, $\mathbf{Q}_{3i}$ and $\mathbf{Q}_{4i}$ are known real constant matrices of appropriate dimensions, and $\boldsymbol{\Delta}_i(t)$ is an unknown matrix function with Lebesgue-measurable elements and satisfies $\boldsymbol{\Delta}_i^T(t)\boldsymbol{\Delta}_i(t) \leq \mathbf{I}$.

For a nonlinear T-S fuzzy system represented by (3), a fuzzy controller is designed to share the same fuzzy sets with the plant. It is based on the PDC concept [2-9]. The output of the PDC-based fuzzy controller is determined by the summation such as

$$u(k) = -\sum_{i=1}^{r} h_i(z(k))\{\mathbf{F}_i\,x(k)\} \quad (6)$$

Substituting (6) into (3), one can obtain corresponding closed-loop system as follows:

$$x(k+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(k))h_j(z(k))$$

$$\times\Big\{\big(\mathbf{H}_{1ij} + \mathbf{D}\Delta(k)\overline{\mathbf{H}}_{1ij}\big)x(k) + \big(\mathbf{H}_{2ij} + \mathbf{D}\Delta(k)\overline{\mathbf{H}}_{2ij}\big)x(k-\tau)$$

$$+\big(\mathbf{H}_{3ij} + \mathbf{D}\Delta(k)\overline{\mathbf{H}}_{3ij}\big)\Big\} + \mathbf{E}v(k) \tag{7}$$

where

$$\mathbf{H}_{1ij} = \frac{\mathbf{G}_{ij} + \mathbf{G}_{ji}}{2},\ \mathbf{H}_{2ij} = \frac{\mathbf{A}_{id} + \mathbf{A}_{jd}}{2},\ \mathbf{H}_{3ij} = \frac{\mathbf{a}_i + \mathbf{a}_j}{2},$$

$$\overline{\mathbf{H}}_{1ij} = \frac{\overline{\mathbf{G}}_{ij} + \overline{\mathbf{G}}_{ji}}{2},\ \overline{\mathbf{H}}_{2ij} = \frac{\mathbf{Q}_{2i} + \mathbf{Q}_{2j}}{2},\ \overline{\mathbf{H}}_{3ij} = \frac{\mathbf{Q}_{4i} + \mathbf{Q}_{4j}}{2},$$

$$\mathbf{G}_{ij} = \mathbf{A}_i - \mathbf{B}_i\mathbf{F}_j \text{ and } \overline{\mathbf{G}}_{ij} = \mathbf{Q}_{1i} - \mathbf{Q}_{3i}\mathbf{F}_j \tag{8}$$

Base on the PDC type fuzzy controller (6), a sufficient condition for ensuring delay-independent stability of controlled time-delay affine T-S fuzzy model (7) is introduced in this paper. Moreover, a H_∞ control performance with $\gamma > 0$ is also considered in this paper. This constraint is of the following form.

$$\sum_{k=0}^{k_f} x^T(k)\mathbf{S}x(k) < \gamma^2 \sum_{k=0}^{k_f} v^T(k)v(k),\ \forall v(k) \neq 0 \tag{9}$$

with zero initial condition for all $v(k) \in L_2[0,\ k_f]$, where k_f is the terminal time of the control, γ is a prescribed value which denotes the worst case effect of $v(k)$ on $x(k)$. Besides, $\mathbf{S} = \mathbf{S}^T > 0$ is a positive definite weighting matrix.

IV. Sufficient Conditions Of Robust Fuzzy Controller Design

Based on the PDC scheme, a fuzzy controller is designed to share the same fuzzy sets with the affine T-S fuzzy model (3). In this section, the delay-independent stability conditions for the affine T–S fuzzy model (7) are described in the following theorem. Note that the proofs of the theorems in this paper are all omitted due to space limit.

Theorem 1

Given a H_∞ attenuation parameter $\gamma > 0$. The affine T–S fuzzy model described in (7) is quadratically stable in the large and the H_∞ control performance (9) is guaranteed for an attenuation γ, if there exist positive definite matrices $\mathbf{P} > 0$, $\mathbf{S} > 0$, $\mathbf{P}_d > 0$, control gains $\mathbf{F}_i$ and scalars $\xi_{ijq} \geq 0$ such that

$$\Upsilon_{ij} < 0 \qquad \text{for } i \in \hat{I}_0 \tag{10}$$

and

$$\overline{\Upsilon}_{ij} - \sum_{q=1}^{n} \xi_{ijq}\mathbf{\Omega}_{ijq} < 0 \qquad \text{for } i \in \hat{I}_1 \tag{11}$$

where

$$\Upsilon_{ij} \triangleq \left\{ \boldsymbol{\varphi} + \begin{bmatrix} \mathbf{H}_{ij}^{\ T} \\ \mathbf{H}_{2ij}^{\ T} \\ \mathbf{E}^T \end{bmatrix} \mathbf{PD}\big(\varepsilon I - \mathbf{D}^T\mathbf{PD}\big)^{-1}\mathbf{D}^T\mathbf{P}\begin{bmatrix} \mathbf{H}_{ij} & \mathbf{H}_{2ij} & \mathbf{E} \end{bmatrix} \right\}$$

$$\boldsymbol{\varphi} = \begin{bmatrix} \mathbf{H}_{1ij}^T\mathbf{PH}_{1ij} - \mathbf{P} + \mathbf{P}_d + \mathbf{S} + \overline{\mathbf{H}}_{1ij}^T\varepsilon\overline{\mathbf{H}}_{1ij} & * & * \\ \mathbf{H}_{2ij}^T\mathbf{PH}_{1ij} + \overline{\mathbf{H}}_{2ij}^T\varepsilon\overline{\mathbf{H}}_{1ij} & \mathbf{H}_{2ij}^T\mathbf{PH}_{2ij} - \mathbf{P}_d + \overline{\mathbf{H}}_{2ij}^T\varepsilon\overline{\mathbf{H}}_{2ij} & * \\ \mathbf{E}^T\mathbf{PH}_{1ij} & \mathbf{E}^T\mathbf{PH}_{2ij} & \mathbf{E}^T\mathbf{PE} - \gamma^2\mathbf{I} \end{bmatrix}$$

$$\overline{\Upsilon}_{ij} \triangleq \overline{\boldsymbol{\varphi}}$$

$$+\left\{ \begin{bmatrix} \mathbf{H}_{ij}^{\ T} \\ \mathbf{H}_{2ij}^{\ T} \\ \mathbf{E}^T \\ \mathbf{H}_{3ij}^{\ T} \end{bmatrix} \mathbf{PD}\big(\varepsilon I - \mathbf{D}^T\mathbf{PD}\big)^{-1}\mathbf{D}^T\mathbf{P}\begin{bmatrix} \mathbf{H}_{ij} & \mathbf{H}_{2ij} & \mathbf{E} & \mathbf{H}_{3ij} \end{bmatrix} \right\}$$

$$\overline{\boldsymbol{\varphi}}_{ij} \triangleq \begin{bmatrix} \boldsymbol{\varphi}_{ij} & * \\ \mathbf{H}_{3ij}^T\mathbf{P}\begin{bmatrix} \mathbf{H}_{1ij} & \mathbf{H}_{2ij} & \mathbf{E} \end{bmatrix} + \overline{\mathbf{H}}_{3ij}^T\varepsilon\begin{bmatrix} \overline{\mathbf{H}}_{1ij} & \overline{\mathbf{H}}_{2ij} & 0 \end{bmatrix} & \mathbf{H}_{3ij}^T\mathbf{PH}_{3ij} + \overline{\mathbf{H}}_{3ij}^T\varepsilon\overline{\mathbf{H}}_{3ij} \end{bmatrix} \tag{12}$$

The stability conditions in Theorem 1 are BMI forms that cannot be solved by LMI technique. Therefore, the ILMI algorithm is applied to solve the BMI condition in next section.

V. Robust Fuzzy Controller Design Via Ilmi Algorithm

In order to solve BMI condition of Theorem1, some modified conditions are derived in this section. The detail formulations are introduced in the following theorem.

Theorem 2

Given a H_∞ attenuation parameter $\gamma > 0$ and the auxiliary constant matrix $\mathbf{R} > 0$. The conditions of Theorem 1 are satisfied if there exist $\alpha < 1$, positive definite matrices $\mathbf{P} > 0$, $\mathbf{S} > 0$, $\mathbf{P}_d > 0$, control gains $\mathbf{F}_i$ and scalars $\xi_{ijq} \geq 0$ such that

$$\begin{cases}\mathbf{\Theta}_{ij} < 0 \\ \mathbf{R}^T\mathbf{P}\mathbf{R} - \mathbf{R} \le 0\end{cases} \quad \text{for } i \in \hat{I}_0 \tag{13}$$

and

$$\begin{cases}\overline{\mathbf{\Theta}}_{ij} < 0 \\ \mathbf{R}^T\mathbf{P}\mathbf{R} - \mathbf{R} \le 0\end{cases} \quad \text{for } i \in \hat{I}_1 \tag{14}$$

where

$$\mathbf{\Theta}_{ij} \triangleq \begin{bmatrix} -\alpha\mathbf{P}+\mathbf{P}_d+\mathbf{S} & * & * & * & * & * \\ \mathbf{H}_{2ij}^T\mathbf{R}^{-1}\mathbf{H}_{1ij}+\overline{\mathbf{H}}_{2ij}^T\varepsilon\overline{\mathbf{H}}_{1ij} & \mathbf{H}_{2ij}^T\mathbf{R}^{-1}\mathbf{H}_{2ij}-\mathbf{P}_d+\overline{\mathbf{H}}_{2ij}^T\varepsilon\overline{\mathbf{H}}_{2ij} & * & * & * & * \\ \mathbf{E}^T\mathbf{R}^{-1}\mathbf{H}_{1ij} & \mathbf{E}^T\mathbf{R}^{-1}\mathbf{H}_{2ij} & \mathbf{E}^T\mathbf{R}^{-1}\mathbf{E}-\gamma^2\mathbf{I} & * & * & * \\ \mathbf{D}^T\mathbf{R}^{-1}\mathbf{H}_{ij} & \mathbf{D}^T\mathbf{R}^{-1}\mathbf{H}_{2ij} & \mathbf{D}^T\mathbf{R}^{-1}\mathbf{E} & -\varepsilon\mathrm{I}+\mathbf{D}^T\mathbf{R}^{-1}\mathbf{D} & * & * \\ \overline{\mathbf{H}}_{1ij} & 0 & 0 & 0 & -\varepsilon^{-1} & * \\ \mathbf{H}_{1ij} & 0 & 0 & 0 & 0 & -\mathbf{R} \end{bmatrix} \tag{15}$$

$$\overline{\mathbf{\Theta}}_{ij} \triangleq \begin{bmatrix} -\alpha\mathbf{P}+\mathbf{P}_d+\mathbf{S}-\xi_{ijq}\mathbf{T}_{ijq} & * & * & * & * & * & * \\ \mathbf{H}_{2ij}^T\mathbf{R}^{-1}\mathbf{H}_{1ij}+\overline{\mathbf{H}}_{2ij}^T\varepsilon\overline{\mathbf{H}}_{1ij} & \mathbf{H}_{2ij}^T\mathbf{R}^{-1}\mathbf{H}_{2ij}-\mathbf{P}_d+\overline{\mathbf{H}}_{2ij}^T\varepsilon\overline{\mathbf{H}}_{2ij} & * & * & * & * & * \\ \mathbf{E}^T\mathbf{R}^{-1}\mathbf{H}_{1ij} & \mathbf{E}^T\mathbf{R}^{-1}\mathbf{H}_{2ij} & \mathbf{E}^T\mathbf{R}^{-1}\mathbf{E}-\gamma^2\mathbf{I} & * & * & * & * \\ \mathbf{H}_{3ij}^T\mathbf{R}^{-1}\mathbf{H}_{1ij}+\overline{\mathbf{H}}_{3ij}^T\varepsilon\overline{\mathbf{H}}_{1ij}-\xi_{ijq}\mathbf{n}^T & \mathbf{H}_{3ij}^T\mathbf{R}^{-1}\mathbf{H}_{2ij}+\overline{\mathbf{H}}_{3ij}^T\varepsilon\overline{\mathbf{H}}_{2ij} & \mathbf{H}_{3ij}^T\mathbf{R}^{-1}\mathbf{E} & \mathbf{H}_{3ij}^T\mathbf{R}^{-1}\mathbf{H}_{3ij}+\overline{\mathbf{H}}_{3ij}^T\varepsilon\overline{\mathbf{H}}_{3ij}-\xi_{ijq}\mathbf{v}_{ij} & * & * & * \\ \mathbf{D}^T\mathbf{R}^{-1}\mathbf{H}_{ij} & \mathbf{D}^T\mathbf{R}^{-1}\mathbf{H}_{2ij} & \mathbf{D}^T\mathbf{R}^{-1}\mathbf{E} & \mathbf{D}^T\mathbf{R}^{-1}\mathbf{H}_{3ij} & -\varepsilon\mathrm{I}+\mathbf{D}^T\mathbf{R}^{-1}\mathbf{D} & * & * \\ \overline{\mathbf{H}}_{1ij} & 0 & 0 & 0 & 0 & -\varepsilon^{-1} & * \\ \mathbf{H}_{1ij} & 0 & 0 & 0 & 0 & 0 & -\mathbf{R} \end{bmatrix} \tag{16}$$

□

Note that the conditions of Theorem 2 are of modified LMI forms which can be solved by using ILMI algorithm [7, 9].

VI. Numerical Simulations

According to the results develop in previous section, this section provides a numerical simulation for the inverted pendulum robot system in order to show the applications of the present fuzzy controller design approach. Considering the inverted pendulum robot system (2) with Fig. 1, one can choose three operating points to obtain the linearized models for the system (2). Let us choose three operating points as follows:

$$\left(x^+,\ x_d^+,\ u^+\right)_{oper1} = \left(0 \quad 0 \quad 88^\circ \quad 0 \,\middle|\, 0 \quad 0 \quad 0^\circ \quad 0 \,\middle|\, 0\right),$$

$$\left(x,\ x_d,\ u\right)_{oper2} = \left(0 \quad 0 \quad 0^\circ \quad 0 \,\middle|\, 0 \quad 0 \quad 0^\circ \quad 0 \,\middle|\, 0\right),$$

$$\left(x^-,\ x_d^-,\ u^-\right)_{oper3} = \left(0 \quad 0 \quad -88^\circ \quad 0 \,\middle|\, 0 \quad 0 \quad 0^\circ \quad 0 \,\middle|\, 0\right) \tag{17}$$

Then, three linear subsystems can be constructed by these three operating points. In which, $\left(x,\ x_d,\ u\right)_{oper2}$ is the maintain equilibrium point and the others are the off-equilibrium points. Through the above three linear subsystems and defining membership functions as Fig. 2, one can obtain the time-delay affine T-S fuzzy model, which is composed by three fuzzy rules as follows:

Rule i: IF $x_3(k)$ is about M_{i1} THEN

$$x(k+1) = \left(\mathbf{A}_i + \Delta\mathbf{A}_i\right)x(k) + \left(\mathbf{A}_{id} + \Delta\mathbf{A}_{id}\right)x(k-\tau) + \mathbf{B}_i u(k) + \mathbf{a}_i + \mathbf{E}v(k), \quad i = 1\cdots 3 \tag{18}$$

where

$$\mathbf{A}_1 = \begin{bmatrix} 0.998 & 0.1 & 0 & 0 \\ 0 & 0.0917 & 0.035 & 0 \\ 0 & 0 & 1 & 0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{A}_2 = \begin{bmatrix} 0.998 & 0.1 & 0 & 0 \\ 0 & 0.0581 & -0.0363 & 0 \\ 0 & 0 & 1 & 0.1 \\ 0 & 2.8983 & 3.127 & 1 \end{bmatrix},$$

$$\mathbf{A}_3 = \begin{bmatrix} 0.998 & 0.1 & 0 & 0 \\ 0 & 0.0917 & 0.035 & 0.0464 \\ 0 & 0 & 1 & 0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\Delta\mathbf{A}_1 = \Delta\mathbf{A}_2 = \Delta\mathbf{A}_3 = \begin{bmatrix} 0.001\sin(t) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.003\cos(t) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{A}_{1d} = \mathbf{A}_{2d} = \mathbf{A}_{3d} = \begin{bmatrix} 0.002 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{a}_1 = \begin{bmatrix} 0 \\ 0.0549 \\ 0 \\ -3.0154 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} 0 \\ -0.0549 \\ 0 \\ 3.0154 \end{bmatrix},$$

$$B_1 = B_2 = B_3 = \begin{bmatrix} 0 \\ 0.0726 \\ 0 \\ -0.2233 \end{bmatrix}, \mathbf{E} = \begin{bmatrix} 0 \\ 0 \\ 0.2 \\ 0 \end{bmatrix}.$$

The corresponding matrices of *S*-procedure are presented as follows:

For ***Rule***$_s$ 11, i.e., $90^\circ \le x_3(k) \le 80^\circ$

$$\mathbf{T}_{111} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \ \mathbf{n}_{111} = \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{2}\left[(80+90)\pi/180\right] \\ 0 \end{bmatrix} \text{ and}$$

$$v_{111} = (90\pi/180)\times(80\pi/180) \quad (19)$$

For ***Rule***$_s$ 33, i.e., $-90^\circ \le x_3(k) \le -80^\circ$, the matrices of *S*-procedure are given as follows:

$$\mathbf{T}_{331} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \ \mathbf{n}_{331} = \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{2}\left[(-80-90)\pi/180\right] \\ 0 \end{bmatrix} \text{ and}$$

$$v_{331} = (-90\pi/180)\times(-80\pi/180) \quad (20)$$

For the above perturbed time-delay affine T-S fuzzy model (18), the fuzzy controller can be designed by applying Theorem 2 and the ILMI algorithm [7, 9]. In this example, it is assumed that the H_∞ control performance is guaranteed for an attenuation $\gamma^2 = 0.32$. Then, we can get a feasible solution after four iterations of the ILMI algorithm. The final decay rate α is 0.9999 and the feasible solutions are obtained as follows:

$$\mathbf{P} = \begin{bmatrix} 4.4530 & 2.0824 & 7.3099 & 1.4670 \\ 2.0824 & 3.3681 & 9.5516 & 1.8320 \\ 7.3099 & 9.5516 & 41.5488 & 7.8826 \\ 1.4670 & 1.8320 & 7.8826 & 1.5450 \end{bmatrix},$$

$$\mathbf{P}_d = \begin{bmatrix} 0.0327 & 0.0186 & 0.0430 & 0.0118 \\ 0.0186 & 0.0238 & 0.0366 & 0.0134 \\ 0.0430 & 0.0366 & 0.1136 & 0.0225 \\ 0.0118 & 0.0314 & 0.0225 & 0.0081 \end{bmatrix}$$

$$\mathbf{S} = \begin{bmatrix} 0.0076 & 0.0046 & 0.0107 & 0.0030 \\ 0.0046 & 0.0059 & 0.0092 & 0.0034 \\ 0.0107 & 0.0092 & 0.0284 & 0.0056 \\ 0.0030 & 0.0034 & 0.0056 & 0.0020 \end{bmatrix},$$

$$\mathbf{R} = \begin{bmatrix} 0.3419 & -0.1082 & 0.0601 & -0.5031 \\ -0.1082 & 0.8916 & -0.1470 & -0.2054 \\ 0.0601 & -0.1470 & 0.7670 & -3.7935 \\ -0.5031 & -0.2054 & -3.7935 & 20.7108 \end{bmatrix},$$

$$\xi_{111} = 186.4773, \ \xi_{331} = 186.4773 \quad (21)$$

And, the fuzzy controller has the following form:

Rule 1: IF $x_3(k)$ is about M_{11} THEN

$u(k) = -[-5.2530 \quad -1.0054 \quad -43.6819 \quad -9.2813]x(k)$

Rule 2: IF $x_3(k)$ is about M_{21} THEN

$u(k) = -[-5.1953 \quad -18.5910 \quad -49.8976 \quad -9.2234]x(k)$

Rule 3: IF $x_3(k)$ is about M_{31} THEN

$u(k) = -[-5.2530 \quad -1.0054 \quad -43.6819 \quad -9.2813]x(k)$ (22)

The disturbance input noise $v(k)$ is given with variance one. The simulation results are shown in Fig. 3 to Fig. 6. From the simulated results, one can find that the controlled nonlinear perturbed time-delay inverted pendulum robot system (2) is globally stable.

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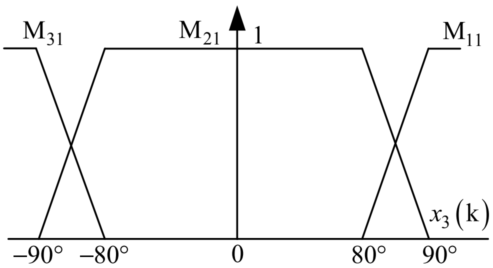

Fig. 2 Membership functions of $x_3(k)$

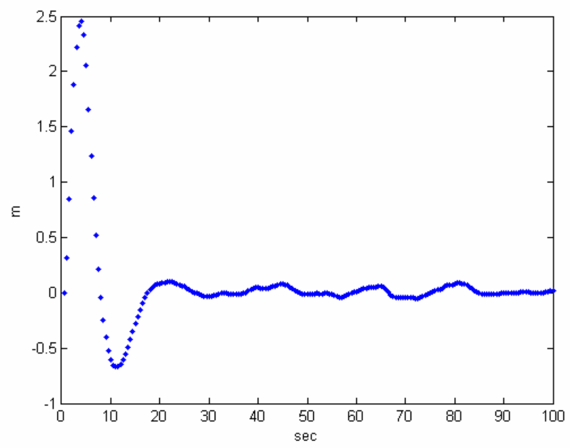

Fig. 3 Responses of $x_1(k)$

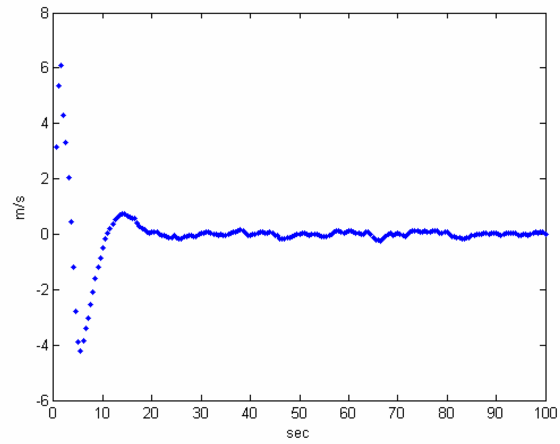

Fig. 4 Responses of $x_2(k)$

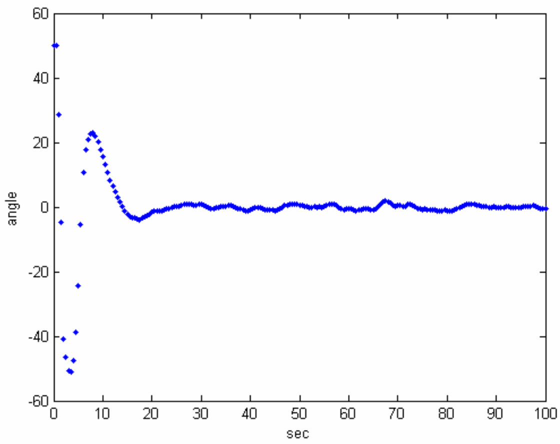

Fig. 5 Responses of $x_3(k)$

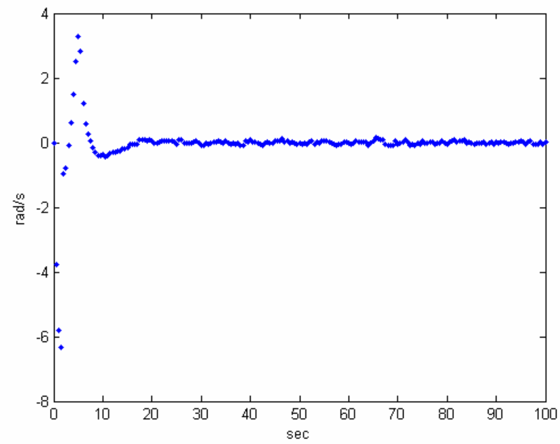

Fig. 6 Responses of $x_4(k)$