Study on Stability of Electric Power Steering System

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***Abstract*—The stability of the electric power steering system is analyzed on the basis of the mathematics model for pinion -and-rack steering system, the interference resistance is also studied by the way of fixing the steering wheel. The result shows that the stability of the system becomes poor gradually along with the increase of power ratio. The stability can be improved by PD control algorithm, but the influence on the system caused by the measure noise of torque sensor cannot be controlled. If PD combines with the H_∞ Optimal Control, the stability and interference resistance can be effectively improved.**

***Keywords*—Automobile; Electric Power Steering; Stability**

I. INTRODUCTION

With the development of the automobile electronic technology, the automobile steering system has developed from traditional mechanical steering, hydraulic power steering and electric-hydraulic power steering to the electric power steering. Due to convenient operation, energy saving, environment-friendly and many other advantages, the electric power steering has become the development current of the automobile power steering technology. As the safety and stability are very important for the electric power steering system, many scholars apply themselves to study how to improve the safety and stability of the electric power steering system. Masahiko Kurishige[1] proposes a new control strategy based on estimation of alignment torque generated by tires and road surfaces. This proposed control strategy requires no supplemental sensors like steering-wheel angle or motor-angle sensors. The experiment shows that it enables improved steering wheel returnability and also better on-center feeling. Mitsubishi [2] develops a method for controlling motor current based on the evaluation and compensation on the interference voltage, by which the current fluctuation of the motor and noise of the steering torque are obviously reduced in the case of using common microprocessor. Kurishiqe.Masahiko[3] proposes a Electric Power Steering control strategy based on steering angular velocity control for specified frequency using a newly developed steering shaft angular-velocity observer, which enables reduced steering torque without any perceived vibration for drivers from slow steering to rapid steering. Chen Yan[4] analyses the effect of introducing yaw angle rate negative response to EPS and built the man-vehicle mathematic model, simulation shows that vehicle's yaw angle rate response is improved by introducing yaw angle rate negative feed and the EPS powered torque response curve arises evenly and slowly, this is useful to vehicle driving at high speed. Mouri, Hiroshi [5] describes a new algorithm of an electric power assistant steering system control. It solved the problem at low damping of the transfer function of steering angle / steering torque at high speed. They solved the controller using a simple model, and in order to maintain the stability of the controller, they approximated a strict solution as a simple controller that is described as a first order non-minimum phase transfer function. Shen Rongwei [6] establishes the seven-degree-of-freedom mathematical model of the electric power steering system and analyzes the steering wheel vibration when aid-power ratio increase.

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II. MATHEMATICAL MODEL OF ELECTRIC POWER STEERING SYSTEM

For the electric power steering system adopting the steering shaft power structure, the output torque of the power motor is amplified and transmitted to the steering shaft. According to the conclusion of the analysis on the steering resistance and the rotation angle of the steering wheel, the relation between the rack and tire (floor) is simplified as the model in which the rack connects with the linear spring fixed at one end. The simplified model is shown as Fig 1, of which the twisting rod is respectively connected to the steering input shaft and steering output shaft, the torque sensor indirectly measures the torque of the steering wheel through measuring the twisted angle of the twisting rod.

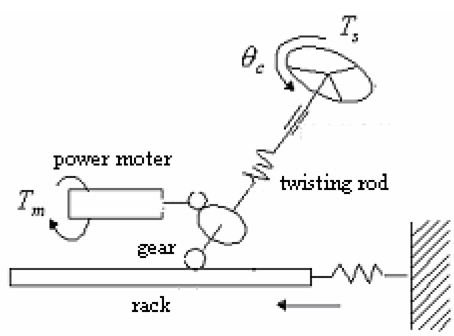

Figure 1. Simplified Model of Electric Power Steering System

The mathematical model of electric power steering system as follows:

$$J_c\ddot{\theta}_c = -b_c\dot{\theta}_c - k_c\theta_c + k_c x_r / r_p \cos\beta + T_s \quad (1)$$

$$
\begin{aligned}
m\ddot{x}_r &= k_c \cos\beta\theta_c / r_p - k_c x_r / r_p^2 - b_r \dot{x}_r \\
&+ k_m G \cos\beta\theta_m / r_p - k_m G^2 x_r / r_p^2 - k_t x_r
\end{aligned} \tag{2}
$$

$$
J_m \ddot{\theta}_m = -b_m \dot{\theta}_m - k_m \theta_m + k_m G x_r / r_p \cos\beta + T_m \tag{3}
$$

Where T_s indicates the input torque of steering wheel; J_c is the assembly rotary inertia of steering wheel and input shaft; k_c is the rigidity coefficient of twisting rod; b_c is the damping coefficient of steering input shaft; T_m is the input torque of power motor; k_m is the assembly rigidity coefficient of power motor armature shaft and reducer driving shaft; J_m is the equivalent rotary inertia of power motor armature shaft and reducer driving gear; b_m is the damping coefficient of power motor armature shaft and reducer driving shaft; m is the quality of rack; b_r is the viscous damping coefficient of rack and steering wheel; k_t is the equivalent spring rigidity; G is the reduction ratio of reducer; r_p is the radius of steering gear; θ_c is the rotation angle of steering shaft; $\dot{\theta}_c$ is the angular speed of steering shaft; x_r is the displacement of steering rack; $\dot{x}_r$ is the speed of rack; θ_m is the rotation angle of power armature shaft; $\dot{\theta}_m$ is the angular speed of power armature shaft; and β is the spiral angle of steering gear.

Equation (1) is the mathematical model of the steering wheel and steering input shaft assembly, (2) is for rack and (3) for power motor.

III. STABILITY ANALYSIS OF ELECTRIC POWER STEERING SYSTEM

A. *The Influence of Dynamic Property of Power Motor*

Under the perfect condition, the power torque of the electric power steering system will completely depend on the power characteristic. The relation between the power torque and steering torque of the steering wheel is linear. It can be seen from the simplified model shown in Fig 1 that the torque of the steering wheel is equivalent to the torque of the twisting rod of the torque sensor.

$$
T_s = k_c(\theta_c - x_r / r_p \cos\beta) \tag{4}
$$

$$
T_a = k_a T_s = k_a k_c (\theta_c - x_r / r_p \cos\beta) \tag{5}
$$

Where *Ta* expresses the power torque. The rack motion equation can be obtained from (2)

$$
m\ddot{x}_r = \frac{k_c \cos\beta}{r_p}\theta_c - \frac{k_c}{r_p^2} x_r - b_r \dot{x}_r - k_t x_r + \frac{T_a \cos\beta}{r_p} \tag{6}
$$

Take (5) into (6)

$$
\begin{aligned}
m\ddot{x}_r &= k_c \cos\beta\theta_c / r_p - k_c x_r / r_p^2 - b_r \dot{x}_r - k_t x_r \\
&- k_a k_c x_r / r_p^2 + k_a k_c \cos\beta\theta_c / r_p
\end{aligned} \tag{7}
$$

The transfer function from the rotation angle θ_c of the steering wheel to the rack displacement x_r can be obtained from (7)

$$
G_1(s) = \frac{(1+k_a)k_c \cos\beta / r_p}{ms^2 + b_r s + k_c / r_p^2 + k_a k_c / r_p^2 + k_t} \tag{8}
$$

As the rotation angle of the steering wheel is equal to the angle of the steering shaft, the transfer function $G_1(s)$ reflects the dynamic relation between the input rotation angle of the steering wheel and the rack displacement. Fig 2 shows the root locus diagram when the power ratio changes from 0 to infinity, it can be seen that the root locus is located at the left half of S plane, which means that the electric power steering system is stable under the perfect condition.

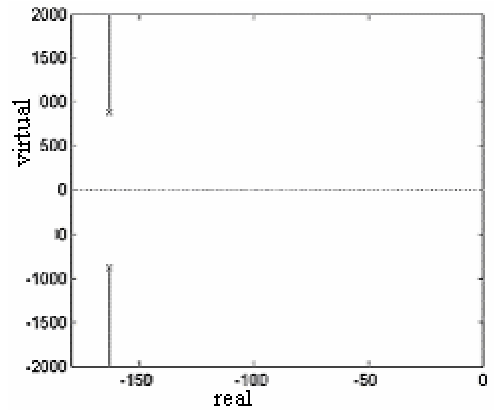

Figure 2. The root locus under the perfect condition

In view of the influence of the dynamic characteristic of the power motor, the actual value of the power torque of the electric power steering system is not completely determined by formula (5). It can be seen from the simplified model of the electric power steering system that the actual power torque is

$$
T_a = k_m G\theta_m - k_m G^2 x_r / r_p \cos\beta \tag{9}
$$

As the output torque of the motor can be controlled directly, regard the output torque as objective control. According to the assistance characteristic, output torque is determined as

$$
T_m = k_a T_s / G = k_a k_c (\theta_c - x_r / r_p \cos\beta) / G \tag{10}
$$

Take (9) into (6)

$$
\begin{aligned}
m\ddot{x}_r &= k_c \cos\beta\theta_c / r_p - [(k_c + k_m G^2) / r_p^2 + k_l\} x_r \\
&- b_r \dot{x}_r + k_m G \cos\beta\theta_m / r_p
\end{aligned} \tag{11}
$$

Take (10) into (3)

$$J_m \ddot{\theta}_m = -b_m \dot{\theta}_m - k_m \theta_m + (k_m G / r_p \cos\beta$$
$$- k_a k_c / G r_p \cos\beta) x_r + \frac{k_a k_c}{G} \theta_c \tag{12}$$

By (11) and (12), the transfer function from the rotation angle θ_c to the displacement x_r of the rack is got as following

$$G_2(s) = \frac{\dfrac{J_m k_c \cos\beta}{r_p} s^2 + \dfrac{b_m k_c \cos\beta}{r_p} s + \dfrac{k_m k_c (1+k_a)\cos\beta}{r_p}}{J_m m s^4 + (J_m b_r + b_m m) s^3 + \Delta_1 s^2 + \Delta_2 s + \Delta_3} \tag{13}$$

Where $\Delta_1 = J_m k_t + b_m b_r + k_m m + (J_m k_c + J_m k_m G) / r_p^2$

$$\Delta_2 = k_m b_r + b_m k_t + \frac{b_m k_c + b_m k_m G^2}{r_p^2}$$

$$\Delta_3 = \frac{(1+k_a) k_m k_c}{r_p^2} + k_m k_t$$

The transfer function $G_2(s)$ reflects the dynamic relation between the input rotation angle of the steering wheel and the rack displacement. The system root locus drawing in term of (13) is showed in Fig3. It can be seen that the poles are located at the virtual axes and the system is in the critical stable state when the power ratio is 0; along with increase of the power ratio, at the right of S plane, two root locus trend to the limitless zeros from the poles located at the original point, which means the system trends to unstable state.

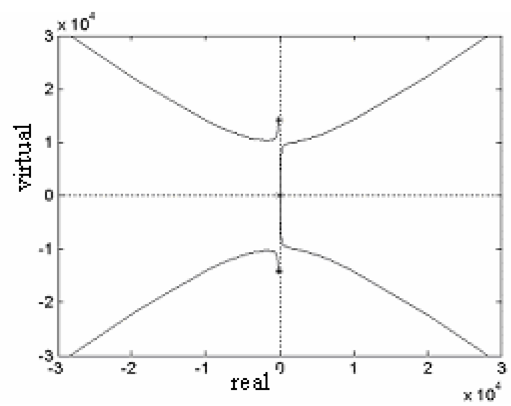

Figure 3. The root locus when the parameter k_a changing

It can be seen from the analyses above that when considering the influence of motor dynamic characteristic, the stability of the electric power steering system changes from the stable state to the critical stable state; along with increase of power ratio, the relative stability of the system will become poor gradually. Therefore, the power torque cannot be determined only by the assistance characteristic, the reasonable control strategy must be adopted.

B. The Influence of Torque sensor Measurement Noise and Road Interference

When steering, the resisting torque acting on the steering small gear transmits to the steering wheel through the steering shaft to enable the steering wheel to receive an anti-torque. The change of the steering wheel anti-torque directly reflects the driving sense, and study of the changes of the anti-torque will help to analyze the stability of the electric power steering system. The dynamic characteristic differential equation of the steering shaft when the steering wheel is fixed as follows:

$$J_c \ddot{\theta}_c = T_e - T_m - b_c \dot{\theta}_c - k_c \theta_c \tag{14}$$

$$T_s = -T_r = -k_c \theta_c \tag{15}$$

Where T_e expresses the steering resistance torque acting on the steering gear and T_r expresses the anti-torque of the steering wheel (other parameters as before). The relation between the power torque T_m and motor current I is shown as follows:

$$T_m = ikI \tag{16}$$

Where i and k is respectively the transmission ratio of reducer and the motor torque coefficient. Adopting the PD control algorithm, the relation between the motor current and angle of twisting rod is as follows:

$$I = k_p \theta_c \tag{17}$$

Where k_p ¡s the ratio coefficient, the transfer function between T_e and T_r is got from (14)-(17)

$$G(s) = k_c / [J_c s^2 + b_c s + (k_c + ikk_p)] \tag{18}$$

This system is the second-order system. Fig4 is the amplitude- frequency characteristic of the transfer function G (jω) when power ratio is 1. It can be seen from the amplitude-frequency characteristics that the gain max is at 61.2rad/s and the bandwidth of the system is about 95rad/s, which means the high frequency components from the road disturbance can be transmitted to the steering wheel and cause the steering wheel vibrate and destroy the stability of the system. It is not appropriate to determine the motor current only by the ratio coefficient kp,. As differential control can increase the damping of the system and improve the stability of the system, PD control algorithm is adopted.

$$I = k_p \theta_c + k_d \dot{\theta}_c \tag{19}$$

Where k_d is the differential coefficient. Fig5 is the amplitude -frequency characteristic when PD control algorithm is adopted. It can be seen from Fig5 that the gain is weakened at 61.2rad/s, and the bandwidth of the system is obviously decreased, so that the transmission of high frequency components of the torque on the steering small gear is inhibited and the influence from load disturbance is reduced.

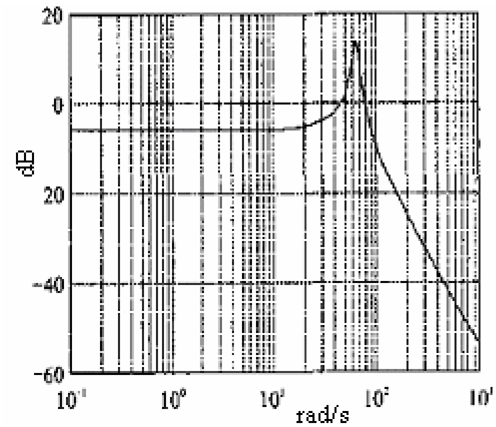

Figure 4. Amplitude- Frequency Characteristic

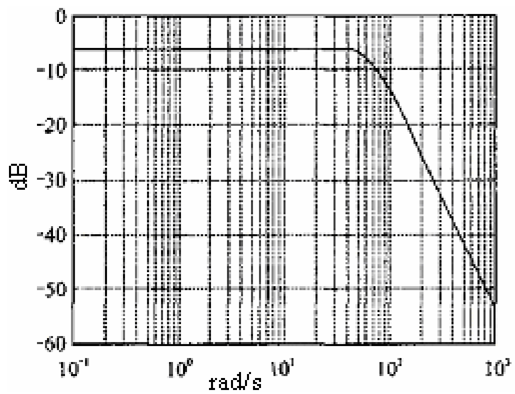

Figure 5. Amplitude -Frequency characteristic when PD control

Fig 6 indicates the time-domain response of twisting rod torque to measurement noise of the torque sensor when unit step-input. It can be seen that the transient response oscillation of twisting rod torque caused by unit step-input measurement noise of torque sensor is violence and the time of trending stability is very long, which means the measurement noise of torque sensor has a significant influence on the electric steering system.

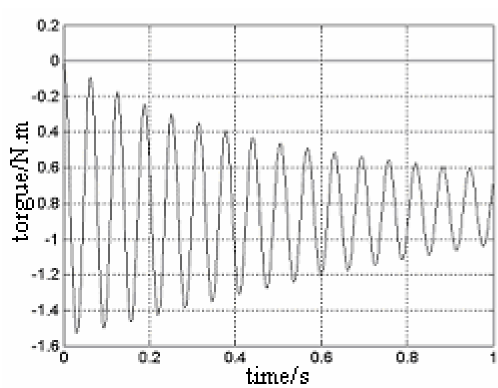

Figure 6. the time-domain response of twisting rod torque to measurement noise of the torque sensor

C. H∞ Controller Design Method Based on PD Control

The H_∞ Optimal Control, the effective method to inhibit the influence of the torque sensor measurement noise and the road disturbance is used for electric power steering system, which takes the H_∞ norm that controls the transfer functions of the system as the optimization target. Taking the measurement noise of steering sensor, road excitation disturbance, steering wheel torque as the system external input signals and the power torque deviation, road disturbance evaluating index as the controlled output signals (or the controlled vectors), The H_∞ controller is designed on the base of PD control. Fig7 is the schematic diagram adopting PD control algorithm and Fig8 is the H_∞ control schematic diagram.

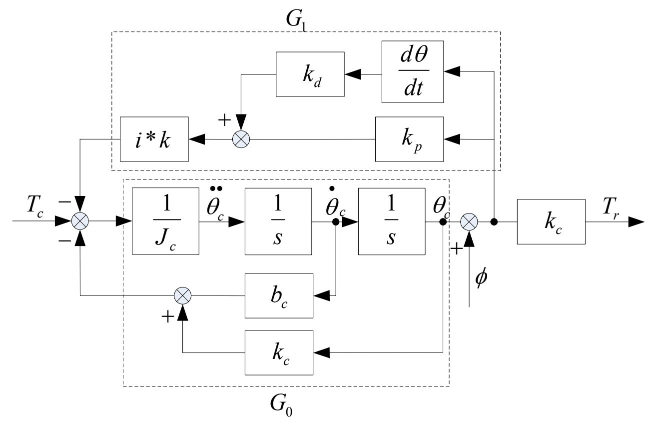

Figure 7. PD Control Schematic Diagram

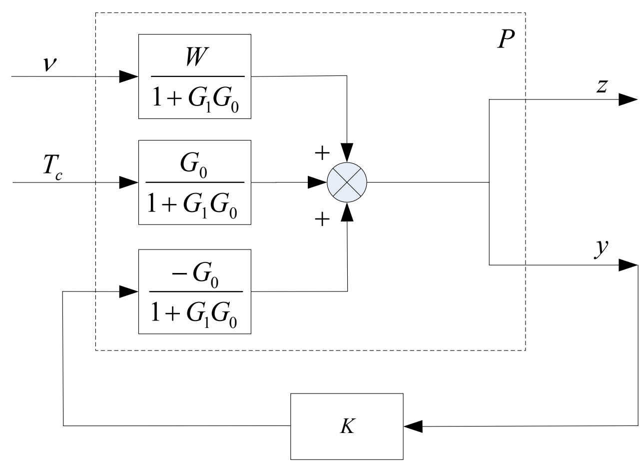

Figure 8. H_∞ Control Schematic Diagram

Take H_∞ norm of the transfer function from the torque sensor measurement noise and steering resisting torque to the torsion angle of the twisting rod as the optimization target of H_∞ controller, the optimal H_∞ controller can be designed in virtual of the tools of MATLAB, which based on the method of solving linear matrix inequality.

Fig.9 indicates the time domain response of the steering wheel anti-torque to the unit impulse of road excitation when adopting controller. It can be seen that the vibration of the steering wheel anti-torque is obviously weakened and the adjusting time is obviously shortened, which means the road disturbance is effectively inhibited. Generally speaking, inhibiting the vibration of the steering wheel anti-torque can effectively weaken the vibration of the steering wheel and improve the comfortableness of the steering.

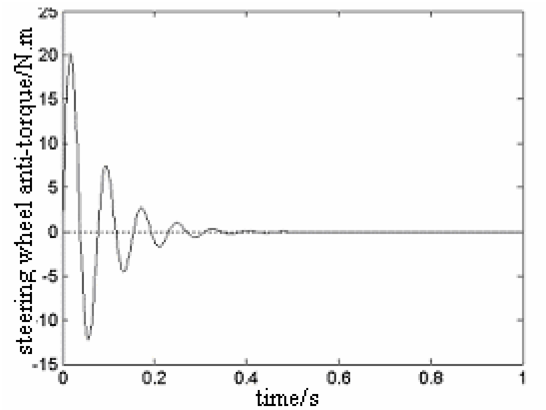

Figure 9. The time domain response of the steering wheel anti-torque to the unit impulse of road excitation when adopting controller K(s)

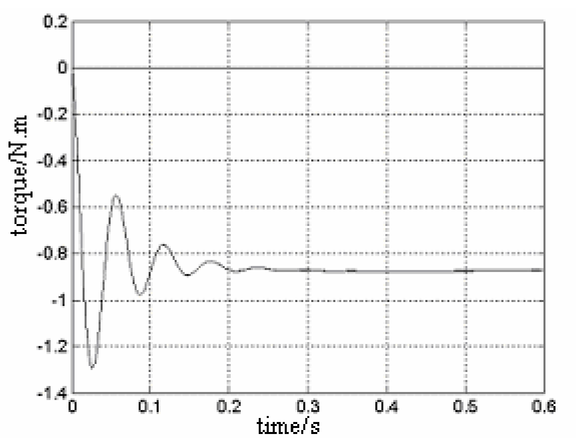

Figure 10. The time-domain response of the twisting rod torque to the step-input measurement noise of torque sensor when adopting controller K(s).

Fig10 indicates the time-domain response of the twisting rod torque to the step-input measurement noise of torque sensor when adopting controller K(s). It can be seen that the torque overshoot caused by step-input of torque sensor measurement noise is obviously decreased and trends to be stable within 0.3 second, which means the adjusting time is greatly shortened and the influence of the torque master measurement noise is effectively inhibited.

D. Conclusion

In order to improve the stability of the electric power steering system, the influence of either the dynamic characteristics of the motor or the torque master measurement noise as well as the road excitation disturbance must be considered. Combination of the PD Control and H_∞Optimal Control can not only change the dynamic characteristic of the system to restrain the disturbance from the road, but also decreased the influence of the torque sensor measurement noise on the system, and improve the stability and anti-interference of the electric power steering system.

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