Genetic Algorithm Based Fuzzy Multi-objective Nonlinear Programming of Regional Water Allocation

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***Abstract*—A genetic algorithm (GA) based approach to solve a fuzzy multi-objective nonlinear programming (FMNP) problem is presented and applied to water resources allocation. A FMNP model of regional water allocation is established originally. The objective functions of the model include three conflicting goals which are economic profits, environment goal and society goal. By turning the FMNP to an optimal level cut problem, the "fuzzy optimal solution" is solved by using a genetic algorithm. An example of the Dongying's water allocation strategy is provided by using the proposed method.**

***Keywords*—fuzzy multi-objective programming, fuzzy optimal solution, water resource optimal allocation, genetic algorithm**

I. Introduction

Water shortage is a serious problem in the north China which affects the people's life and economic development directly. Thus water management is one of the most important problems for Chinese government. To satisfy the growing of the demand, a scientific programming is required to allocate the limited water resource to meet the demand for some regions.

Many researchers have applied mathematical programming to water management. The traditional optimization methods attempted to obtain the optimal value to satisfy the objective function and constrains[2,5,6,11]. Grounder water quality management is investigated in [3,7]. Water supply capacity expansion problem is studied in [1]. Water distribution optimization problem is discussed in [4,5]. Generally, the mentioned works are deterministic linear or nonlinear programming problems. Recently, some researchers are studying the water management by using fuzzy programming[8, 12].

In this paper, considering some uncertainty and the different benefits of water allocation among industry, agriculture, residental consumption and environment in a region, a FMNP model is established. A GA based solving approach for FMNP is presented. By using the optimal level cut idea, the FMNP is changed to be a single objective nonlinear programming problem which is easily solved with GA. To show the validity of the method, a FMNP model of Dongying city's water allocation is built and the "fuzzy optimal solutions" are provided.

II. A FMNP Model of Regional Water Resource Allocation

A FMNP model of regional water resource allocation is established as follows.

A. Objective functions

(i)Economic profit function

$$\max \tilde{f}_1(\mathbf{x}) = \sum_k \sum_j [\sum_i (b_{ij}^k - a_{ij}^k) x_{ij}^k \alpha_{ij}^k + \sum_p (b_{pj}^k - a_{pj}^k) x_{pj}^k \alpha_{pj}^k] \quad (1)$$

where

$\tilde{f}(\mathbf{x})$ -a function which can be fuzzified, means that the value of objective function can vary in some domain. The domain is called the admissible error variable region.

$f_1(\mathbf{x})$ - net profit of all regions water allocated.

$\mathbf{x}$ -the decision variables.

k -the *k*th sub-area.

i -the *i*th independent water resource of some subarea.

p-the pth common water resource.

x_{ij}^k -the amount of water released from *i* source of *k*th sub-area to *j*th plant, the outflow of water.

b_{ij}^k -the profit of per unit released water from *i* source of *k*th sub-area to *j*th plant.

a_{ij}^k -the cost of per unit released water from *i* source of *k*th sub-area to *j*th plant.

(ii)Environmental objective function

The environmental objective is determined by the water quality and can be formulated by the amount of released pollution water. Generally, the objective function is to minimize the amount of some important matters such as Biochemical Oxygen Demand (BOD) and Chemical Oxygen Demand (COD).

$$\text{Min } \widetilde{f}_2(\mathbf{x}) = \sum_k \sum_j (\sum_i P_{ij}^k x_{ij}^k + \sum_p P_{pj}^k x_{pj}^k) \quad (2)$$

where

$f_2(\mathbf{x})$ -the amount of pollution matter (BOD and COD) of the region.

P_{ij}^k -the pollution matter of per unit released water from i source of kth sub-area to jth plant.

(iii) Social objective function

The social objective function is formulated by minimizing water shortage of the residents and plants.

$$\text{Min } \widetilde{f}_3(\mathbf{x}) = \sum_k [D_j^k - (\sum_i x_{ij}^k + \sum_p x_{pj}^k)] \quad (3)$$

where

$f_3(\mathbf{x})$ -water shortage of the residental life and plants.

D_j^k -the planning required water of jth plant of kth sub-area.

B. Constraints condition

(i) Capacity constraints of water resources

Constraints of independent water resources

$$\sum_k x_{ij}^k \le W_i \ . \quad (4)$$

Constraints of common water resources

$$\sum_k \sum_j x_{pj}^k \le W_p \ , \quad (5)$$

where W_i is the upper limit of the amount of i th water resource.

(ii) Constraints of water transportations

$$L_j^k \le \sum_i x_{ij}^k + \sum_p x_{pj}^k \le H_j^k \quad (6)$$

(iii) Constraints of regional harmonious development.

$$\sqrt{\mu_1 \mu_2} \ge 0.8 \ , \quad (7)$$

where $\sqrt{\mu_1 \mu_2}$ is the coefficient of harmonious development, and μ_1, μ_2 is defined as follows,

$$\mu_1(\sigma_1) = \exp(-\omega_1(\sigma_1 - \sigma_1^*)^2) \ , \quad (8)$$

$$\mu_2(\sigma_2) = \exp(-\omega_2(\sigma_2 - \sigma_2^*)^2) \ . \quad (9)$$

IN (8) and (9), σ_1, σ_2 are defined as follows,

$$\sigma_1 = (\sum_k E_j^k)/k$$

$$E_j^k = \begin{cases} 1 - \dfrac{D_j^k - \sum_k (\sum_i x_{ij}^k + \sum_p x_{pj}^k)}{D_j^k - L_j^k}, & D_j^k \ge \sum_k (\sum_i x_{ij}^k + \sum_p x_{pj}^k) \\ 1 & D_j^k < \sum_k (\sum_i x_{ij}^k + \sum_p x_{pj}^k) \end{cases} , \quad (10)$$

$$\sigma_2 = \frac{\dfrac{P}{P_0}}{\dfrac{GA}{GA_0}} \ , \quad (11)$$

where

D_j^k -the planning required water of jth plant of kth sub-area.

P_0 -the amount of BOD and COD yielding in the reference year.

P- the amount of BOD and COD yielding in the planning year.

GA_0 -the gross regional product per capita in the reference year.

GA -the gross regional product per capita in the planning year.

σ_1^*, σ_2^* -the optimal ratios pre-assigned.

ω_1, ω_2 -the weights.

From(10), we hope that the amount of consumed water of j th plant can meet the planed level. Equation (11) means that the relative increment of polluted matters is less than that of the gross regional product per capita.

(iv) Non-negative constraints

All decision variables should be non-negative.

$$x_{ij}^k \ge 0 \quad (12)$$

III. Fuzzy Nonlinear Programming (FNP) Solving via a Genetic Algorithm

Generally, a FNP problem can be described in the following form,

$$\begin{aligned} &\max \tilde{f}(x) = f(x_1, x_2, \cdots x_n) \\ &\text{s.t.} \quad g(x) \lesssim b, \\ &\qquad x \ge 0 \end{aligned} \quad (13)$$

where $x = (x_1, x_2, \cdots x_n)^T$ is $n-$ dimensional decision variable vector, $g(x)$ is a vector of constraint conditions, i.e.

$$g(x) = (g_1(x), g_2(x), \cdots g_m(x))^T ,$$

b is a constant vector, $b = (b_1, b_2, \cdots, b_m)^T$.

The FNP (13) can be turned to be the following deterministic nonlinear programming model,

$$\begin{cases} \text{Max } \alpha \\ \text{s.t. } \quad \mu_0(x) \geq \alpha, \\ \qquad \mu_i(x) \geq \alpha, \quad i = 1, 2, \cdots, m \\ \qquad x \geq 0, \quad 0 \leq \alpha \leq 1, \end{cases} \tag{14}$$

Where α is the optimal level cut (or degree) defined as follows,

$$\alpha = \min\{\mu_0(x), \mu_i(x), i = 1, 2, \cdots, m\}.$$

Before solving (13) and (14), we need explain the following notations.

Definition 1. A fuzzy optimum of problem (13) is defined as a following fuzzy set $\tilde{S}$,

$$\tilde{S} = \{(x, \mu_{\tilde{S}}(x)) \mid x \in (\mathbb{R}^n)^+,$$
$$\mu_{\tilde{S}}(x) = \min\{\mu_0(x), \mu_i(x), i = 1, 2, \cdots, m\}\}.$$

Let $S_\alpha = \{x \in (\mathbb{R}^n)^+ \mid \mu_{\tilde{S}}(x) \geq \alpha\}$, $\alpha \in [0,1]$. Then, S_α, which is called the α level cut of $\tilde{S}$, is an ordinary set.

Definition 2. α^* is called the optimal level cut, if α^* satisfies $\forall\ 0 \leq \alpha \leq \alpha^*$, S_α is nonempty, and $\forall\ \alpha > \alpha^*$, S_α is empty.

For the level cut, with the augmentation of α, the domain of S_α will be reduced till there exists an α^* such that $\forall \delta > 0, \alpha = \alpha^* + \delta, S_\alpha$ is an empty set. α^* is the optimal level cut.

The idea of solving FNP via a genetic algorithm is to find a small neighborhood of α^*, such that for all x corresponding to the α in neighborhood of α^* is the required optimal decision variable, i.e. fuzzy optimal solution. Thus the GA based approach can be expressed as follows. One can first set an optimal level cut α_0, and randomly produce a population including np individuals. By defining a fitness function, the individual with degree of membership no less than α_0 is selected. The selected individuals consist of a sub-population. For reducing computing time, the individuals which have degree of membership less than α_0 will be set much small degree of membership, so that they have little chance to be chosen in the next step.

After some iterative steps, the individuals with degree of membership less than α_0 will be eliminated. The rest is a *k*th sub-population in which each individual is with degree of membership α_k no less than α_0. Thus $S_{\alpha_k} \subseteq S_{\alpha_0}$, every element in S_{α_k} is the desired optimal solution.

For individual x, let

$$\mu_{\min}(x) = \min\{\mu_0(x), \mu_1(x), \cdots \mu_m(x)\}.$$

The fitness function is defined as follows.

$$F(x) = \mu_{\tilde{S}}(x) = \begin{cases} \mu_{\min}(x), & \mu_{\min}(x) \geq \alpha_0 \\ \varepsilon\mu_{\min}(x), & \mu_{\min}(x) < \alpha_0 \end{cases} \tag{15}$$

where α_0 is a pre-assigned degree of membership, and $\varepsilon \in [0,1]$.

From (15), if $x_j \notin S_{\alpha_0}$, a much small degree of membership will be set to it.

The GA based method is outlined as follows.

Step 1. Initialize the population in the search space.

Substep 1.1. Input the pre-assigned degree of membership α_0, and the maximal iterative steps *NG*, and the number of individuals *NP*.

Substep 1.2. Input the index set $CS = \{0, 1, 2, \cdots\}$ which represent the index of the objective function and constraints respectively. For each index $r \in CS$, input the minimum and maximum of corresponding objective function or constraint.

Step 2. Produce a population and determine a membership function randomly.

Step 3. Set $k = 1$.

Step 4. For individual $j (j = 1, 2, \cdots, NP)$, calculate the fitness function $F(j)$ and selection probability $P(j)$,

$$F(j) = \mu_{\tilde{S}}(x_j),\ P(j) = \frac{F(j)}{\sum_{i=1}^{NP} F(j)}.$$

Step 5. Perform crossover according to the crossover probability, new individuals $x_j (j = 1, 2, \cdots, NP)$ will be obtained after mutation.

Step 6. For each new individual, calculate the value of the memebership function $\mu_{\tilde{S}}(x_j)$. Update the optimal level cut $\mu_{\max}$, and adjust the minimum and maximum of objective function or constraints.

Step 7. $k + 1 \to k$, if $k \leq NG$,Then go to Step 4 or go to Step 8.

Step 8. Output the optimal level cut $\mu_{\max}$ and the minimum and maximum of objective function or constraints.

If some constraints are inequality, penalty function can be applied to these inequalities.

IV. Model Solving of FMNP of Water Resource Allocation

A. Fuzzification of the model

For (1)-(3), the following membership functions are defined.

For economic profit, the membership function is defined as

$$\mu_1(x)=\begin{cases}1, & f_1(x)\ge z_{1g}\\ \dfrac{f_1(x)-l_1}{z_{1g}-l_1}, & l_1\le f_1(x)\le z_{1g}\\ 0, & f_1(x)\le z_{1g}\end{cases}$$

For environment objective, the membership function is defined as

$$\mu_2(x)=\begin{cases}1, & f_2(x)\le z_{2g}\\ \dfrac{h_2-f_2(x)}{h_2-z_{2g}}, & z_{2g}\le f_2(x)\le h_2\\ 0, & f_2(x)\ge z_{2g}\end{cases}$$

For social objective, the membership function is defined as

$$\mu_3(x)=\begin{cases}1, & f_3(x)\le z_{3g}\\ \dfrac{h_3-f_3(x)}{h_3-z_{3g}}, & z_{3g}\le f_3(x)\le h_3\\ 0, & f_3(x)\ge z_{3g}\end{cases}$$

where

z_{1g} -the optimal value of economic profit,

l_1 -the lower limit of economic profit,

z_{2g} -the optimum value of COD,

z_{3g} -the optimum value of water shortage,

h_2 -the upper limit of COD,

h_3 -the upper limit of water shortage.

Then, according to above section, the fuzzy programming (1)-(12) can be turned to be a following deterministic programming model,

$$\begin{cases}\max\ \alpha\\ \text{s.t.}\\ \mu_1(x)\ge\alpha\\ \mu_2(x)\ge\alpha\\ \mu_3(x)\ge\alpha\\ G(x)\le b\\ x\ge 0,\quad 0\le\alpha\le 1.\end{cases}$$

where α is the optimal level cut, and $G(x)$ denotes the constraints of (4)-(12).

B. Computation of $z_{ig}, i=1,2,3$ and h_2, h_3.

The upper limit or lower limit of each goal can be obtained by solving the following single goal programming.

$$\max\ f_1(\mathbf{x})=\sum_k\sum_j[\sum_i(b_{ij}^k-a_{ij}^k)x_{ij}^k\alpha_{ij}^k+\sum_p(b_{pj}^k-a_{pj}^k)x_{pj}^k\alpha_{pj}^k]$$

$$or\quad \min\ f_2(\mathbf{x})=\sum_k\sum_j(\sum_i P_{ij}^k x_{ij}^k+\sum_p P_{pj}^k x_{pj}^k)\le h_2$$

$$or\quad \min\ f_3(\mathbf{x})=\sum_k[D_j^k-(\sum_i x_{ij}^k+\sum_p x_{pj}^k)]\le h_3$$

s.t.

$$\sum_k x_{ij}^k\le W_i\ ,$$

$$\sum_k\sum_j x_{pj}^k\le W_p\ ,$$

$$L_j^k\le\sum_i x_{ij}^k+\sum_p x_{pj}^k\le H_j^k\ ,$$

$$\sqrt{\mu_1\mu_2}\ge 0.8\ ,$$

$$x_{ij}^k\ge 0\ .$$

C. Applications to the city of Dongying

The Dongying city locates in the delta of the yellow river which is a water-short city. According to the practical situation, the supplying water area can be divided into 6 sub-areas which are the Dongying district, the Hekou district, the Guangrao county, the Lijin County, the Kenli county and the Shengli oilfield. In the Dongying district, the Hekou district, the Guangrao county, the Lijin County and the Kenli county, the water consumption includes industrial water, agricultural water, residental life water of the towns, residental life water of the countryside and environmental water. Only industrial water is considered in the Shengli oilfield. In the Dongying city, there are three water resources which are the yellow river, the rainfall and the groundwater. The yellow river and the rainfall are common water resources and the groundwater is an independent resource only in the Guangrao county.

For the water allocation of the Dongying city, we set the lower limit of the economic profit to be $h_1=1.32\times10^8$ yuan, the upper limit of the environmental goal to be $l_2=3.6\times10^4$ ton, the lower limit of social goal to be the 10% of the residental consumption water. The other parameters in the model are displayed in the following table.

V. Conclusions

This paper establishes a multi-objective fuzzy nonlinear programming problem of regional water allocation. By some manipulation, the multi-objective fuzzy nonlinear programming is turned to be an optimal level cut problem which is a single goal nonlinear programming problem and can be solved by using a genetic algorithm. Applying this method to the Dongying's water allocation, a satisfaction solution among multi-objectives is obtained. The optimization result could be applied to aid in the police making of the city's water allocation.

TABLE I. THE PARAMETERS IN THE MODEL (2.1)-(2.12)

Sub-area	Paramters	Industrial water	Agricultural water	Life water of the towns	Life water of the countryside	Environmental water
Dongying distrint	P1	425	23.8	——	——	——
	P2	3.35	0.05	2.25	2.25	3.35
	P3	0．08	——	0．06	——	——
	P4	2820	5425	——	——	——
	P5	2207	4611	——	——	——
	P6	——	——	3400	340	600
Hekou district	P1	307	8.7	——	——	——
	P2	3.35	0.05	2.25	2.25	3.35
	P3	0．1	——	0．11	——	——
	P4	1768	11626	——	——	——
	P5	1383	9882	——	——	——
	P6	——	——	1000	170	200
Kenli county	P1	322	13.6	——	——	——
	P2	3.35	0.05	2.25	2.25	3.35
	P3	0．1		0．11	——	——
	P4	4775	9419	——	——	——
	P5	3737	8006	——	——	——
	P6	——	——	600	300	300
Lijin county	P1	296	9.1	——	——	——
	P2	3.35	0.05	2.25	2.25	3.35
	P3	0．1	——	0．11	——	——
	P4	3524	24074	——	——	——
	P5	2758	20463	——	——	——
	P6	——	——	550	580	600
Guangrao county	P1	552	20.2	——	——	——
	P2	3.35	0.05	2.25	2.25	3.35
	P3	0．13	——	0．1	——	——
	P4	5803	16212	——	——	——
	P5	4541	13780	——	——	——
	P6	——	——	780	1000	300
Shengli oilfield	P1	625	——	——	——	——
	P2	3.35	0.05	2.25	2.25	3.35
	P3	0.1	——	——	——	——
	P4	15569	——	——	——	——
	P5	12184	——	——	——	——

P1- the profit per ton water（yuan/m^3, P2- The cost per ton water（yuan/m^3）, P3- The resulted amount of COD per ton water (kg/m^3), P4- H_j in (6) (ten thousand m^3), P5- L_j in (6)(ten thousand m^3), P6- Environmental water (ten thousand m^3)(ten thousand m^3).

TABLE II. CAPACITY CONSTRAINTS OF THE THREE INDEPENDENT WATER RESOURCES

Capacity constraints	90% of the maximal storage （ten thousand m^3）
Groundwater	9759
Rainfall	12134
Yellow river	78000

TABLE III. THE OPTIMUM VALUE OF THE OBJECTIVES

The objective	Economic profit (Ten thousand yuan)	Environment goal (Ten thousand ton)	Social goal (Ten thousand m^3)
The value of the objective	14934000	3.391	335

TABLE IV. THE OPTIMUM DECISION VARIABLES

Sub-area	Water resources	Industrial water	Agricultural water	residental life water of town	residental life water of countryside	Environmental water
Dongying district	rainfall	823	862	184	121	79
	Yellow river	1686	4517	3150	219	521
Hekou district	rainfall	455	613	201	55	126
	Yellow river	969	9992	734	115	74
Kenli county	rainfall	704	780	156	109	1
	Yellow river	3066	8136	396	191	303
Lijin county	rainfall	727	744	146	171	78
	Yellow river	2051	20519	346	409	522
Guangrao county	rainfall	787	756	59	82	11
	Yellow river	1724	8058	239	460	35
	groundwater	2205	6456	385	458	255
Shengli oilfield	rainfall	3305				
	Yellow river	9567				

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