Stabilization and Path Following of a Spherical Robot

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***Abstract*—In this paper, we present a spherical mobile robot BYQ_III, for planetary surface exploration and security tasks. The driving torque for the rolling robot is generated by a new type of mechanism equipped with a counter-pendulum. This robot is nonholonomic in nature, and underactuated. In this paper, the three-dimensional (3-D) nonlinear dynamic model is developed, then decoupled to the longitudinal and lateral motions by linearization. Two sliding-mode controllers are proposed to asymptotically stabilize the tracking errors in lean angle and spinning angular velocity, respectively, and indirectly to stabilize the desired path curvature, because the robot steers only by leaning itself to a predefined angle. For the task of path following, a path curvature controller, based on a geometrical notion, is employed. The stability and performance analyses are performed, and also the effectiveness of the controllers is shown by numerical simulations. To the best of author's knowledge, similar results could not be obtained in the previous spherical robot control system based on the dynamics. The work is of significance in understanding and developing this type of planning and controlling motions of nonholonomic systems.**

***Keywords*—nonholonomic system, dynamics, sliding–mode control, path following, spherical robot**

I. INTRODUCTION

The BYQ project is an effort to design and develop the omni-directional spherical mobile robots, for planetary surface exploration, security surveillance, inspection of disaster areas, and human search and rescue. BYQ_III in Fig.1 is the third prototype originally developed in our laboratory [1]. The actuation mechanism consists of two separate actuators: (1) *a steer motor*, which mainly controls the steering motion of the robot by tilting the counter-weight pendulum; and (2) *a drive motor*, which causes forward and/or backward acceleration by swinging the counter-weight pendulum. And the main axes of the two motors are perpendicular. The concept of the spherical mobile robot allows the entire system to be enclosed within the shell to provide mechanical and environmental protection for the equipment and actuation mechanism inside. This configuration conveys significant advantages over multi-wheel, statically stable vehicles. These advantages include good dynamic stability, high maneuverability, low rolling resistance, ability to omni-directionally roll, and amphibious capability. Most important, the robot can resume stability even if a collision happened.

Figure 1. Photograph of the third prototype BYQ_III

At the same time, spherical robot concept also brings a number of challenging problems in modeling, stabilization and path following. First, there are highly coupled dynamics among the shell, the gimbal and the counter-weight pendulum (Fig.1). This is similar to the case when a manipulator is mounted on a satellite [7]. Second, it is subject to a kind of underactuated non-holonomic system that can control more degrees of freedom (**DOF**) using less drive inputs. The constraints mainly arise when the robot rolls on the ground without slipping. Therefore, when the problem of controlling the position and attitude of the sphere is considered, the system must be treated as a nonholonomic system. This type of motion control of a sphere, referred to as the *plate-ball* problem [2-6], has been of particular interest to researchers over the past years, since its kinematic model can not be reduced to chained-form; this renders regimented nonholonomic control techniques inapplicable[8]. But, it has been shown by Li and Canny [9] that the ball-plate system is completely controllable. T. Otani [6] constructed two feedback controllers respectively, one for the kinematic model based on Lyapunov control, and the other for the dynamic model based on Backstepping technique. It is shown that the both controllers make the system converge to a small neighborhood of the desired point. R. Mukherjee [8] presented a sweep-tuck algorithm for four dimensional reconfiguration of the spherical robot. The sphere is reconfigured by repeated application of a pair of control actions: moving along straight line and circular path, respectively. In this case, the posture needs to be constrained and, accordingly, arbitrary trajectories can not be globally followed.

This paper considers the problem of stabilization and path following of the spherical robot, BYQ_III, using two independent torque inputs. Two feedback controllers, based on the sliding-mode control method, are then employed for the stabilization of the linearized model with the uncertainties. One controller (*velocity controller*) makes the system converge to an arbitrarily small neighborhood of the desired spinning angular velocity. The other controller (*position controller*) stabilizes the lean angle to a small neighborhood of the desired point, and indirectly guarantees the asymptotic stabilization of the heading direction. For the path following of the robot, we design a steer function to track any desired straight line based on the path curvature. By proposed method, the geometric constraints in [8] are eliminated and both tracking and stabilization problem in the research of the spherical robot are solved. The proposed approach does not generate any oscillatory and/or chattering behavior to the output/control variables. The stability and performance of each control system are analyzed. By combining these controllers, the global asymptotic performance of path tracking can be achieved even with large initial tracking errors in both position and heading direction, which are shown through numerical simulation.

II. MATHEMATICAL MODEL OF BYQ_III

In derivation of the equations of the robot motion, we assume that the shell is a rigid, homogeneous ball which rolls over a perfectly flat surface without slipping, and the counter-weight pendulum is a particle (Fig. 2).

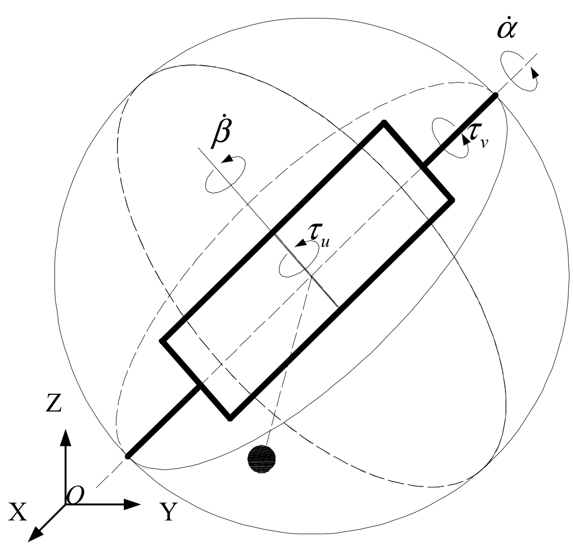


Figure 2. Configuration of the system BYQ_III

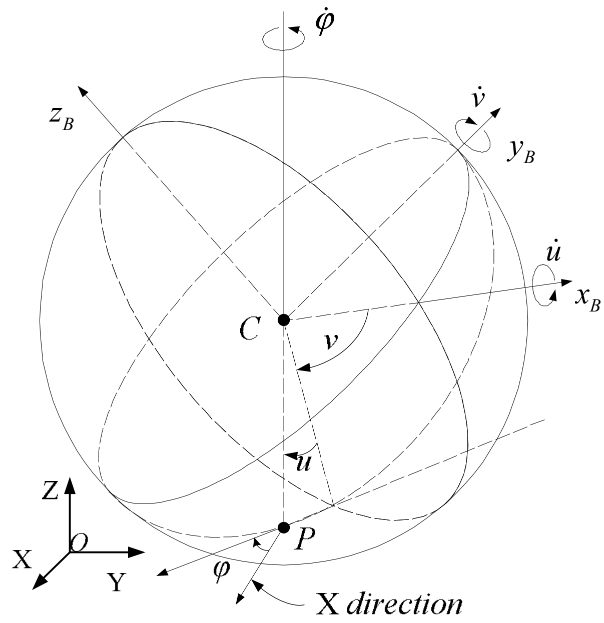


Figure 3. Orientation of the shell

A. Kinematic Model

Spherical robots require six independent coordinates to determine their complete configuration (position and orientation) when the configuration of the internal mechanism is not considered. Like Marine vessels [11], the six different motion components can be defined as surge, sway, heave, roll, pitch, and yaw. But the heave displacement holds constant, and the heave motion does not exist because the shell is a rigid, homogeneous ball which rolls over the surface. It is common to reduce the general six-DOF of the model to motion in roll, pitch, and yaw only by neglecting the sway and surge modes which can be formulated by the roll, pitch and yaw states. The state vector $\eta = \mathbf{SE}(2)$ is then defined by

$$\boldsymbol{\eta} = [x, \quad y, \quad \varphi]^T \tag{1}$$

where $(x, y) \in \mathbb{R}^2$ is the position of geometrical center of the spherical robot in an inertial frame, and $\varphi \in [0, 2\pi)$ is the heading angle of the robot(Fig.3). Denote by u and v the coordinates (latitude and longitude respectively) of the contact point (**P**) on the ball. The kinematic model resulting from the nonholonomic constrains can be written as

$$\dot{\boldsymbol{\eta}} = R \cdot \mathbf{R}(\varphi)\boldsymbol{\upsilon} \tag{2}$$

where $\mathbf{R}$ is the rotation matrix in yaw and roll, $\boldsymbol{\upsilon} \in \mathbb{R}^3$ is a vector containing the body-fixed angular velocities and R is the radius of the shell. $\mathbf{R}$ is defined as

$$\mathbf{R} = \begin{bmatrix} \sin\varphi & \cos\varphi\cos u & 0 \\ \cos\varphi & -\sin\varphi\cos u & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

The velocity vector $\boldsymbol{\upsilon}$ is defined by

$$\boldsymbol{\upsilon} = \begin{bmatrix} \dot{u} & \dot{v} & \dot{\varphi} \end{bmatrix}^T \tag{4}$$

Here, $\dot{u}$ is the transverse angular velocity (roll), $\dot{v}$ is the forward angular velocity (pitch), and $\dot{\varphi}$ is the angular velocity (yaw) decomposed in the body-fixed frame. There is a representation singularity when $u = \pm\pi/2$, where v and φ are undefined. We assume $-\pi/2 < u < \pi/2$ and $-\pi < v < \pi$, so that the contact point belongs always to the same coordinate patch for the ball.

B. Dynamic Model

The dynamic equations under nonholonomic constraints can be described by Euler-Lagrange formulation [10] as

$$M(q)\ddot{q} + V(q,\dot{q})\dot{q} + G(q) = A^T(q)\lambda + B(q)\tau. \tag{5}$$

where $q \in \mathbb{R}^n$ is generalized coordinates, $\lambda \in \mathbb{R}^m$ is a constraint force vector, $\tau \in \mathbb{R}^{n-m}$ is a torque control input vector, $M(q) \in \mathbb{R}^{n\times n}$ is a symmetric and positive definite inertial matrix. $V(q,\dot{q}) \in \mathbb{R}^{n\times n}$ is a centripetal and coriolis matrix, $G(q) \in \mathbb{R}^n$ is a gravitation vector, $B(q) \in \mathbb{R}^{n\times(n-m)}$ is an input transformation matrix, and $A(q) \in \mathbb{R}^{m\times n}$ is a matrix related with nonholonomic constraints.

In derivation of the equations of BYQ_III, noting that there are only two control torques available: the tilt torque τ_v and drive torque τ_u (Fig.2), and that the relative angular displacements (i.e. the swinging α and tilting β angle) of the counter-pendulum are not important from the perspective of our control objects, they should be omitted in the simplified model. Using the Euler-Lagrange method, and simple algebraic manipulations, the complete kinematic and dynamical vector equations describing the motion of a spherical robot can be obtained are written as

$$\dot{\boldsymbol{\eta}} = R \cdot \mathbf{R}(\varphi)\boldsymbol{\upsilon} \tag{6}$$

$$(mR^2 + I)\ddot{u} = (mR^2 + I)\cos u\dot{v}\dot{\varphi} + \tau_u \tag{7}$$

$$\begin{aligned}(mR^2\cos^2 u + I)\ddot{v} + I\sin u\ddot{\varphi} \\ = (mR^2 + I)\cos u\dot{u}\dot{\varphi} - \frac{1}{2}mR^2\sin 2u\dot{u}\dot{v} + \tau_v\end{aligned} \tag{8}$$

$$I\ddot{\varphi} + I\sin u\ddot{v} = -I\cos u\dot{u}\dot{\varphi} \tag{9}$$

where m is the total mass of the robot, and τ_{su}, τ_{sv} are the control torques under the actions of the counter-weight pendulum, respectively. Equations (6)-(9) show the reduced dynamic model of the robot, the lean angle u is not coupled with the spinning angle v and the steering angle φ at the acceleration level; they are coupling at the velocity level through the cross terms $\dot{v}$ and $\dot{\varphi}$.

III. CONTROL OF ROBOT MOTION

There exists no continuous time-invariant state feedback that renders the system (6)-(9) asymptotically stable about the origin[8], but the system is real analytic, there exists a piecewise analytic feedback law which can stabilize the closed loop system to a given equilibrium. In the problem of the motion control, the lean angle of the robot can be controlled indirectly by tilting the counter-pendulum, then, the spherical robot steers by leaning at different angles. Therefore, it is necessary to design a controller that stabilizes the robot at any desired lean angle to control the steering velocity.

On the other hand, for a typical unicycle, it should be noted that the longitudinal and lateral motions are highly coupled to each other, and it is not suitable to decompose the motions through a linearization method [12] and [13]. However, like the single wheel robot [10], because of the stabilizing effect of the counter-pendulum, the effect of the coupling/cross terms between the longitudinal and lateral motions of the robot become less significant for the spherical robot. It is feasible to decouple them by linearizing the dynamic model [10], and then by designing a linear state feedback law to control the lean angle and spinning speed of the robot. In the process of linearization, the approximate uncertainties are conducted, and sliding-mode control methods are employed to solve this problem.

A. Linearized Model

In derivation of the linearized model, we make the following assumptions: 1) the terms δu , $\delta\dot{v}$ are sufficiently small, and 2) $u = \delta u$, $\dot{v} = \Omega_o + \delta\dot{v}$, Ω_o is the nominal value. The linearized model of the system (10)-(12) can be represented as

$$\delta\ddot{u} = \Omega_o\dot{\varphi} + \delta\dot{v}\dot{\varphi} + \tau_u/(mR^2 + I) = \Omega_o\dot{\varphi} + \Delta f_u + \tau_1 \tag{10}$$

$$\delta\ddot{v} = \delta\dot{u}\dot{\varphi} + \tau_v/(mR^2 + I) = \Delta f_v + \tau_2 \tag{11}$$

$$\ddot{\varphi} = -\Omega_o\delta\dot{u} \tag{12}$$

where the approximate uncertainties Δf_u and Δf_v can be treated as the disturbances, and both are bounded because $\delta\dot{u}$ and $\delta\dot{v}$ are controllable by steer torque and drive torque, respectively.

B. Velocity Controller

Because $\delta\dot{v}$ is independent of the roll and yaw dynamics (10) and (12), we can decompose the longitudinal motion (11) and establish a closed–loop control for controlling the spinning velocity $\dot{v}$ to the nominal value Ω_o.

Proposition 1: Consider the system (11), and design τ_2 to robustly stabilize the origin $\delta\dot{v} = 0$. This can be achieved with

$$\tau_2 = -k_{v1}\delta\dot{v} - k_v \cdot sign(\lambda_v\delta\dot{v}) \tag{13}$$

where $k_{v1} > 0$, $k_v > \sup|\Delta f_v|$,and $\lambda_v > 0$. The sliding surface is $s_v = \lambda_v\delta\dot{v}$.

Proof: This controller globally stabilizes the origin, because

$$\begin{aligned} s_v\dot{s}_v &= \lambda_v s_v(\Delta f_v - k_{v1}\delta\dot{v} - k_v\cdot sign(s_v)) \\ &= -\lambda_v^2 k_{v1}\delta\dot{v}^2 + \lambda_v s_v(\Delta f_v - k_v\cdot sign(s_v)) \\ &\le -\lambda_v^2 k_{v1}\delta\dot{v}^2 - \lambda_v(k_v\cdot|s_v| - \sup|\Delta f_v|s_v) \\ &\le -\lambda_v^2 k_{v1}\delta\dot{v}^2 \end{aligned} \tag{14}$$

Choosing Lyapunov functions and taking the time derivative, we have

$$V = \frac{1}{2}s_v^2 \quad \dot{V} \le -\lambda_v^2 k_{v1}\delta\dot{v}^2 \tag{15}$$

C. Position Controller

When the tracking errors are chosen as

$$e_u = \delta u - u_r \tag{16}$$

and if $\lambda_u > 0$, the convergence of (10) to zero can be achieved by making the following variables converge to zero

$$s_u = \dot{e}_u + \lambda_u e_u \tag{17}$$

Using the computed-torque method, a torque control input τ_1 can be chosen from (10) as

$$\begin{aligned} &\tau_1 = -k_{u1}\dot{e}_u - k_{u2}e_u - \beta(\delta u)\cdot sign(s_u) - \Omega_o\dot{\varphi} \\ &\beta(\delta u) = a\,|\,\delta\dot{u}\,| + b\,|\,\delta u\,| + \eta\,|\,R\,| + \sup(\Delta f_u) \\ &R = -\delta\ddot{u}_r - (k_{u1}+\lambda_u)\delta\dot{u}_r - k_{u2}\delta u_r \end{aligned} \tag{18}$$

where a , b , η , k_{u1} and k_{u2} are all the positive parameters that satisfy $\lambda_u - k_{u1} \le a$, $-k_{u2} \le b$ and $1 < \eta$. Then the control system satisfies the property in the following theorem.

Proposition 2: under the condition that the bounded disturbance Δf_u exists in the system (10), control input (18) stabilizes the sliding surface (17). Then, position-tracking errors converge to zero and heading-direction is bounded.

Proof: From the yaw dynamics (12), it can be obtain that

$$\begin{aligned} &\dot{\varphi} = -\Omega_o\delta\dot{u} + \dot{\varphi}_0 = -\Omega_o\delta\dot{u} \quad (\dot{\varphi}_0 = 0) \\ &\varphi = -\frac{1}{2}\Omega_o\delta u^2 + \varphi_0 \end{aligned} \tag{19}$$

where φ_0 and $\dot{\varphi}_0$ are the initial value. And we can assume that

$$|\,\Omega_o\dot{\varphi}\,| = |\,-\Omega_o^2\delta\dot{u}\,| \le D \tag{20}$$

$$|\,\Delta f_u\,| = |\,\delta\dot{v}\dot{\varphi}\,| = |\,-\delta\dot{v}\Omega_o\delta\dot{u}\,| \le \sup|\,\Delta f_u\,| \tag{21}$$

Differentiating the sliding surface (17) yields

$$\begin{aligned} \dot{s}_u &= \ddot{e}_u + \lambda_u\dot{e}_u \\ &= \delta\ddot{u} - \delta\ddot{u}_r + \lambda_u(\delta\dot{u} - \delta\dot{u}_r) \\ &= \Omega_o\dot{\varphi} + \Delta f_u + \tau_1 - \delta\ddot{u}_r + \lambda_u(\delta\dot{u} - \delta\dot{u}_r) \end{aligned} \tag{22}$$

Substituting (18) into (22) and multiplying both sides by s_u gives

$$\begin{aligned} s_u\dot{s}_u &= s_u[\Delta f_u - (k_{u1}-\lambda_u)\delta\dot{u} - k_{u2}\delta u - \beta(\delta u)sign(s_u) + R] \\ &= \Delta f_u s_u - \sup(\Delta f_u)\,|\,s_u\,| \\ &\quad - (k_{u1}-\lambda_u)\delta\dot{u}s_u - a\,|\,\delta\dot{u}s_u\,| \\ &\quad - k_{u2}\delta u s_u - b\,|\,\delta u s_u\,| \\ &\quad - \eta\,|\,R s_u\,| + R s_u \\ &\le 0 \end{aligned} \tag{23}$$

Choosing Lyapunov functions and taking the time derivative, we have

$$V = \frac{1}{2}s_u^2 \ , \ \dot{V} \le 0 \tag{24}$$

IV. PATH FOLLOWING

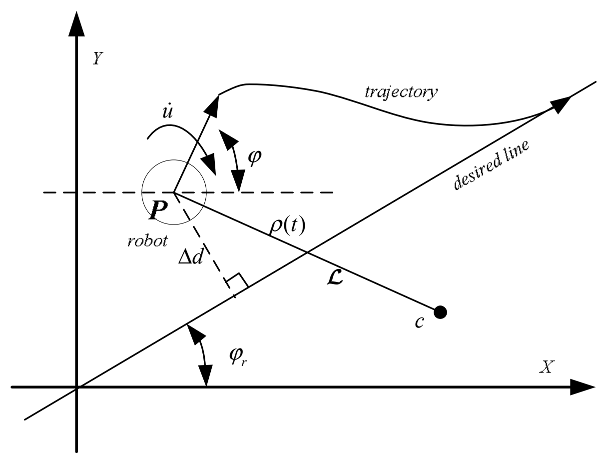


Figure 4. Principle of line following. (Top view).

The spherical robot steers by leaning itself to a predefined angle. Therefore, the main difficulty in solving the path-following problem of the spherical robot is that we must control the position and the orientation using two control inputs.Here, we employ an approach to the path following problem in controlling the path curvature [16]. Like [10], we redefine the system configuration of the robot based on the geometrical notion and characteristics of the nonholonomic motion.

A. Robot Configuration

For the path following, because the shell is a rigid, homogeneous ball, it is not different to use the point of contact on the ground, or use the geometrical center of the shell, to describe the position of the robot.The status of the contact point (or the geometrical center of the shell) of the robot can be described by an alternative set of configurations based upon [10] and [17].

$$q = (p, \varphi, k) = ((x, y), \varphi, k) \quad (25)$$

where (x, y), φ, and k are the position, the heading orientation and the path curvature of the robot with respect to the inertial frame, respectively. The new configuration of the robot is shown in Fig. 4.

We can only indirectly control the path curvature to steer the robot by changing its lean angle. Fortunately, the path curvature of the contact point can be expressed as

$$k(t) = 1/\rho(t) = \tan u(t)/R \quad (26)$$

where $\rho(t)$ is the radius of the curvature from the center of rotation $\boldsymbol{c}$ to the contact point (**P**). If the center c of the rotation is at infinity, the robot is moving on a straight line and the path curvature and steering velocity are zero. From (26), for the given linear velocity v_y, we can control the path curvature k by controlling the lean angular u, and it is not identical to the single wheel robot which controlled the path curvature by controlling the steering velocity directly[10].

B. Line Following

Based on [17], we designed a line following controller for the robot to track a desired straight line. Assuming that the linear velocity v_y of the robot is fixed/controlled to a nominal value, we consider the lean angle as the only position input for the robot. Using the position control law (18), we can stabilize the robot to a predefined lean angle which corresponds to the desired path curvature.

We consider the derivative of the path curvature and then express the derivative of the path curvature dk/ds with respect to the path length s as

$$\frac{dk}{ds} = -k_a \cdot k - k_b \cdot (\varphi - \varphi_r) - k_c \cdot \Delta d \quad (27)$$

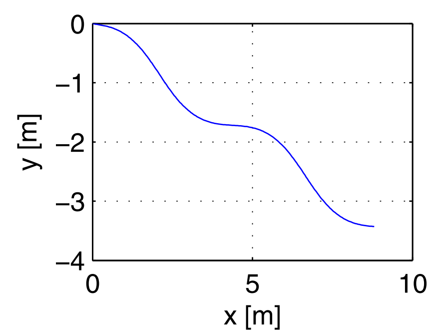
where k_a, k_b and k_c are positive constants, φ_r and δd are the direction of the desired line and the perpendicular distance between the robot and the desired line, respectively, (Fig. 4). Equation (26) is called a steering function [17]. The first term, $-k_a \cdot k$ is a feedback term (a damping factor) for the curvature, the second term $-k_b \cdot (\varphi - \varphi_r)$ is a feedback term for the angle error, and the third term $-k_c \cdot \Delta d$ is a feedback term for the positional error. The positional error Δd is the signed distance from (**P**) to **L**, where $\Delta d > 0$ if the robot is on the left side of the directed line, $\Delta d < 0$ if it is on the right side, and $\Delta d = 0$ if it is on the line. If (k_a, k_b, k_c) are selected such that (27) is asymptotically stable, the continuity of the path curvature can be ensured, and $(dk/ds) \to 0$ as the path length s increases, i.e., $d \to 0$, $k \to 0$ and $\varphi \to \varphi_r$ as s increases. Hence, the robot converges to the desired straight line asymptotically. In order to design the position input (lean angle), we first find the path curvative feedback by integrating the (27) in each instant. Using the path curvature feedback, we can obtain the corresponding desired lean angle input for the robot according to (27). The equilibrium point **0** of Equation (27) is uniformly asymptotically stable if k_a, k_b and k_c are positive constants and $k_a \cdot k_b > k_c$. And the solution for critically damped condition is $k_a = 3k_{cur}$, $k_b = 3k_{cur}^2$, $k_c = k_{cur}^3$, where k_{cur} is the gain of the curvature control law. The detailed proof is described in [17].

V. SIMULATION RESULTS

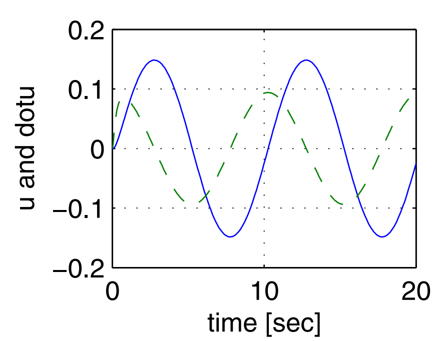
In this section, simulation results on the spherical robot, BYQ_III, using the proposed method are presented to demonstrate the effectiveness of the controllers and verify the path following performance. System (6)-(9) is always used in the simulation instead of (10)-(12) for dynamics. Consider the spherical robot with the model parameters $m = 25\ kg$, $R = 0.3\ m$, and $I = 0.27\ kg \bullet m^2$. The design parameters of (13), (18), and (27) are $\lambda_u = 5$, $\lambda_v = 1$, $k_{v1} = 1$, $k_v = 1$, $k_{u1} = 10$, $k_{u2} = 10$, $a = 4$, $b = 1$, $\eta = 2$, $\sup(\Delta f_u) = 10$, and $k_{cur} = 1$. The signum functions in the controllers (13), (18) are replaces by saturation functions to reduce the chattering phenomenon. Assume the initial condition of the system is as follow

$$[x, y, u, v, \varphi, \dot{u}, \dot{v}, \dot{\varphi}] = [0, 0, 0, 0, 0, 0, 0, 0]$$

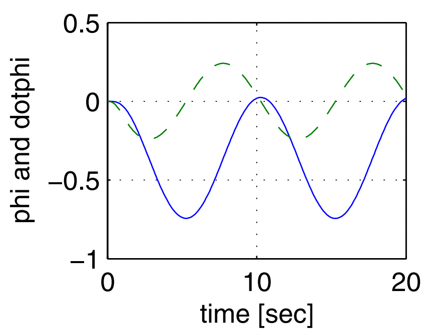
Fig.5 shows the evolutions of the variables for tracking the desired curvature: sine wave and square wave, respectively. The reference tangent linear velocity is $v_{yr} = 0.5 m/s$. As expected, simulations reveal that the spherical robot converges globally uniformly to the desired point with acceptable dynamic performances.



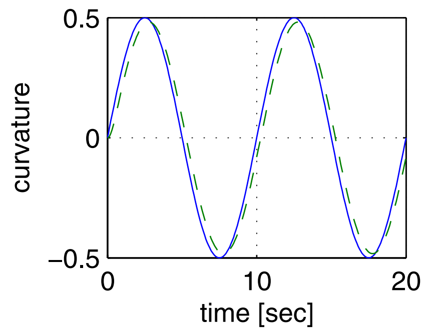
(a) x-y plot



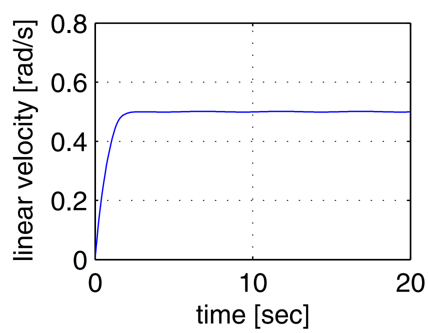
(b) Lean angle (solid)(rad) and angular velocity (dashed)(rad/s)



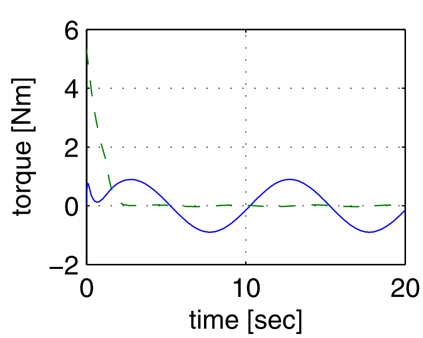
(c) Steer angle (solid)(rad) and angular velocity (dashed)(rad/s)



(d) Tracking curvature (k)(1/m)(solid:reference, dashed:actual)



(e) Tracking velocity v_y



(f) Input torque(solid: τ_u ,dashed: τ_v)

Figure 5. Stabilization performance of the proposed controllers

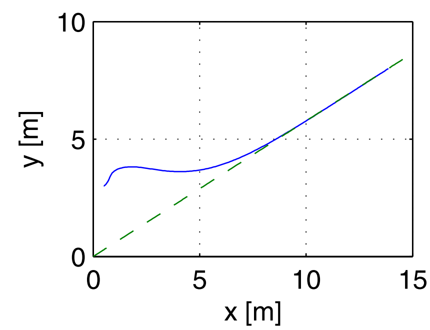
Fig.6 shows the results of the robot in tracking a straight line. The reference trajectory is a line with $y = \tan(\pi/6)\cdot x$, with

$$\varphi_r = \frac{\pi}{6}, \quad \Delta d = \cos(\pi/6)\cdot(x - \tan(\pi/6)\cdot y)$$

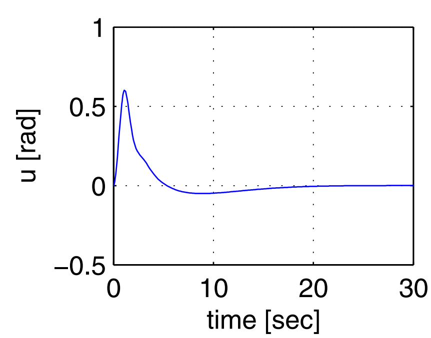
The reference tangent linear velocity $v_{yr} = 0.5\,m/s$, and the initial conditions of position and heading direction are assumed to exist as follow

$$[x, y, u, v, \varphi, \dot{u}, \dot{v}, \dot{\varphi}] = [0.5, 1, 0, 0, \pi/2, 0, 0, 0]$$

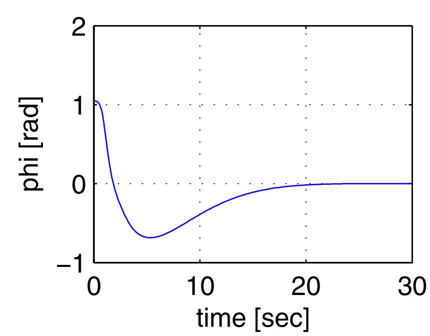
Fig.6 shows that a straight line can be exactly followed as posture-tracking errors converge to zero as in Fig.6 (b),(c) and (d). Also, the linear and angular velocities do not contain chattering phenomenon, which are omitted here.



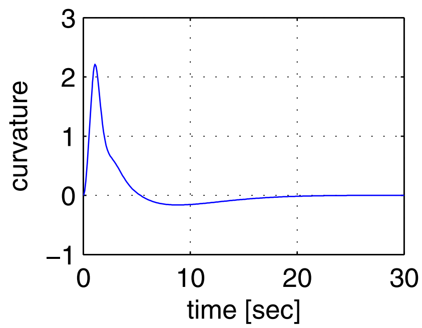
(a) x-y plot(dashed: reference, solid: actual model)



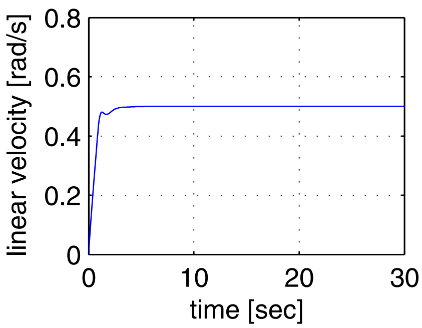
(b) Tracking errors (u)



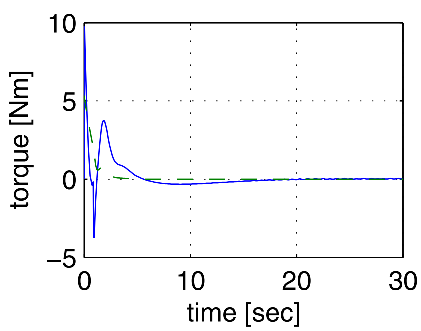
(c) Tracking errors (φ)



(d) Tracking errors (k)(1/m)



(e) Tracking velocity (v_y)



(f) Input torque(solid: τ_{su} ,dashed: τ_{sv})

Figure 6. Path following performance of the proposed controller

VI. CONCLUSION

In this paper, we have formulated the kinematic model and dynamic model using the constrained generalized Lagrangian formulation, and verified it through simulations. Two discontinuous feedback control laws have been derived using the sliding-mode control method for stabilization of nonholonomic spherical robot, BYQ_III. Both controllers make the system converge to a neighborhood of the desired point. Furthermore, we developed a line following controller for tracking any desired straight line. Then the robot can be stabilized for tracking a lean angle trajectory in which the desired path curvature is identical to the desired value. Global asymptotic stabilization and tracking result are obtained, and the effectiveness of the controllers is shown by numerical simulations.

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