

# Tracking Control of Unmanned Trimaran Surface Vehicle: Using Adaptive Unscented Kalman Filter to Estimate the Uncertain Parameters

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**Abstract** - This paper proposes an Adaptive unscented Kalman filter (UKF) based tracking controller to force underactuated nonlinear autonomous ships to follow a reference path under constant disturbances induced by wave, wind and ocean-current. The controller development is based on MIT based UKF and backstepping techniques. The AUKF is used to update the estimation of the uncertain parameters online to avoid the parameters' drift due to time-varying added mass matrices. Along the way of tracking control, we obtain a new stability result for nonlinear underactuated systems with non-vanishing uncertainties. Simulations conducted with respect to the tracking control of unmanned ship illustrate the effectiveness of our proposed controller.

**Index Terms** – Adaptive Unscented Kalman filter, Tracking control, Dynamic disturbances, Underactuated ships, Parameter estimation

## I. INTRODUCTION

Stabilization and tracking control of position (sway and surge) and orientation (yaw) of underactuated surface ships, have recently received considerable attention from the control community. One challenge of these problems is due to the fact that the dynamics of the ship differ from the dynamics of a rigid body on a plane because of hydrodynamical effects (behavior of ambient water) and the presence of friction terms from the motion in the water with both linear and quadratic velocity dependencies; another challenge is that the motion of the underactuated ship in question possesses three degrees of freedom (yaw, sway and surge neglecting the motion in roll, pitch and heave) while there are only two available controls (surge force and yaw moment) under a nonintegrable second order nonholonomic constraint.

Several linear controllers for path-tracking control using linear ship dynamics were proposed in [1] and [2] where loss of stability due to linearization was analyzed. The trajectory-tracking control problem of nonlinear underactuated ships has recently studied in [3], [4] and [5]. The main limitation of these designs is that the yaw velocity was required to be nonzero, i.e., a straight-line cannot be tracked. A high gain-based global practical tracking controller was developed in [6] by transforming the ship tracking system to a skew-symmetric form. Seemingly, Pettersen and Lefeber [7] were

the first to solve the problem of tracking a straight-line. The problem of trajectory-tracking control without imposing a nonzero yaw velocity was solved in [8]. Relevant independent work also includes [9], [10], [11] and [12] on local tracking control, trajectory planning and fuzzy logic approaches. The path-following problem for a fourth order ship model in the Serret-Frenet frame was addressed in [13] under constant, known direction ocean-current disturbance. Output maneuvering of a class of strict feedback nonlinear systems with an application to fully actuated ships was studied in [14]. Using a line-of-sight projection algorithm, Fossen et al. [15] proposed a controller to force an underactuated ship to follow a sequence of way-points connected by straight-line segments.

The dynamic equations of such vehicles when they move in water are coupled and highly nonlinear. The added mass matrices and hydrodynamic parameters, which characterize this motion, are always assumed to be constant. These restrictive assumptions imply that the ship must be same-submerge sphere all the time. Indeed, in underactuated ship control system, control design not only depends on the chosen design approach, but also depends on the precision of the available mathematical model, so these assumptions do not hold for real ships and will affect the tracking results. In this paper we use the AUKF to give more accurate estimations for these parameters. In AUKF, a cost function is developed based on the error between the covariance matrices of innovation and their corresponding estimations. The normal UKF is enhanced by the adaptive mechanism that tries to minimize the cost function by the MIT-principle. This improves the robustness of conventional UKF with respect to the uncertain and time-varying noise distribution in the real system.

From the above discussion, it is clear that design of a controller which can give accurate estimations for these time-varying parameters will improve the tracking result. The note is organized as follows. In Section II, we present the kinematic and dynamic model for an underactuated surface vessel and then transform the open-loop tracking dynamics into a more convenient form for the subsequent controller development. In Section III, we present the AUKF tracking control design. The corresponding simulations of ship

tracking control are given in Section IV. Concluding remarks are presented in Section V.

## II. KINEMATIC AND DYNAMIC MODEL DEVELOPMENT

We seek to control the ship motion in the horizontal plane, so we neglect the dynamics associated with the motion in heave, roll, and pitch when modeling the ship. Moreover, we do not include the environmental forces due to wind, currents, and waves in the model. Furthermore, we assume that the inertia, added mass and damping matrices are diagonal. The notion equation of a model helicopter can be written with respect to the body frame with x-axis pointing to its head, y-axis going to the right of the body, and z is defined by the right-handed rule (Fig. 1).

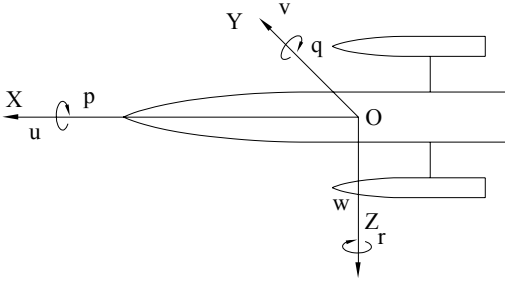


Fig. 1 Definition of the six-degree-of-freedom motion

Following the results in [11], the kinematics of the system can be written as

$$\begin{cases} \dot{x} = u \cos \varphi - v \sin \varphi \\ \dot{y} = u \sin \varphi + v \cos \varphi \\ \dot{\varphi} = r \end{cases} \quad (1)$$

where  $(x, y)$  denotes the coordinate of the center of mass of the surface vessel in the earth-fixed frame,  $\varphi$  is the orientation of the vessel, and  $u, v$  and  $r$  are the velocities in surge, sway and yaw, respectively.

The dynamics of the surface vessel is described as [11]

$$\begin{cases} \dot{u} = \frac{m_{22}}{m_{11}} vr - \frac{d_{11}}{m_{11}} u + \frac{1}{m_{11}} \tau_1 \\ \dot{v} = -\frac{m_{11}}{m_{22}} ur - \frac{d_{22}}{m_{22}} v \\ \dot{r} = \frac{m_{11} - m_{22}}{m_{33}} uv - \frac{d_{33}}{m_{33}} r + \frac{1}{m_{33}} \tau_3 \end{cases} \quad (2)$$

where  $m_{ii} > 0$  are given by the vessel inertia and the added mass effects,  $d_{ii} > 0$  are given by the hydrodynamic damping,  $m_{ii}$  is assumed to be time-varied and  $d_{ii}$  are assumed to be constant.  $\tau_1$  and  $\tau_3$  are the surge control force and the yaw control moment, respectively.

A wide range of control methodologies have been applied to previous underactuated ships, but the control of trimans has

less been mentioned. In this part we present the methodology of AUKF-based tracking control.

For future uses, our desired trajectory is generated by the dynamic equations of a virtual underactuated ship:

$$\begin{bmatrix} \dot{x}_d \\ \dot{y}_d \\ \dot{\varphi}_d \end{bmatrix} = \begin{bmatrix} \cos \varphi_d & -\sin \varphi_d & 0 \\ \sin \varphi_d & \cos \varphi_d & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

$$\begin{cases} \dot{u}_d = \frac{m_{22}}{m_{11}} v_d r_d - \frac{d_{11}}{m_{11}} u_d + \frac{1}{m_{11}} \tau_{1d} \\ \dot{v}_d = -\frac{m_{11}}{m_{22}} u_d r_d - \frac{d_{22}}{m_{22}} v_d \\ \dot{r}_d = \frac{m_{11} - m_{22}}{m_{33}} u_d v_d - \frac{d_{33}}{m_{33}} r_d + \frac{1}{m_{33}} \tau_{3d} \end{cases} \quad (4)$$

where  $(x_d, y_d, \varphi_d)$  denote the desired position and orientation of the virtual ship,  $(u_d, v_d, r_d)$  stand for the desired velocities, and  $(\tau_{1d}, \tau_{2d})$  are the reference inputs in surge and yaw. Throughout the remainder of the paper, it is assumed that the reference signals  $(x_d, y_d, u_d, v_d, r_d)$  (except the reference and  $\varphi_d$ ) and  $(\tau_{1d}, \tau_{2d})$  are bounded over  $[0, \infty)$ . This assumption is realistic from the physics of the problem.

To facilitate the controller design, we give (1) and (2) coordinate transformations. Applying the following state and input transformation [11]:

$$\begin{cases} x_1 = x \cos \varphi + y \sin \varphi \\ x_2 = v \\ x_3 = -x \sin \varphi + y \cos \varphi + \frac{m_{11}}{d_{22}} v \\ x_4 = \varphi \\ x_5 = r \\ x_6 = -\frac{m_{11}}{d_{22}} u - x_1 \end{cases} \quad (5)$$

$$\begin{cases} u_1 = \frac{m_{11} - m_{22}}{m_{33}} uv - \frac{d_{33}}{m_{33}} r + \frac{\tau_2}{m_{33}} \\ u_2 = \frac{d_{11} - d_{22}}{d_{22}} u - x_3 x_5 + \frac{\tau_1}{d_{22}} \end{cases} \quad (6)$$

Given a bounded feasible reference trajectory  $(x_d, y_d, \varphi_d, u_d, v_d, r_d)$  with reference input  $(\tau_{1d}, \tau_{2d})$  which satisfies (1) and (2), using the same transformation as (5), we have

$$\begin{cases} x_{1d} = x_d \cos \varphi_d + y_d \sin \varphi_d \\ x_{2d} = v_d \\ x_{3d} = -x_d \sin \varphi_d + y_d \cos \varphi_d + \frac{m_{11}}{d_{22}} v_d \\ x_{4d} = \varphi_d \\ x_{5d} = r \\ x_{6d} = -\frac{m_{11}}{d_{22}} u_d - x_{1d} \end{cases} \quad (7)$$

The tracking error is

$$e = [e_1, e_2, e_3, e_4, e_5, e_6]^T =$$

$$[x_1 - x_{1d}, x_2 - x_{2d}, x_3 - x_{3d}, x_4 - x_{4d}, x_5 - x_{5d}, x_6 - x_{6d}]$$

The control inputs

$$\begin{aligned} \tau_1 &= (d_{11} - d_{22})u - d_{22}x_3x_5 + d_{22} \left( w_{2d} - k_3 x_{5d} e_3 - k_3 x_{5d} e_3 \right. \\ &\quad \left. - k_4 e_6 - (k_3 k_4 + 1) z_{5d} e_3 - e_3 e_5 \right) \\ \tau_2 &= (m_{22} - m_{11})uv - d_{33}r + m_{33} (w_{1d} - (k_1 + k_2) e_5 - k_1 k_2 e_4) \end{aligned}$$

### III. ADAPTIVE AUKF-BASED ESTIMATION FOR MODEL PARAMETERS

Recently, researchers are focusing on the sequential estimation and its applications on active modeling and model-reference control [10]. The classical state estimator for nonlinear system is the extended Kalman Filter (EKF). The EKF has some deficiencies because of its linearization to the nonlinear dynamics. The UKF, on the other hand, has the same computational complexity with the EKF, but directly use the nonlinear models instead of linearizing it and can give the joint estimation of states and parameters. However, since the UKF is in the framework of Kalman filter, it can only achieve good performance under the assumption that some information has to be known a priori [7], but in practice this priori knowledge may not always maintain accurate because it is influenced by the dynamics and working environment of a mobile vehicle, both of which are time-varying and uncertain. One of the efficient ways to overcome the above mentioned weakness is to use an adaptive algorithm. In this paper we use the enhanced UKF which adds the adaptive mechanism named MIT-principle to minimize the cost function. This improves the robustness of conventional UKF with respect to the uncertain and time-varying noise distribution in the real system.

#### A. Standard UKF Approach

Consider a discrete-time nonlinear dynamic system:

$$\begin{cases} x_{k+1} = f(x_k, u_k) + w_k \\ y_k = h(x_k) + v_k \end{cases} \quad (8)$$

where  $x_k \in \mathfrak{R}^n$  is system state vector;  $y_k \in \mathfrak{R}^m$  is the output vector;  $u_k \in \mathfrak{R}^r$  is the input vector;  $w_k, v_k$  are uncorrelated zero-mean white noises.

Definition of Standard UKF:

##### 1) Initialization:

$$\begin{cases} \bar{x}_0 = E[x_0] \\ P_0 = E[(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T] \end{cases} \quad (9)$$

##### 2) Time Update:

$$\begin{cases} \mathcal{X}_{k-1} = [\bar{x}_{k-1}, \bar{x}_{k-1} + \sqrt{(n+\lambda)P_{k-1}}, \bar{x}_{k-1} - \sqrt{(n+\lambda)P_{k-1}}] \\ \mathcal{X}_{k|k-1}^* = f(\mathcal{X}_{k-1}) \\ \bar{x}_{k|k-1} = \sum_{i=0}^{2n} w_i^m \mathcal{X}_{i,k|k-1}^* \\ P_{k|k-1} = \sum_{i=0}^{2n} w_i^c (\mathcal{X}_{i,k|k-1}^* - \bar{x}_{k|k-1}) \\ \quad (\mathcal{X}_{i,k|k-1}^* - \bar{x}_{k|k-1})^T + Q \\ \mathcal{X}_{k|k-1} = [\bar{x}_{k|k-1}, \bar{x}_{k|k-1} + \sqrt{(n+\lambda)P_{k|k-1}}, \\ \quad \bar{x}_{k|k-1} - \sqrt{(n+\lambda)P_{k|k-1}}] \\ \gamma_{k|k-1} = h(\mathcal{X}_{k|k-1}) \\ \bar{y}_{k|k-1} = \sum_{i=0}^{2n} w_i^m \gamma_{i,k|k-1} \end{cases} \quad (10)$$

where  $Q$  is noise covariance, weights  $w_i^m, w_i^c$  are calculated as:

$$\begin{cases} w_0^m = \frac{\lambda}{n+\lambda} \\ w_0^c = \frac{\lambda}{n+\lambda} + (n - \alpha^2 + \beta) \\ w_i^m = w_i^c = \frac{\lambda}{2(n+\lambda)} \quad i=1, \dots, 2n \\ \eta = \sqrt{(n+\lambda)} \\ \lambda = n(\alpha^2 - 1) \end{cases} \quad (11)$$

where  $\alpha$  determines the spread of the sigma point;  $\beta$  is used to incorporate prior knowledge of the distribution of  $x$ ,  $n$  is the dimension of augmented state.

##### 3) Measurement Update:

$$\begin{cases} P_{\bar{y}_k \bar{y}_k} = \sum_{i=0}^{2n} w_i^c (\gamma_{i,k|k-1} - \bar{y}_{k|k-1}) \\ \quad (\gamma_{i,k|k-1} - \bar{y}_{k|k-1})^T + R \\ P_{\bar{x}_k \bar{y}_k} = \sum_{i=0}^{2n} w_i^c (\mathcal{X}_{i,k|k-1} - \bar{x}_{k|k-1})(\gamma_{i,k|k-1} - \bar{y}_{k|k-1})^T \\ K_k = P_{\bar{x}_k \bar{y}_k} P_{\bar{y}_k \bar{y}_k}^{-1} \\ P_k = P_{k|k-1} - K_k P_{\bar{y}_k \bar{y}_k} K_k^T \\ \bar{x}_k = \bar{x}_{k|k-1} + K_k (y_k - \bar{y}_{k|k-1}) \end{cases} \quad (12)$$

where  $R$  is the measurement noise covariance.

### B Adaptive UKF Approach

The traditional MIT rule is used in this paper as the adaptive law. With the MIT rule, the parameter can be adjusted in the negative gradient direction of the criterion function, and leads to the following recursive scheme:

$$q_k^m = q_{k-1}^m - \eta \frac{\partial V_k}{\partial q_k^m} \cdot dt \quad (13)$$

where :

$$\begin{aligned} \frac{\partial V_k}{\partial q_k^m} &= \frac{\partial}{\partial q_k^m} \left[ \text{tr}(\Delta S_k^2) \right] = \text{tr} \left( \frac{\partial \Delta S_k^2}{\partial q_k^m} \right) \\ &= \text{tr} \left( \frac{\partial \Delta S_k}{\partial q_k^m} \Delta S_k + \Delta S_k \frac{\partial \Delta S_k}{\partial q_k^m} \right) \end{aligned} \quad (14)$$

We can get

$$\frac{\partial \Delta S_k}{\partial q_k^m} = \frac{\partial}{\partial q_k^m} (S_k - \hat{S}_k) = \frac{\partial S_k}{\partial q_k^m} - \frac{\partial \hat{S}_k}{\partial q_k^m} \quad (15)$$

We can get the first term of (15):

$$\begin{aligned} \frac{\partial S_k}{\partial q_k^m} &= \frac{1}{N} \sum_{i=k-N+1}^k \left( \frac{\partial v_k}{\partial q_k^m} v_k^T + v_k \frac{\partial v_k^T}{\partial q_k^m} \right) \\ &= \frac{1}{N} \sum_{i=k-N+1}^k \left( \begin{array}{c} -\frac{\partial \bar{y}_{k|k-1}}{\partial q_k^m} (y_k - \bar{y}_{k|k-1})_k^T - \\ (y_k - \bar{y}_{k|k-1}) \frac{\partial \bar{y}_{k|k-1}^T}{\partial q_k^m} \end{array} \right) \end{aligned} \quad (16)$$

and the second term of (15) can be obtained :

$$\frac{\partial \hat{S}_k}{\partial q_k^m} = \sum_{i=0}^{2l} w_i^c \left[ \begin{array}{c} -\frac{\partial \bar{y}_{k|k-1}}{\partial q_k^m} (\gamma_{i,k|k-1} - \bar{y}_{k|k-1})^T - \\ (\gamma_{i,k|k-1} - \bar{y}_{k|k-1}) \frac{\partial \bar{y}_{k|k-1}^T}{\partial q_k^m} \end{array} \right] \quad (17)$$

To implement (16) and (17)  $\frac{\partial \bar{y}_{k|k-1}}{\partial q_k^m}$  is required.

Using the standard UK, we can get the recursive algorithm of  $\frac{\partial \bar{y}_{k|k-1}}{\partial q_k^m}$  as following:

1) Initialization:

$$\begin{cases} \frac{\partial \bar{x}_0}{\partial q_k^m} = 0 \\ \frac{\partial P_0}{\partial q_k^m} = 0 \end{cases} \quad (18)$$

2) Time Update:

$$\left. \begin{aligned} \frac{\partial \chi_{i,k-1}}{\partial q_k^m} &= \frac{\partial \bar{x}_{k-1}}{\partial q_k^m} + \sqrt{n+\lambda} \cdot \left( \frac{\partial \sqrt{P_{k-1}}}{\partial q_k^m} \right)_i, i=1, \dots, n \\ \frac{\partial \chi_{i,k-1}}{\partial q_k^m} &= \frac{\partial \bar{x}_{k-1}}{\partial q_k^m} - \sqrt{n+\lambda} \cdot \left( \frac{\partial \sqrt{P_{k-1}}}{\partial q_k^m} \right)_i, i=n+1, \dots, 2n \\ \frac{\partial \chi_{i,k|k-1}^*}{\partial q_k^m} &= \frac{\partial f}{\partial x} \Big|_{x=\chi_{i,k-1}} \cdot \frac{\partial \chi_{i,k-1}}{\partial q_k^m} \\ \frac{\partial \bar{x}_{k|k-1}}{\partial q_k^m} &= \sum_{i=0}^{2n} w_i^m \frac{\partial \chi_{i,k|k-1}^*}{\partial q_k^m} \\ \frac{\partial P_{k|k-1}}{\partial q_k^m} &= \sum_{i=0}^{2n} w_i^c \left\{ \left( \frac{\partial \chi_{i,k|k-1}^*}{\partial q_k^m} - \frac{\partial \bar{x}_{k|k-1}}{\partial q_k^m} \right) (\chi_{i,k|k-1}^* - \bar{x}_{k|k-1})^T \right. \\ &\quad \left. + (\chi_{i,k|k-1}^* - \bar{x}_{k|k-1}) \left( \frac{\partial \chi_{i,k|k-1}^*}{\partial q_k^m} - \frac{\partial \bar{x}_{k|k-1}}{\partial q_k^m} \right)^T \right\} + \frac{\partial Q}{\partial q_k^m} \\ \frac{\partial \chi_{i,k|k-1}}{\partial q_k^m} &= \frac{\partial \bar{x}_{k|k-1}}{\partial q_k^m} + \sqrt{n+\lambda} \left( \frac{\partial \sqrt{P_{k|k-1}}}{\partial q_k^m} \right)_i, i=1, \dots, n \\ \frac{\partial \chi_{i,k|k-1}}{\partial q_k^m} &= \frac{\partial \bar{x}_{k|k-1}}{\partial q_k^m} - \sqrt{n+\lambda} \left( \frac{\partial \sqrt{P_{k|k-1}}}{\partial q_k^m} \right)_i, i=n+1, \dots, 2n \\ \frac{\partial \gamma_{i,k|k-1}}{\partial q_k^m} &= \frac{\partial h}{\partial x} \Big|_{x=\chi_{i,k|k-1}} \cdot \frac{\partial \chi_{i,k|k-1}}{\partial q_k^m} \\ \frac{\partial \bar{y}_{k|k-1}}{\partial q_k^m} &= \sum_{i=0}^{2n} w_i^m \frac{\partial \gamma_{i,k|k-1}}{\partial q_k^m} \end{aligned} \right\} \quad (19)$$

3) Measurement Update:

$$\left. \begin{aligned} \frac{\partial P_{\bar{x}_k \bar{y}_k}}{\partial q_k^m} &= \sum_{i=0}^{2n} w_i^c \left\{ \left( \frac{\partial \chi_{i,k|k-1}}{\partial q_k^m} - \frac{\partial \bar{x}_{k|k-1}}{\partial q_k^m} \right) (\gamma_{i,k|k-1} - \bar{y}_{k|k-1})^T \right. \\ &\quad \left. + (\gamma_{i,k|k-1} - \bar{y}_{k|k-1}) \left( \frac{\partial \gamma_{i,k|k-1}}{\partial q_k^m} - \frac{\partial \bar{y}_{k|k-1}}{\partial q_k^m} \right)^T \right\} \\ \frac{\partial P_{\bar{y}_k \bar{y}_k}}{\partial q_k^m} &= \sum_{i=0}^{2n} w_i^c \left\{ \left( \frac{\partial \gamma_{i,k|k-1}}{\partial q_k^m} - \frac{\partial \bar{y}_{k|k-1}}{\partial q_k^m} \right) (\gamma_{i,k|k-1} - \bar{y}_{k|k-1})^T \right. \\ &\quad \left. + (\gamma_{i,k|k-1} - \bar{y}_{k|k-1}) \left( \frac{\partial \gamma_{i,k|k-1}}{\partial q_k^m} - \frac{\partial \bar{y}_{k|k-1}}{\partial q_k^m} \right)^T \right\} \\ \frac{\partial P_{\bar{x}_k \bar{y}_k}^{-1}}{\partial q_k^m} &= -P_{\bar{x}_k \bar{y}_k}^{-1} \frac{\partial P_{\bar{x}_k \bar{y}_k}}{\partial q_k^m} P_{\bar{x}_k \bar{y}_k}^{-1} \\ \frac{\partial K_k}{\partial q_k^m} &= \frac{\partial P_{\bar{x}_k \bar{y}_k}}{\partial q_k^m} P_{\bar{x}_k \bar{y}_k}^{-1} - P_{\bar{x}_k \bar{y}_k} P_{\bar{x}_k \bar{y}_k}^{-1} \frac{\partial P_{\bar{x}_k \bar{y}_k}}{\partial q_k^m} P_{\bar{x}_k \bar{y}_k}^{-1} \\ \frac{\partial P_k}{\partial q_k^m} &= \frac{\partial P_{k|k-1}}{\partial q_k^m} - \frac{\partial K_k}{\partial q_k^m} P_{\bar{x}_k \bar{y}_k}^T - K_k \left( \frac{\partial P_{\bar{x}_k \bar{y}_k}}{\partial q_k^m} \right)^T \\ \frac{\partial \bar{x}_k}{\partial q_k^m} &= \frac{\partial \bar{x}_{k|k-1}}{\partial q_k^m} + \frac{\partial K_k}{\partial q_k^m} (y_k - \bar{y}_{k|k-1}) - K_k \frac{\partial \bar{y}_{k|k-1}}{\partial q_k^m} \end{aligned} \right\} \quad (20)$$

## IV. SIMULATIONS OF SHIP TRACKING CONTROL

In this section we apply the proposed UKF based tracking controller design method to design the trajectory

tracking controller of an underactuated Trimaran, and study the effectiveness of the proposed control laws by simulation. Consider an underactuated surface vessel with the model parameters [16]  $m_{11} = 200kg$  ,  $m_{22} = 250kg$  ,  $m_{33} = 80kg$  ,  $d_{11} = 70kg/s$  ,  $d_{22} = 100kg/s$  ,  $d_{33} = 50kg/s$  . Assume the initial condition of the system is  $(x(0), y(0), \varphi(0), u(0), v(0), r(0)) = (30, -1, 0.2, 0, 0, 0)$ . The reference trajectory to be tracked is similar to that in [14]. Assume the initial reference states are  $x_d(0) = -2m$  ,  $y_d(0) = -20m$  ,  $\varphi_d(0) = 0rad$  ,  $u_d(0) = 1m/s$  ,  $v_d(0) = -0.1m/s$  ,  $r_d(0) = 0.05rad/s$  and  $u_d(t) = 1m/s$  ,  $r_d(t) = 0.05rad/s$  , the reference trajectory  $(x_d, y_d, \varphi_d, u_d, v_d, r_d)$  can be generated with proper  $\tau_{1d}$  and  $\tau_{2d}$  .

A. Tracking Control Simulations

Figs 2-5 show the estimation results for the active model estimation on Matlab simulation from which we can see the robust improvement due to the introduction of the proposed adaptive mechanism. The blue line is the UKF estimation, and the black line is the actual state. The main contributions of AUKF function are Estimating the uncertain parameters, avoiding the parameters' drift due to time-varying added mass matrices and removing the noise of the observed states.

Fig.6 and Fig.7 show the tracking control input of the underactuated surface ships, we can get that the control input of the AUKF-based tracking control is much stable than the tracking control without UKF.

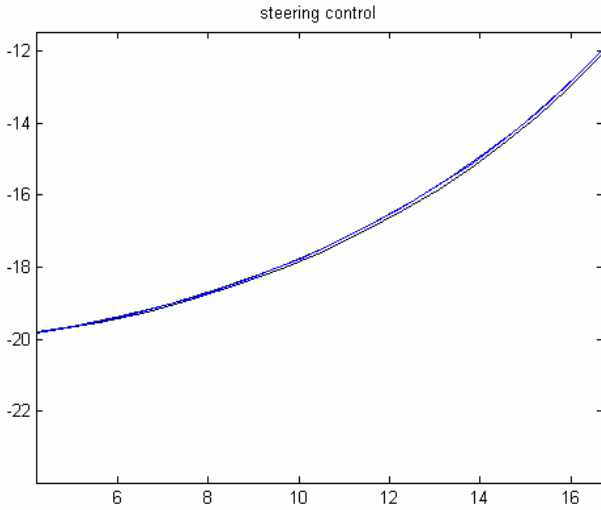


Fig. 2 Tracking control with Adaptive UKF.

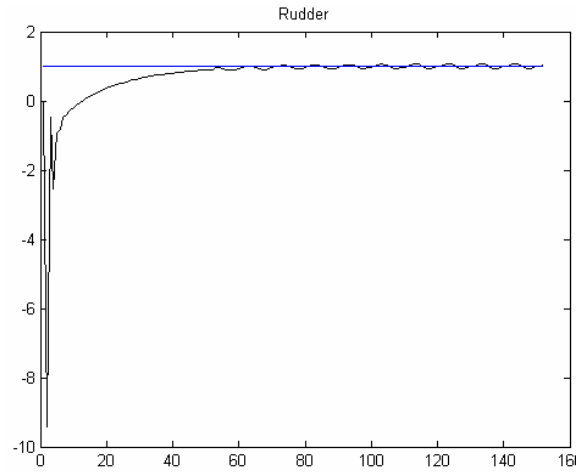


Fig. 3 Tracking control of surge velocity

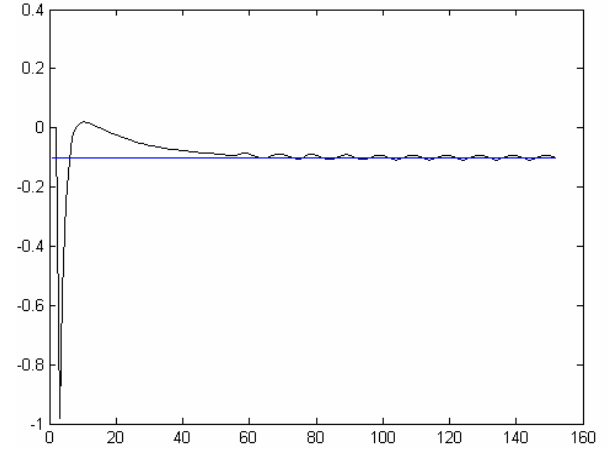


Fig. 4 Tracking control of sway velocity

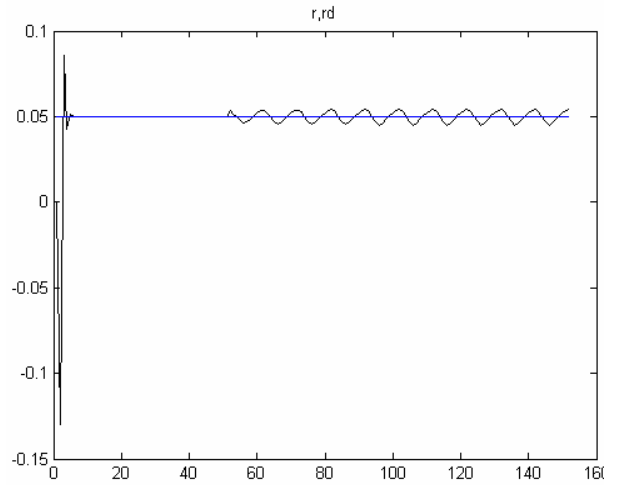


Fig. 5 Tracking control of yaw velocity

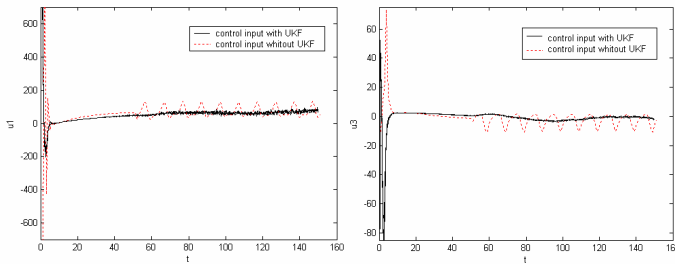


Fig. 6 Control input of  $\tau_1$

(..... is the control input without AUKF, — is the control input with AUKF)

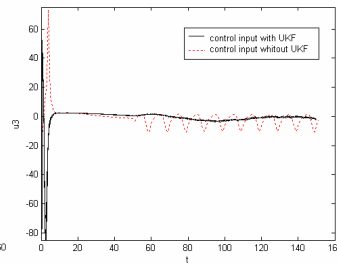


Fig. 7 Control input of  $\tau_2$

## B. Parameter Estimation

Added mass  $m_{11}$  has sinusoidal variation at 50s:

$$\begin{cases} m_{11} = 200 & t \leq 50s \\ m_{11} = 200 + 20\sin(\omega t) & t > 50s \end{cases}$$

$$\omega = 2\pi/10 \text{ rad/s}$$

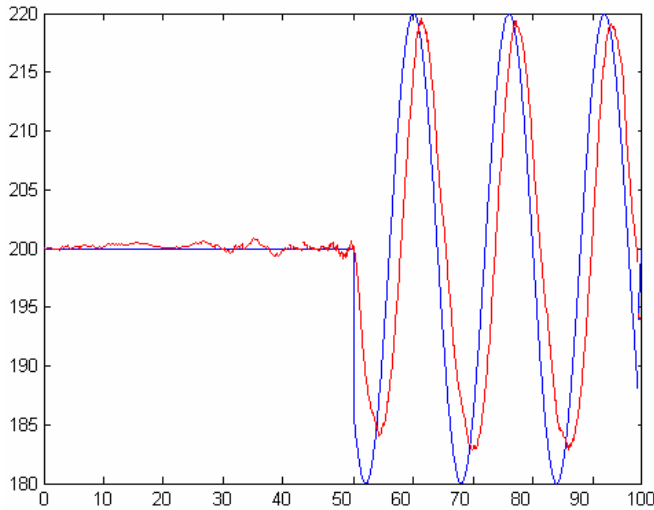


Fig.8 Estimation of sinusoidal varied  $m_{11}$

It is noted that the above simulation results illustrate the effectiveness of our proposed AUKF-based tracking control of unmanned ship. The proposed controller can estimate the uncertain parameters, avoid the parameters' drift due to time-varying added mass matrices and remove the noise of the observed states.

## V. CONCLUSION

In this paper, we introduced an Adaptive unscented Kalman filter (UKF) based tracking controller for online updating the time-varying added mass matrices and online modelling of the dynamic states and model uncertainty of underactuated surface ships. The normal UKF is enhanced by the adaptive mechanism that tries to minimize the cost function by the MIT-principle. A cost function is developed based on the error between the covariance matrices of innovation and their corresponding estimations. This improves the robustness of conventional UKF with respect to the uncertain and time-varying noise distribution in the real

system. This tracking controller can force the position/orientation tracing error to an arbitrarily small neighbourhood about zero. Simulations illustrate the effectiveness of our proposed controller.

## REFERENCES

- [1] F. A. Papoulias, "Cross track error and proportional turning rate guidance of marine vehicles," *J. Ship Res.*, vol. 38, pp. 123–132, 1994.
- [2] F. A. Papoulias and Z. O. Oral, "Hopf bifurcations and nonlinear studies of gain margins in path control of marine vehicles," *Appl. Ocean Res.*, vol. 17, pp. 21–32, 1995.
- [3] —, "Underactuated ship tracking control: theory and experiments," *Int. J. Contr.*, vol. 74, pp. 1435–1446, 2001.
- [4] E. Lefeber, Tracking control of nonlinear mechanical systems," Ph.D. dissertation, Dept. Mech. Eng., Univ. Twente, Twente, The Netherlands, 2000.
- [5] Z. P. Jiang, "Global tracking control of underactuated ships by Lyapunov's direct method," *Automatica*, vol. 38, pp. 301–309, 2002.
- [6] A. Behal, D. M. Dawson, W. E. Dixon, and Y. Fang, "Tracking and regulation control of an underactuated surface vessel with nonintegrable dynamics," *IEEE Trans. Autom. Control*, vol. 47, no. 4, pp. 495–500, Apr. 2002.
- [7] K. Y. Pettersen and E. Lefeber, "Way-point tracking control of ships," in *Proc. 40th IEEE Conf. Decision Control*, 2001, pp. 940–945.
- [8] K. D. Do, Z. P. Jiang, and J. Pan, "Underactuated ship global tracking under relaxed conditions," *IEEE Trans. Autom. Control*, vol. 47, no. 12, pp. 1529–1536, Dec. 2002.
- [9] L. Morawski and J. Pomirski, "Ship track-keeping: experiments with a physical tanker model," *Contr. Eng. Practice*, vol. 6, pp. 763–769, 1998.
- [10] H. Sira-Ramirez, "On the control of the underactuated ship: a trajectory planning approach," *Proc. 38th Conf. Decision and Control*, 1999, pp. 2192–2197.
- [11] G. J. Toussaint, T. Basar, and F. Bullo, "Tracking for nonlinear underactuated surface vessels with generalized forces," *Proc. IEEE Int. Conf. Control Applications*, 2000, pp. 355–360.
- [12] J. Velagic, Z. Vukic, and E. Omerdic, Adaptive fuzzy ship autopilot for track-keeping," *Contr. Eng. Practice*, vol. 11, pp. 433–443, 2003.
- [13] F. Repoulias, and E. Papadopoulos, "Trajectory planning and tracking control of underactuated AUVs," *Proceedings of the 2005 IEEE International Conference on Robotics and Automation*, Barcelona, Spain, April 2005, p1610-1615.
- [14] R. Skjetne, T. I. Fossen, and P. V. Kokotovic, "Output maneuvering for a class of nonlinear systems," *Automatica*, vol. 40, pp. 373–383, 2004.
- [15] C. Vuilmet, "A MIMO backstepping control with acceleration feedback for torpedo," *Proceedings of the 38th Southeastern Symposium on System Theory*, Tennessee Technological University Cookeville, TN, USA, March, 2006, p390-395
- [16] T. I. Fossen, Marine Control Systems. Trondheim, Norway: Marine Cybernetics, 2002.