Optimal Control of Polymer Flooding Injection Under Nonlinear Inequality Constraints

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***Abstract*—In order to implement the exploration schemes of polymer flooding economically, the optimal control problem (OCP) is investigated to determine the best injection strategies. The performance criterion is chosen as profit grained from enhanced oil recovery, and the average cost is considered as a nonlinear inequality constraint. The OCP is governed by a distributed parameter system (DPS), which is described by two-dimensional (2-D) fluid equations in porous media. A control vector parameterization (CVP) based approach is presented for solving the OCP numerically and the nonlinear inequality constraint is handled by penalty function method. An example of a 2-D polymer flooding process is solved demonstrating the effectiveness of the given method.**

***Keywords*—optimal control, distributed parameter systems, noninear inequality constraints, injection strategies, polymer flooding**

I. Introduction

Polymer flooding is a kind of techniques with high investment in enhanced oil recovery. It is important to optimize the injection strategies to maximize the profit gained from oil recovery. Since the oil recovery can be regarded as a continuous process with time, one should make a dynamic injection sequence in time for polymer flooding. Therefore, an optimal control based injection strategy is discussed here.

Many people have applied the optimal control theory in petroleum engineering. Ramirez and Fathi firstly established an optimal control model to design the injection process of surfactant flooding [1-3]. Brouwer and Sarma studied the dynamic optimization of water flooding with smart wells [4-7].Ye studied the optimal gas-cycling decision problem of a condensate reservoir [8]. Zhang and Li studied the optimal control of injection strategies for polymer flooding in enhanced oil recovery [9].

In the Zhang and Li's work [9], the optimal objective was to maximize the net present value (NPV) of profit within a given time. The governed equations are a set of partial differential equations (PDEs) which are the pressure equation, water saturation equation and polymer concentration equation of porous flow. By applying variational method, the necessary conditions of optimal control are deduced.

However, in practical situation, besides the governed PDEs, some inequality constraints may exist such as the amount of injection liquid per hour, the amount of polymer, etc. Thus in this paper, the optimal control of polymer flooding injection problem under nonlinear inequality constraints is investigated which is an improvement of the research in [9]. The constraint added is the average cost of per unit oil output from polymer flooding. In order to maximize the profit, the injection concentrate and total dosage of polymer will be increased and it will also raise the cost for oil production. So the average cost constraint is added to the optimal control model of polymer flooding. The solving method is based on control vector parameterization method [10], which converts an infinite dimensional optimal control problem into a nonlinear programming problem (NLP). The nonlinear constraints of optimal control are also transferred to the same constraints in the NLP. With the idea of penalty function, the constrained NLP is numerically solved by using negative gradient method. The gradient of optimal performance is derived from necessary conditions of optimality for optimal control problem governed by PDEs, which are reviewed in section II in this paper. A polymer flooding example is solved to demonstrate the practicality and effectiveness of the proposed approach.

II. Maximum Principle for 2-D DPS

Let $\Omega \subset R^2$ denote a 2-D domain with boundary $\partial\Omega$, $(x, y) \in \Omega$. Consider the optimal control problem of a distributed parameter systems defined on Ω, which can be formulated as follows,

$$\min_{v} \ J = \int_0^{t_f} \iint_\Omega F(u, u_x, u_y, u_{xx}, u_{yy}, v) d\sigma dt \quad (1)$$

$$s.t. \ f(\dot{u}, u, u_x, u_y, u_{xx}, u_{yy}, v) = 0 \quad (2)$$

where $u(x, y, t) \in R^n$ is the state vector, $v(x, y, t) \in R^m$ is the control vector. u_x, u_y, u_{xx} and u_{yy} are defined as,

$$u_x = \frac{\partial u}{\partial x}, u_y = \frac{\partial u}{\partial y}, u_{xx} = \frac{\partial^2 u}{\partial x^2}, u_{yy} = \frac{\partial^2 u}{\partial y^2}. \quad (3)$$

To find the minimum of the cost functional (1) under the dynamic constraint (2), let us introduce the Lagrange multipliers $\lambda \in R^n$, then the augmented performance functional is defined as,

$$J_A = J + \int_0^{t_f} \iint_\Omega \lambda^T f(\dot{u}, u, u_x, u_y, u_{xx}, u_{yy}, v) d\sigma dt . (4)$$

Define the Hamiltonian as $H = F + \lambda^T f$, then the augmented performance functional can be simplified.

$$J_A = \int_0^{t_f} \iint_\Omega H(\dot{u}, u, u_x, u_y, u_{xx}, u_{yy}, v) d\sigma dt . (5)$$

The necessary condition for an extremum of J_A is the first variation of augmented performance functional $\delta J_A = 0$, and δJ_A is,

$$\delta J_A = \int_0^{t_f} \iint_\Omega [(\frac{\partial H}{\partial u})^T \delta u + (\frac{\partial H}{\partial u_x})^T \delta u_x + (\frac{\partial H}{\partial u_{xx}})^T \delta u_{xx} + (\frac{\partial H}{\partial u_y})^T \delta u_y + (\frac{\partial H}{\partial u_{yy}})^T \delta u_{yy} + (\frac{\partial H}{\partial \dot{u}})^T \delta \dot{u} + (\frac{\partial H}{\partial v})^T \delta v] d\sigma dt \quad (6)$$

The variations δu_x , δu_{xx} , δu_y , δu_{yy} , and $\delta \dot{u}$ are not independent and can be expressed in terms of the variations δu by integrating the following terms by parts.

$$(\frac{\partial H}{\partial u_x})^T \delta u_x = \frac{\partial}{\partial x}[(\frac{\partial H}{\partial u_x})^T \delta u] - \frac{\partial}{\partial x}(\frac{\partial H}{\partial u_x})^T \delta u \quad (7)$$

$$(\frac{\partial H}{\partial u_y})^T \delta u_y = \frac{\partial}{\partial y}[(\frac{\partial H}{\partial u_y})^T \delta u] - \frac{\partial}{\partial y}(\frac{\partial H}{\partial u_y})^T \delta u \quad (8)$$

$$\begin{aligned}(\frac{\partial H}{\partial u_{xx}})^T \delta u_{xx} &= \frac{\partial}{\partial x}[(\frac{\partial H}{\partial u_{xx}})^T \delta u_x] - \frac{\partial}{\partial x}(\frac{\partial H}{\partial u_{xx}})^T \delta u_x \\ &= \frac{\partial}{\partial x}[(\frac{\partial H}{\partial u_{xx}})^T \delta u_x] - \frac{\partial}{\partial x}[\frac{\partial}{\partial x}(\frac{\partial H}{\partial u_{xx}})^T \delta u] \\ &+ \frac{\partial^2}{\partial x^2}(\frac{\partial H}{\partial u_{xx}})^T \delta u]\end{aligned} \quad (9)$$

$$\begin{aligned}(\frac{\partial H}{\partial u_{yy}})^T \delta u_{yy} &= \frac{\partial}{\partial y}[(\frac{\partial H}{\partial u_{yy}})^T \delta u_y] - \frac{\partial}{\partial y}(\frac{\partial H}{\partial u_{yy}})^T \delta u_y \\ &= \frac{\partial}{\partial y}[(\frac{\partial H}{\partial u_{yy}})^T \delta u_y] - \frac{\partial}{\partial y}[\frac{\partial}{\partial y}(\frac{\partial H}{\partial u_{yy}})^T \delta u] \\ &+ \frac{\partial^2}{\partial y^2}(\frac{\partial H}{\partial u_{yy}})^T \delta u]\end{aligned} \quad (10)$$

$$(\frac{\partial H}{\partial \dot{u}})^T \delta \dot{u} = \frac{\partial}{\partial t}[(\frac{\partial H}{\partial \dot{u}})^T \delta u] - \frac{\partial}{\partial t}(\frac{\partial H}{\partial \dot{u}})^T \delta u \quad (11)$$

By using the above expressions (7)-(11), the first variation of J_A can be written as follows,

$$\delta J = \int_{t_0}^{t_f} \iint_\Omega (H_u - \frac{\partial}{\partial x}\frac{\partial H}{\partial u_x} - \frac{\partial}{\partial y}\frac{\partial H}{\partial u_y} + \frac{\partial^2}{\partial x^2}\frac{\partial H}{\partial u_{xx}} + \frac{\partial^2}{\partial y^2}\frac{\partial H}{\partial u_{yy}} - \frac{\partial}{\partial t}\frac{\partial H}{\partial \dot{u}})^T \delta u \, d\sigma dt \quad (12)$$

$$+\int_{t_0}^{t_f} \iint_\Omega \frac{\partial}{\partial x}[(\frac{\partial H}{\partial u_x} - \frac{\partial}{\partial x}\frac{\partial H}{\partial u_{xx}})^T \delta u + (\frac{\partial H}{\partial u_{xx}})^T \delta u_x] d\sigma\, dt$$

$$+\int_{t_0}^{t_f} \iint_\Omega \frac{\partial}{\partial y}[(\frac{\partial H}{\partial u_y} - \frac{\partial}{\partial y}\frac{\partial H}{\partial u_{yy}})^T \delta u + (\frac{\partial H}{\partial u_{yy}})^T \delta u_y] d\sigma\, dt$$

$$+\int_{t_0}^{t_f} \iint_\Omega \frac{\partial}{\partial t}[(\frac{\partial H}{\partial \dot{u}})^T \delta u] d\sigma dt + \int_{t_0}^{t_f} \iint_\Omega (\frac{\partial H}{\partial v})^T \delta v d\sigma\, dt$$

Applying Pontryagin's Maximum Principle, the necessary conditions for an extremum of J_A are given as:

1) The adjoint equation

$$\frac{\partial H}{\partial u} - \frac{\partial}{\partial x}\frac{\partial H}{\partial u_x} + \frac{\partial^2}{\partial x^2}\frac{\partial H}{\partial u_{xx}} - \frac{\partial}{\partial y}\frac{\partial H}{\partial u_y} + \frac{\partial^2}{\partial x^2}\frac{\partial H}{\partial u_{yy}} - \frac{\partial}{\partial t}\frac{\partial H}{\partial \dot{u}} = 0 \quad (13)$$

2) Transversality Boundary Conditions

$$\iint_\Omega \frac{\partial}{\partial x}[(\frac{\partial H}{\partial u_x} - \frac{\partial}{\partial x}\frac{\partial H}{\partial u_{xx}})^T \delta u + (\frac{\partial H}{\partial u_{xx}})^T \delta u_x] d\sigma = 0 \quad (14)$$

$$\iint_\Omega \frac{\partial}{\partial y}[(\frac{\partial H}{\partial u_y} - \frac{\partial}{\partial y}\frac{\partial H}{\partial u_{yy}})^T \delta u + (\frac{\partial H}{\partial u_{yy}})^T \delta u_y] d\sigma = 0 \quad (15)$$

3) Transversality Final Conditions

$$\frac{\partial H}{\partial \dot{u}}=0,\ t=t_f \tag{16}$$

4) Optimal Control

With the first three necessary conditions being satisfied, the first variation becomes

$$\delta J_A=\int_0^{t_f}\iint_\Omega (\frac{\partial H}{\partial v})^T \delta v\, d\sigma\, dt \tag{17}$$

If the variation δv is not constrained, then the necessary condition for an extremum is $\partial H/\partial v=0$.

If the variation δv is constrained, which means that the control is at a constraint boundary, then the necessary condition for minimizing the performance functional is

$$\min_v H \tag{18}$$

III. Optimal Control Problem of Polymer Flooding

A. Optimal control model

The mathematic model of the dynamic procedure of polymer flooding is described by the PDEs of porous flow. Let $\mathbf{u}(x,y,t)=[u_1,u_2,u_3]^T$ denote the states of system, u_1,u_2 and $u_3(x,y,t)$ separately represent the pressure, water saturation and polymer concentration in the oil reservoir of an 2-D domain Ω. Then the optimal control model of polymer flooding can be formulated as follows,

$$\min_v J=\int_0^{t_f}\iint_\Omega [r\, q_{in} v-(1-f_w)q_{out}]d\sigma dt \tag{19}$$

$$s.t.\quad \sum_{l=x,y}\frac{\partial}{\partial l}(k\frac{\partial u_1}{\partial l})+q_{in}-q_{out}-\alpha\frac{\partial u_1}{\partial t}=0 \tag{20}$$

$$\sum_{l=x,y}\frac{\partial}{\partial l}(k_w\frac{\partial u_1}{\partial l})+q_{in}-q_{out}f_w -\beta_1\frac{\partial u_1}{\partial t}-\beta_2\frac{\partial u_2}{\partial t}=0 \tag{21}$$

$$\sum_{l=x,y}[\frac{\partial}{\partial l}(k_d\frac{\partial u_3}{\partial l})+k_w\frac{\partial u_1}{\partial l}\frac{\partial u_3}{\partial l}]+q_{in}(v-u_3) -\gamma_1\frac{\partial u_1}{\partial t}-\gamma_2\frac{\partial u_3}{\partial t}=0 \tag{22}$$

$$g(u,v,t)=\frac{\int_0^{t_f}\iint_\Omega C_{pol}\, q_{in} v\, d\sigma dt}{\int_0^{t_f}\iint_\Omega (1-f_w)q_{out}d\sigma dt-Q_{wf}}-\xi\le 0 \tag{23}$$

$$\frac{\partial \mathbf{u}}{\partial n}=0,\quad \mathbf{u}(x,y,0)=\mathbf{u}_0(x,y) \tag{24}$$

The objective of polymer flooding is to maximize the profit obtained by oil recovery, that is

$$\max J_p=\int_0^{t_f}\iint_\Omega [C_{oil}(1-f_w)q_{out}-C_{pol}\, q_{in} v]d\sigma dt \tag{25}$$

The performance functional (19) converts the maximization problem (25) to a minimization problem. r is a coefficient defined as C_{pol}/C_{oil}, C_{oil} is the price of oil, C_{pol} is the cost of polymer injection, q_{in} and q_{out} represent the flux of water injection and liquid production respectively. v denotes the inner boundary control of the injection concentrate of polymer. f_w is the water cut of the liquid phase in production well.

The mathematic model of polymer flooding consists of three PDEs, the pressure equation(20), the water saturation equation (21) and the polymer concentration equation(22).

The nonlinear constraint (23) means the average cost for unit oil production added from polymer flooding must be less a given value. Q_{wf} is the accumulative production of oil from water flooding with no polymer injection.

The following equations will be used to calculate of the coefficients in (20)-(23), the meaning of these equations can be found in [3, 11].

$$k=(\frac{K\, k_{ro}}{\mu_o}+\frac{K\, k_{rw}}{R_f\mu_p}),\ k_w=\frac{K\, k_{rw}}{R_f\mu_p} \tag{26}$$

$$k_D=D\phi S_w,\quad k_c=c_p\frac{K\, k_{rw}}{R_f\mu_p} \tag{27}$$

$$\alpha=\phi C_f,\beta_1=\phi S_w C_{fw},\ \beta_2=\phi, \tag{28}$$

$$\gamma_1=-\phi c_p c_{fp},\gamma_2=\phi C_{fc} \tag{29}$$

$$C_{fo}=\frac{\phi^0}{\phi}C_R+C_o\ ,\ C_{fw}=\frac{\phi^0}{\phi}C_R+C_w \tag{30}$$

$$c_{fc} = (S_w - A_d) + \frac{A_d}{\phi}, c_{fp} = \frac{\phi^0 C_R}{\phi}(S_w - A_d) \quad (31)$$

$$A_d = \frac{a\rho_R}{(1+b\,c_p)^2} \quad (32)$$

$$f_w = \frac{k_{rw}/\mu_w}{k_{rw}/\mu_w + k_{ro}/\mu_o} \quad (33)$$

$$k_{rw} = k_{rwro}\left(\frac{S_w - S_{wc}}{1 - S_{wc} - S_{or}}\right)^{n_w} \quad (34)$$

$$k_{ro} = k_{rocw}\left(\frac{1 - S_w - S_{or}}{1 - S_{wc} - S_{or}}\right)^{n_o} \quad (35)$$

$$\mu_w = \mu_{w0}[1 + (a_{p1}C_p + a_{p2}C_p^2 + a_{p3}C_p^3)] \quad (36)$$

$$R_k = 1 + \frac{(R_{k\max} - 1)\cdot brkp \cdot c_p}{1 + brkp \cdot c_p} \quad (37)$$

B. Necessary conditions for the OCP of polymer flooding

Applying the maximum principle of DPS to the OCP of polymer flooding (19)-(24), we can get the necessary conditions of the optimality.

1) The adjoint equations

$$\sum_{l=x,y}[\frac{\partial}{\partial l}(k\frac{\partial \lambda_1}{\partial l}) + \frac{\partial}{\partial l}(k_w\frac{\partial \lambda_2}{\partial l}) - \frac{\partial}{\partial l}(k_w\frac{\partial u_3}{\partial l}\lambda_3)] = -\alpha\frac{\partial \lambda_1}{\partial t} - \beta_1\frac{\partial \lambda_2}{\partial t} - \gamma_1\frac{\partial \lambda_3}{\partial t} \quad (38)$$

$$q_{out}\frac{\partial f_w}{\partial u_2} = -\beta_2\frac{\partial \lambda_2}{\partial t} \quad (39)$$

$$-\gamma_2\frac{\partial \lambda_3}{\partial t} = \sum_{l=x,y}[\frac{\partial}{\partial l}(k_d\frac{\partial \lambda_3}{\partial l}) - \frac{\partial}{\partial l}(k_w\frac{\partial u_1}{\partial l}\lambda_3)] - q_{in}\lambda_3 + q_{out}\frac{\partial f_w}{\partial u_3} \quad (40)$$

2) Boundary Conditions

$$\frac{\partial \boldsymbol{\lambda}}{\partial n} = 0, \text{ at boundary } \partial\Omega. \quad (41)$$

3) The terminal conditions at $t = t_f$

$$\boldsymbol{\lambda}(x, y, t_f) = 0 \quad (42)$$

4) The gradient of the functional J_A

$$\nabla J(v) = \iint_\Omega \frac{\partial H}{\partial v} d\sigma \quad (43)$$

$$\frac{\partial H}{\partial v} = q_{in}(r + \lambda_3) \quad (44)$$

IV. Numerical Solution

A. Control vector parameterization method

In CVP method, the time domain $[0, t_f]$ is divided into time grids $0 = t_0 < t_1 < ... < t_{N-1} < t_N = t_f$. In each time interval, the control trajectories are approximated by orthogonal functions or polynomials. That is

$$v(t) \approx \sum_{i=0}^{n} a_i g_i(t) \quad (45)$$

where $\{a_i\}$ are unknown parameters to be determined and $\{g_i(t)\}$ is a family of n orthogonal functions of polynomials functions [10]. Typical examples of the orthogonal functions are the Chebyshev etc. In this paper, piecewise constants are adapted for simplicity. Consequently, the original optimal control problem is converted into an NLP problem with the following form,

$$\min_{\mathbf{P}} J(\mathbf{p}) \quad (46)$$

$$s.t. \quad \mathbf{g}(\mathbf{p}) \le \mathbf{0} \quad (47)$$

$$\mathbf{p}_a \le \mathbf{p} \le \mathbf{p}_b \quad (48)$$

where $\mathbf{p} = (p_1, p_2, ..., p_N)^T$ is the vector of free parameter $\{a_i\}$, and $\mathbf{g}(\mathbf{p})$ are nonlinear inequality constraints, $\mathbf{p}_a, \mathbf{p}_b$ are the box constrains.

B. Solving the NLP

In order to solve the NLP, many algorithms can be adopted and they are mostly gradient based method, the gradient of the objective function can be derived by forward differences method, sensitivity equations method [10] or adjoint equations method. Since the model of polymer flooding is too complex; it is time consuming to use the first two approaches. The adjoint equations method is used to calculate the gradient of objective in this paper. The gradient of the NLP objective function is,

$$\nabla J(\mathbf{p}) = \int_0^{t_f} \iint_\Omega (\frac{\partial H}{\partial v}) \frac{\partial v}{\partial \mathbf{p}} d\sigma\, dt \tag{49}$$

The box constraints are easily handled by choosing the boundary values when the decision variables exceed the up and down limitations. Penalty methods are used to deal with the nonlinear constraints, which approximate the constrained problem by solving a sequence of unconstrained method. In the original optimal control problem (19)-(24), the nonlinear constraint (23) is a function of both control and state variables, so the converted performance is formulated as follows,

$$\begin{aligned}\min_v J_k = &\int_0^{t_f} \iint_\Omega [r\, q_{in} v - (1-f_w) q_{out}] d\sigma dt \\ &+ M_k\, h(u,v,t)\end{aligned} \quad . (50)$$

Here M_k is known as the penalty parameter, $h(u,v,t)$ is the penalty function of $g(u,v,t)$. Because the performance is changed when constraint is violated, some modification must be made for the adjoint problem and the gradient calculation. For convenience we transform the constraint (23) to a similar form to(19),

$$\int_0^{t_f} \iint_\Omega C_{pol}\, q_{in} v - \xi(1-f_w) q_{out}\, d\sigma dt - \xi Q_{wf} \le 0 \quad .(51)$$

We use (51) to calculate the new adjoint system and the gradient $\partial H / \partial v$. By using penalty method, there is only one adjoint system has to be solved at each iteration.

During the solution of the NLP, two sets of PDEs must be solved, the states equations and the adjoint equation. These two set of PDEs are solved by finite difference method in this paper.

C. Solving Steps

The steps for solving the OCP for polymer flooding are given below:

1) Select a trial parameter vector $\mathbf{p} = \mathbf{p}_0$, let $k = 0$ and chose a initial penalty parameter $M_k = M^0$.

2) Get the control variables $v(t)$ from the vector $\mathbf{p}$, and then integrate the states equations forward in time with the initial condition and the boundary conditions.

3) Integrate the adjoint equations backward in time with the specified final condition the boundary conditions.

4) Solving the unconstrained optimization problem (46)-(48) with M_k .

5) If the final convergence test satisfied stop, otherwise Go to step 2).Here the stop criteria is $M_k \left| g(u,v) \right| < \varepsilon$, where k is the current iterative counts, ε is a small positive number.

V. Case Study

Considering the optimal control problem of a two dimensional polymer flooding process, the length and width of the oil reservoir are also 300 meters, and height is 3 meters. For the simulation of the polymer flooding process, the reservoir is discretized into 10×10 blocks, and there is a water injection well at block (1, 1) and a oil production well at bock (10, 10). The optimal control problem is formulate by the performance index (19), states equations (20)-(24), and the adjoint equations (38)-(42). The control variable is v , which represents the injection concentration of polymer. The control variables are approximated by piecewise constants with fixed time grids. The time domain is (0, 9000) (day) and the time interval of time grid in CVP is 60 days. The initial states are denoted as,

$$u_1(x,y,0) = u_1^0, u_2(x,y,0) = u_2^0, u_3(x,y,0) = u_3^0 \tag{52}$$

The injection of polymer began from the water cut of production well reach 95% (3650 days), and ended in 1200 days later. The initial injection concentrate is 1.5 g/L and the corresponding profit is ￥8758.98×10^4 .

The data for polymer flooding model are given in the following table.

By using the given method, the optimal injection strategies are obtained. Fig.1 shows the initial and optimal control trajectories of injection concentration. Fig.2 compares the initial and optimal water cut curves of production well. The optimal profit is ￥10915.02×10^4 and with the average cost for unity oil production added from polymer flooding reached 100 ￥$/ton$

TABLE I. DATA OF POLYMER FLOODING MODEL

Symbol	Value	Symbol	Value
q_{in}	20 m^3/day	q_{out}	20 m^3/day
u_1^0	12 MPa	u_2^0	0.35
u_3^0	0 g/L	K	2.5 μm^2
μ_o	50 $mPa \cdot s$	μ_{w0}	1 $mPa \cdot s$
D	1e-5 m^2/s	C_w	0.0046 MPa^{-1}
C_r	0.00938 MPa^{-1}	ξ	100 ￥$/ton$
C_{oil}	3850￥$/kg$	C_{pol}	15000￥$/kg$
k_{rwro}	0.5228	k_{rocw}	0.9
S_{wc}	0.22	S_{or}	0.25
n_w	2.3447	n_o	4.287
ρ_R	2 g/cm^3	a_{p1}	15.426 $(g/L)^{-1}$
a_{p2}	0.4228 $(g/L)^{-2}$	a_{p3}	0.2749 $(g/L)^{-3}$
a	0.03 cm^3/g	b	3.8 $(g/L)^{-1}$
$R_{k\max}$	1.15	$brkp$	1.2

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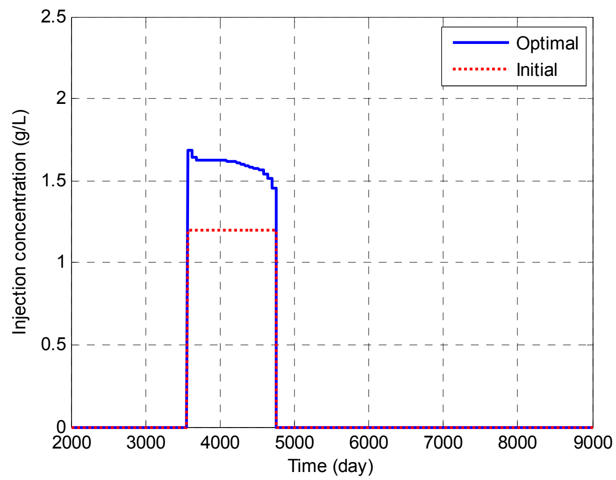

Figure 1. Polymer concentration of the water injection wel

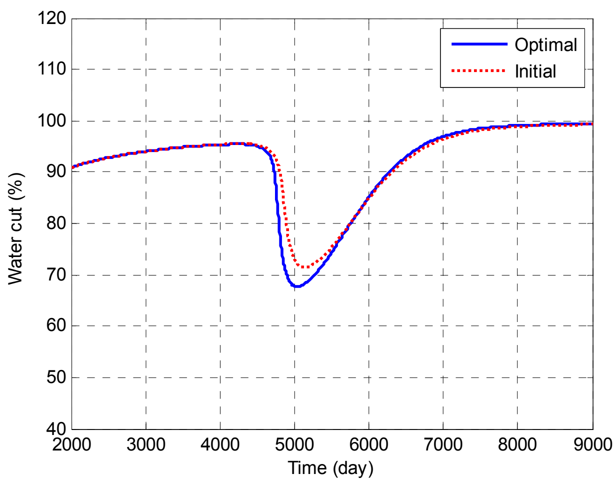

Figure 2. Water cut of the production well

VI. CONCLUSIONS

Optimal control approach provide an effective way to determining the injection strategies of polymer flooding in enhanced oil recovery. The infinite optimal control problem is converted to NLP by using control vector parameterization, and the NLP is solved by the gradient based algorithms rather than intelligent algorithms for the consideration of computation cost. In order to calculate the gradient, the adjoint method is adopted in this paper which applying the necessary conditions of 2-D distributed parameter systems to the OCP of the polymer flooding.

Considering the practical situation, the average cost per unit oil production is added as a nonlinear inequality constraint, which is handled by penalty function method. This work can be improved to involve other constraints for the factual polymer flooding process. It is also worthy to improved the efficiency of computation for solving the optimal control problem of large scale polymer flooding models.

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