The Nonlinear Controller Design of Generator System with Dynamic Uncertainties

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***Abstract*—By combining backstepping techniques and small-gain theorem, a nonlinear controller for generator system with dynamic uncertainties is proposed to make rotor angle and voltage stable. For the high-order model of generator system, the excitation control is designed by the modified backstepping techniques and dynamic uncertainties are controlled by the small-gain method, which render the whole closed-loop system asymptotically stable. Since the controller design is based completely on the nonlinear dynamic system without any linearization, the nonlinear property of the system is used to design the nonlinear robust controller. Simulations have shown the effectiveness of the proposed design method.**

***Index Terms*—dynamic uncertainty, small-gain, backstepping, generator system**

I. Introduction

The stability problem of power systems is always important, and the development of the large-scale power systems requires a high degree of reliability. Power systems are nonlinear dynamic systems with complicated structures [2], [3], [4], where possibly there exist various uncertain factors. The obtained control laws are focused on the research of the system with static uncertainties [5], [6], [7]. However, the system with dynamic uncertainties is studied rarely. In the generator system, the part of dynamic uncertainties comes from modelling simplifications and modelling error, for which the general robust controllers are often ineffective. Therefore, the design approach to the dynamically uncertain system is needed.

The small-gain technique is an important design idea in the field of nonlinear control. For the interconnected systems, the nonlinear robust controller based on small-gain technique can make closed-loop system stable. In 1963, Zames presented the thesis about the small-gain principle [8], which becomes one important basis of modern robust control theory. With the improvement of small-gain theorem [1], [9], it becomes an effective tool in the nonlinear control. Recently, the small-gain techniques are joined with other design methods to design nonlinear systems with dynamic uncertainties [10].

In this paper, for the high-order model of generator system, we combine the Backstepping techniques and small-gain method to design the excitation controller, which renders the generator system stable. The design procedure is divided into two steps. Firstly, the certain part of generator system is designed by Backstepping techniques, which make power angle and voltage stable. Secondly, according to the small-gain theorem, we deal with the dynamic uncertainties, which ensures that the whole system is stable. Finally, simulations show that the design result is effective to the system.

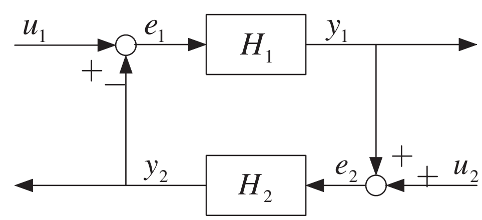

Fig. 1. Feedback system

II. Preliminaries and Key Lemmas

As shown in Fig. 1, there are two systems $H_1 : L_{pe} \to L_{qe}$ and $H_2 : L_{qe} \to L_{pe}$. Suppose both systems are finite-gain L stable, that is

$$\|y_{1\tau}\|_L \le \gamma_1 \|e_{1\tau}\|_L + \beta_1, \quad \forall e_1 \in L_{pe}, \quad \forall \tau \in \mathbb{R}_+ \tag{1}$$

$$\|y_{2\tau}\|_L \le \gamma_2 \|e_{2\tau}\|_L + \beta_2, \quad \forall e_2 \in L_{qe}, \quad \forall \tau \in \mathbb{R}_+ \tag{2}$$

Suppose further that the feedback system is well defined in the sense that for every pair of inputs $u_1 \in L_{pe}$ and $u_2 \in L_{qe}$, there exist unique outputs $e_1, y_2 \in L_{pe}$ and $e_2, y_1 \in L_{qe}$. Define

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \quad y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}, \quad e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

The question of interest is whether the feedback connection, when viewed as a mapping from the input u to the output e or a mapping from input u to the output y, is finite-gain L stable. The following theorem, known as the Small-Gain Theorem, gives a sufficient condition for finite-gain L stability of the feedback connection.

Theorem 1: (Small-Gain Theorem) [1] Under the preceding assumptions, if

$$\gamma_1 \gamma_2 < 1$$

then the feedback connection is finite-gain L stable, and for all inputs $u_1 \in L_{pe}$ and $u_2 \in L_{qe}$,

$$\|e_{1\tau}\|_L \leq \frac{1}{1-\gamma_1\gamma_2}(\|u_{1\tau}\|_L + \gamma_2\|u_{2\tau}\|_L + \beta_2 + \gamma_2\beta_1) \quad (3)$$

$$\|e_{2\tau}\|_L \leq \frac{1}{1-\gamma_1\gamma_2}(\|u_{2\tau}\|_L + \gamma_1\|u_{1\tau}\|_L + \beta_1 + \gamma_1\beta_2) \quad (4)$$

for all $\tau \in \mathbb{R}_+$; if $u_1 \in L_p$ and $u_2 \in L_q$, then $e_1, y_2 \in L_p$ and $e_2, y_1 \in L_q$.

Proof: See reference [1] ■

On the basis of Small-Gain Theorem, a lot of research work has done. Now, the small-gain technique has become an effective tool in the field of nonlinear control. In this paper, the Lyapunov formulation of nonlinear Small-Gain Theorem is used to design the interconnected systems, which is given as follows:

Theorem 2: [9] Consider the interconnected systems

$$\begin{cases} \dot{x}_1 = f_1(x_1, x_2, u_1) \\ \dot{x}_2 = f_2(x_1, x_2, u_2) \end{cases} \quad (5)$$

where, for $i = 1, 2$, $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{p_i}$ and $f_i : \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \times \mathbb{R}^{p_i} \to \mathbb{R}^{n_i}$ is locally Lipschitz. Assume that there exists an input-to-state practically stable (ISpS) Lyapunov function $V_i(\cdot)$, for the x_i-subsystem such that the following hold:

- there exist functions $\varphi_{i1}(\cdot), \varphi_{i2}(\cdot) \in K_\infty$ such that

$$\varphi_{i1}(\|x_i\|) \leq V_i(x_i) \leq \varphi_{i2}(\|x_i\|), \quad \forall x_i \in \mathbb{R}^{n_i} \quad (6)$$

- there exist functions $\alpha_i(\cdot) \in K_\infty$, $\chi_i(\cdot), \gamma_i(\cdot) \in K$ and some constant $c_i \geq 0$ such that $V_1(x_1) \geq \max\{\chi_1(V_2(x_2)), \gamma_1(\|u_1\|) + c_1\}$ implies

$$\nabla V_1(x_1) f_1(x_1, x_2, u_1) \leq -\alpha_1(V_1) \quad (7)$$

and $V_2(x_2) \geq \max\{\chi_2(V_1(x_1)), \gamma_2(\|u_2\|) + c_2\}$ implies

$$\nabla V_2(x_2) f_2(x_1, x_2, u_2) \leq -\alpha_2(V_2) \quad (8)$$

if there exists some $c_0 \geq 0$ such that

$$\chi_1 \circ \chi_2(r) < r, \quad \forall r > c_0 \quad (9)$$

then the interconnected system (5) is ISpS. Furthermore, if $c_0 = c_1 = c_2 = 0$, the system is input-to-state stable (ISS).

Proof: See reference [9] ■

In addition, Young's inequality is given as follows:

Lemma 3: (Young's inequality) For any two vectors x and y, the following holds:

$$x^T y \leq \frac{\varepsilon^p}{p}\|x\|^p + \frac{1}{q\varepsilon^q}\|y\|^q \quad (10)$$

where $\varepsilon > 0$ and the constants $p > 1$ and $q > 1$ satisfy $(p-1)(q-1) = 1$.

III. Nonlinear Controller Design

A. Single Machine Model

The model of single machine infinite-bus system is presented by the following dynamic functions:

$$\begin{cases} \dot{\delta} = \omega - \omega_0 \\ \dot{\omega} = -\frac{D}{H}(\omega - \omega_0) + \frac{\omega_0}{H}(P_m - P_e) \\ \dot{E}'_q = -\frac{1}{T'_d}E'_q + \frac{1}{T_{d0}}\frac{X_d - X'_d}{X'_{d\Sigma}}V_s \cos\delta + \frac{1}{T_{d0}}V_f \end{cases} \quad (11)$$

where

$$P_e = \frac{E'_q V_s}{X'_{d\Sigma}}\sin\delta + \frac{V_s^2}{2}\left(\frac{X'_{d\Sigma} - X_{q\Sigma}}{X'_{d\Sigma}X_{q\Sigma}}\right)\sin 2\delta \quad (12)$$

List of symbols is as follows:
δ is the power angle, ω the relative speed, ω_0 the synchronous machine speed ($\omega_0 = 2\pi f_0$), P_m the mechanical input power, P_e the electric power, E'_q the q-axis internal transient voltage, D the per-unit damping constant, H the inertia constant, V_s the infinite bus voltage, X'_d the d-axis transient reactance, X_d the d-axis reactance, X_q the q-axis transient reactance, V_f the excitation control variable, T_{d0} the d-axis open circuit transient time constant, V_t the end voltage, $T'_d = T_{d0}\frac{X'_{d\Sigma}}{X_{d\Sigma}}$, $X'_{d\Sigma} = X'_d + X_T + X_L$, $X_{q\Sigma} = X_q + X_T + X_L$, $X_{d\Sigma} = X_d + X_T + X_L$, where X_L the reactance of transmission line, X_T the reactance of transformer.

B. Control Model

Firstly, the generator system with dynamic uncertainties is presented by the following model:

$$\dot{\mu} = Q(\mu, \delta) \quad (13)$$

$$\begin{aligned} \dot{\delta} &= \omega - \omega_0 \\ \dot{\omega} &= -\frac{D}{H}(\omega - \omega_0) + \frac{\omega_0}{H}(P_m - P_e) \\ \dot{E}'_q &= -\frac{1}{T'_d}E'_q + \frac{1}{T_{d0}}\frac{X_d - X'_d}{X'_{d\Sigma}}V_s\cos\delta + \frac{1}{T_{d0}}V_f \end{aligned} \quad (14)$$

where $\mu \in \mathbb{R}^{n_0}$, the integer $n_0 \geq 1$. The system structure consists of two parts: the equations (14) is main part of the system, which is certain; μ-subsystem (13) is the uncertain part, which has the influence on the machine power P_m. These two parts are interconnected to one complex system.

The equations (13)-(14) are rewritten into the incremental form. Setting the work point $(\mu_0, \delta_0, \omega_0, E'_{q0})$, the system is changed into

$$\Delta\dot{\mu} = \Delta Q(\Delta\mu, \Delta\delta) \quad (15)$$

$$\begin{aligned} \Delta\dot{\delta} &= \Delta\omega \\ \Delta\dot{\omega} &= -\frac{D}{H}\Delta\omega - \frac{\omega_0}{H}\left[\frac{V_s}{X'_{d\Sigma}}E'_{q0}(\sin(\delta_0 + \Delta\delta) - \sin\delta_0) \right. \\ &\quad + \frac{V_s}{X'_{d\Sigma}}\Delta E'_q \sin(\delta_0 + \Delta\delta) + \frac{V_s^2}{2}\left(\frac{X'_{d\Sigma} - X_{q\Sigma}}{X'_{d\Sigma}X_{q\Sigma}}\right) \\ &\quad \left.(\sin 2(\delta_0 + \Delta\delta) - \sin 2\delta_0) - \Delta P_m(\Delta\mu)\right] \\ \Delta\dot{E}'_q &= -\frac{1}{T'_d}\Delta E'_q + \frac{1}{T_{d0}}\frac{X_d - X'_d}{X'_{d\Sigma}}V_s(\cos(\delta_0 + \Delta\delta) \\ &\quad - \cos\delta_0) + \frac{1}{T_{d0}}V_f \end{aligned} \quad (16)$$

where $\sin(\delta_0+\Delta\delta)\neq 0$, the machine input power satisfies the following assumption:

Assumption 1: There exists a known smooth function $P(\cdot)$, $P(0)=0$, which satisfies

$$|\Delta P_m(\Delta\mu)|\leq P(|\Delta\mu|) \tag{17}$$

The dynamic uncertainties satisfy the following assumption:

Assumption 2: The μ-subsystem (15) has the ISS-Lyapunov function $V_0(\cdot)$, which satisfy

$$\frac{\partial V_0}{\partial\Delta\mu}\Delta Q(\Delta\mu,\Delta\delta)\leq -\alpha_z(V_0(\Delta\mu))+\upsilon_z(|\Delta\delta|) \tag{18}$$

where, $\alpha_z(\cdot)$ and $\upsilon_z(\cdot)\in K_\infty$.

C. Controller Design

Set the states $x_1=\Delta\delta$, $x_2=\Delta\omega$, $x_3=\Delta E'_q$, the state of uncertain part $z=\Delta\mu$, the input $u=V_f$; Let $a_1=\frac{\omega_0 V_s}{HX'_{d\Sigma}}$, $a_2=\frac{V_s}{T_{d0}}\frac{X_d-X'_d}{X'_{d\Sigma}}$, $a_3=\frac{\omega_0 V_s^2}{2H}\left(\frac{X'_{d\Sigma}-X_{q\Sigma}}{X'_{d\Sigma}X_{q\Sigma}}\right)$, the following equations are obtained

$$\begin{aligned}
\dot z &= \Delta Q(z,x_1) \qquad (19)\\
\dot x_1 &= x_2\\
\dot x_2 &= -\frac{D}{H}x_2-a_1E'_{q0}(\sin(\delta_0+x_1)-\sin\delta_0)\\
&\quad -a_1x_3\sin(\delta_0+x_1)-a_3(\sin 2(\delta_0+x_1)-\sin 2\delta_0)\\
&\quad +\frac{\omega_0}{H}\Delta P_m(z)\\
\dot x_3 &= -\frac{1}{T'_d}x_3+a_2(\cos(\delta_0+x_1)-\cos\delta_0)+\frac{1}{T_{d0}}u \qquad (20)
\end{aligned}$$

Based on small-gain method, the design produre is divided into two steps. Firstly, x-subsystem with the lower-triangular structure is designed by the Backstepping technique. Secondly, according to Small-Gain Theorem, we deal with the dynamic uncertainties.

D. Design x-Subsystem by Using Backstepping Technique

Set $b_1(x_1)=a_1E'_{q0}(\sin(\delta_0+x_1)-\sin\delta_0)+a_3(\sin 2(\delta_0+x_1)-\sin 2\delta_0)$, $b_2(x_1)=a_2(\cos(\delta_0+x_1)-\cos\delta_0)$.

Firstly, let $\xi_1=x_1$, the Lyapunov function $V_1=\frac{\xi_1^2}{2}$ and the derivative of it is

$$\dot V_1=\xi_1x_2$$

The virtual controller is taken as $\alpha_2=-C_1\xi_1$, we have

$$\dot V_1=-C_1\xi_1^2+\xi_1(x_2-\alpha_2)$$

Secondly, let $\xi_2=x_2-\alpha_2$, then $\dot\xi_2=\dot x_2+C_1x_2$. The Lyapunov function is $V_2=V_1+\frac{\xi_2^2}{2}$. Its derivative is

$$\begin{aligned}
\dot V_2 &= -C_1\xi_1^2+\xi_2\Big[-a_1x_3\sin(\delta_0+x_1)-\frac{D}{H}x_2\\
&\quad +\frac{\omega_0}{H}\Delta P_m(z)-b_1(x_1)+\xi_1+C_1x_2\Big] \qquad (21)
\end{aligned}$$

Based on Assumption 1 and Young's inequality (10), it is obtained

$$\xi_2\frac{\omega_0}{H}\Delta P_m(z)\leq|\xi_2|\frac{\omega_0}{H}P(|z|)\leq\frac{\omega_0^2}{H^2}\xi_2^2+\frac{1}{4}P(|z|)^2 \tag{22}$$

Substituting (22) into (21) yields

$$\begin{aligned}
\dot V_2 &\leq -C_1\xi_1^2+\xi_2\Big[-a_1\sin(\delta_0+x_1)x_3-b_1(x_1)+\xi_1\\
&\quad +\Big(C_1-\frac{D}{H}\Big)x_2+\frac{\omega_0^2}{H^2}\xi_2\Big]+\frac{1}{4}P(|z|)^2 \qquad (23)
\end{aligned}$$

From $\sin(\delta_0+x_1)\neq 0$, take the virtual controller as

$$\alpha_3=\frac{\Big(C_2+\frac{\omega_0^2}{H^2}\Big)\xi_2+\Big(C_1-\frac{D}{H}\Big)x_2-b_1(x_1)+\xi_1}{a_1\sin(\delta_0+x_1)}$$

which renders

$$\dot V_2\leq-\sum_{i=1}^{2}C_i\xi_i^2-a_1\sin(\delta_0+x_1)\xi_2(x_3-\alpha_3)+\frac{1}{4}P(|z|)^2 \tag{24}$$

Thirdly, let $\xi_3=x_3-\alpha_3$, $V_3=V_2+\frac{\xi_3^2}{2}$, we have

$$\begin{aligned}
\dot V_3 &\leq -\sum_{i=1}^{2}C_i\xi_i^2+\frac{1}{4}P(|z|)^2+\xi_3\Big[\frac{1}{T_{d0}}u-\frac{1}{T'_d}x_3\\
&\quad +b_2(x_1)-a_1\sin(\delta_0+x_1)\xi_2-\frac{\partial\alpha_3}{\partial x_1}x_2\\
&\quad -\frac{\partial\alpha_3}{\partial x_2}\Big(-\frac{D}{H}x_2-b_1(x_1)-a_1x_3\sin(\delta_0+x_1)\\
&\quad +\frac{\omega_0}{H}\Delta P_m(z)\Big)\Big] \qquad (25)
\end{aligned}$$

Similar to (22), from Assumption 1 and Young's inequality (10), we have

$$-\xi_3\frac{\partial\alpha_3}{\partial x_2}\frac{\omega_0}{H}\Delta P_m(z)\leq\frac{\omega_0^2}{H^2}\xi_3^2\Big(\frac{\partial\alpha_3}{\partial x_2}\Big)^2+\frac{1}{4}P(|z|)^2 \tag{26}$$

Therefore, (25) is rewritten into

$$\begin{aligned}
\dot V_3 &\leq -\sum_{i=1}^{2}C_i\xi_i^2+\frac{1}{2}P(|z|)^2+\xi_3\Big[\frac{1}{T_{d0}}u\\
&\quad -a_1\sin(\delta_0+x_1)\xi_2-\frac{1}{T'_d}x_3+b_2(x_1)\\
&\quad -\frac{\partial\alpha_3}{\partial x_1}x_2-\frac{\partial\alpha_3}{\partial x_2}\Big(-\frac{D}{H}x_2-b_1(x_1)\\
&\quad -a_1x_3\sin(\delta_0+x_1)-\frac{\omega_0^2}{H^2}\frac{\partial\alpha_3}{\partial x_2}\xi_3\Big)\Big] \qquad (27)
\end{aligned}$$

With the choice of the state feedback controller

$$\begin{aligned}
u=V_f &= T_{d0}\Big[-C_3\xi_3+a_1\sin(\delta_0+x_1)\xi_2+\frac{1}{T'_d}x_3\\
&\quad -b_2(x_1)+\frac{\partial\alpha_3}{\partial x_1}x_2+\frac{\partial\alpha_3}{\partial x_2}\Big(-\frac{D}{H}x_2-b_1(x_1)\\
&\quad -a_1x_3\sin(\delta_0+x_1)-\frac{\omega_0^2}{H^2}\frac{\partial\alpha_3}{\partial x_2}\xi_3\Big)\Big] \qquad (28)
\end{aligned}$$

we have

$$\dot V_3\leq-\sum_{i=1}^{3}C_i\xi_i^2+\frac{1}{2}P(|z|)^2\leq-C_xV_3+\frac{1}{2}P(|z|)^2 \tag{29}$$

where $C_x = \min\{2C_i; i = 1, 2, 3\}$.

Therefore, x-subsystem is ISS under the nonlinear controller and all the states are asymptotically stable. Next, the dynamic uncertainties are handled by using the small-gain technique.

1) Deal with Dynamic Uncertainties by Small-Gain Technique: Form the above design, we have (29), and there exists $\upsilon_x(\cdot) \in K_\infty$ such that

$$\frac{1}{2}P(|z|)^2 \le \upsilon_x(|z|^2)$$

then

$$\dot{V}_3 \le -C_x V_3 + \upsilon_x(|z|^2) \quad (30)$$

From Assumption 2, it is known that the dynamic uncertainties are ISS, that is, there exist $\varphi_{z1}(\cdot)$ and $\varphi_{z2}(\cdot) \in K_\infty$, which satisfy

$$\varphi_{z1}(|z|) \le V_0(z) \le \varphi_{z1}(|z|)$$

Given any $0 < D_1 < C_x$, (30) ensure

$$\dot{V}_3 \le -D_1 V_n \quad (31)$$

as long as $V_3 \ge \{\frac{2}{C_x - D_1}\upsilon_x(|\varphi_{z1}^{-1}(V_0(z))|^2)\}$.

On the other hand, from Assumption 2 we have

$$\frac{\partial V_0}{\partial z}\Delta Q(z, x_1) \le -\alpha_z(V_0) + \upsilon_z(|x_1|) \quad (32)$$

In addition, $V_3 = \frac{1}{2}\sum_{i=1}^{3}\xi_i^2$ implies that

$$|x_1| \le \sqrt{2V_3}$$

Hence

$$\frac{\partial V_0}{\partial z}\Delta Q(z, x_1) \le -\alpha_z(V_0) + \upsilon_z(\sqrt{2V_3}) \quad (33)$$

Similarly, taking any $0 < D_2 < 1$, from (33) we obtain

$$\dot{V}_0 \le -D_2\alpha_z(V_0) \quad (34)$$

as long as $V_0 \ge \{\alpha_z^{-1} \circ \frac{2}{1 - D_1}\upsilon_x(\sqrt{2V_n})\}$.

According to Theorem 2, it is known that

$$\begin{aligned} \chi_1(s) &= \frac{2}{C_x - D_1}\upsilon_x(|\varphi_{z1}^{-1}(V_0(z))|^2) \\ \chi_2(s) &= \alpha_z^{-1} \circ \frac{2}{1 - D_1}\upsilon_x(\sqrt{2V_n}) \end{aligned}$$

From the condition (9), we have

$$\frac{2}{C_x - D_1}\nu_x\left(\left(\varphi_{z1}^{-1} \circ \alpha_z^{-1} \circ \frac{2}{1 - D_2}\nu_z(\sqrt{2s})\right)^2\right) < s, \quad \forall s > 0 \quad (35)$$

Via the appropriate choice of the design parameters, we are able to make the inequality (35) hold. Then the interconnected systems are ISS.

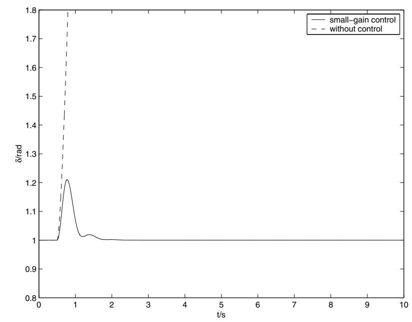

Fig. 2. Responses of δ to three-phase short circuit fault with dynamic uncertainties

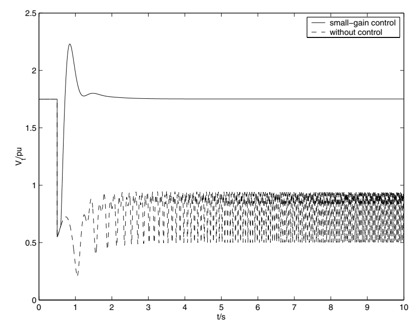

Fig. 3. Responses of V_t to three-phase short circuit fault with dynamic uncertainties

2) Design Result.: The main theorem of this paper is presented as follows:

Theorem 4: Consider the generator system with dynamic uncertainties (13)-(14), under Assumption 1 and Assumption 2, the excitation control is designed as

$$\begin{aligned} V_f \;=\; & T_{d0}\Bigg[-C_3\xi_3 + \frac{\omega_0 V_s}{HX'_{d\Sigma}}\sin(\delta_0 + x_1)\xi_2 + \frac{1}{T'_d}x_3 \\ & + \frac{\partial\alpha_3}{\partial x_1}x_2 - \frac{V_s}{T_{d0}}\frac{X_d - X'_d}{X'_{d\Sigma}}(\cos(\delta_0 + x_1) - \cos\delta_0) \\ & + \frac{\partial\alpha_3}{\partial x_2}\Bigg(-\frac{D}{H}x_2 - \frac{\omega_0^2}{H^2}\frac{\partial\alpha_3}{\partial x_2}\xi_3 \\ & - \frac{\omega_0 V_s}{HX'_{d\Sigma}}E'_{q0}(\sin(\delta_0 + x_1) - \sin\delta_0) \\ & - \frac{\omega_0 V_s}{HX'_{d\Sigma}}x_3\sin(\delta_0 + x_1) - \frac{\omega_0 V_s^2}{2H}\left(\frac{X'_{d\Sigma} - X_{q\Sigma}}{X'_{d\Sigma}X_{q\Sigma}}\right) \\ & (\sin 2(\delta_0 + x_1) - \sin 2\delta_0)\Bigg)\Bigg] \end{aligned} \quad (36)$$

where $\xi_1 = x_1$, $\xi_2 = x_2 - \alpha_2$, $\xi_3 = x_3 - \alpha_3$, and choosing the appropriate parameters makes (35) hold. Then the whole system is ISS and all the states are asymptotically stable.

IV. Simulation Results

The simulation of single machine infinite-bus system is performed for the following parameters: $X_d = 2.534$, $X_q = 2.534$, $X'_d = 0.318$, $X_T = 0.1$, $X_L = 1.46$, $T_{d0} = 10$, $T_{H\Sigma} = 0.1$, $C = 1.0$, $D = 5$, $H = 8$. The dynamically uncertain part of system is $\Delta\dot{\mu} = -2\Delta\mu + 2\Delta\delta$, which is a ISS subsystem.

Simulation process: three-phase short circuit fault is assumed to occur during $t = 0.5-0.6s$. In the simulation figures, the responses of system under the small-gain controller are shown by real lines, and the responses of system without the control are shown by broken lines.

Figure 2 and Figure 3 show the performance of power angle δ and end voltage V_t. From the above figures, we can see that, the generator system with dynamic uncertainties is unstable without the control, while the closed-loop system holds stable under the small-gain controller. Therefore, the design method guarantees the stability of the power system and enhances the ability of disturbance attenuation.

V. Conclusions

In this paper, the nonlinear controller is designed by using Small-Gain Theorem for the generator system with dynamic uncertainties, which renders the states of closed-loop system asymptotically stable. For the high-order model of generator system, the Backstepping techniques are joined with small-gain technique. The obtained controller not only makes power angle and output voltage stable, but also controls the dynamic uncertainties. In the end, the simulations are provided to illustrate the effectiveness of the design.

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