# The Nonlinear Accuracy Model of Electro-Hydrostatic Actuator

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*Abstract*—The Electro-Hydrostatic Actuator (EHA) is a kind of Power-By-Wire (PBW) actuator. In this paper, a typical architecture of EHA is described and the block diagram model of EHA is established directly from the mathematic equations without transfer functions, which is a nonlinear accuracy model. The influence of refeeding circuit on the EHA system is discussed based on the comparison with conventional linear model. A gainvariable PID controller is introduced to this model to compensate the friction.

# Keywords-EHA, block diagram, modeling, control

# I. INTRODUCTION

The demand for conventional hydraulic actuation is gradually decreasing due to its limitations such as: low energy efficiency, leakage, noise, low maintainability. The Power-By-Wire (PBW) technology is becoming an attractive direction of future airborne actuation system. A PBW flight control system would simplify the secondary power generation, eliminate the need for a central hydraulic power supply, and replace the hydraulic pipes by electric power cables. As a result, the reliability, survivability, efficiency and maintainability of the aircraft would be greatly improved.

The Electro-Hydrostatic Actuator (EHA) is one kind of PBW actuator which uses a hydraulic pump to transfer the rotational motion of electric motor to the actuator output. EHA is based on the principle of closed-circuit hydrostatic transmission, so that there are no requirements for oil reservoir and electro-hydraulic servo-valves.

A lot of research papers have modeled the EHA system by transfer function in the past, however, this linear modeling method has some disadvantages: neglecting the refeeding circuit which contains some nonlinearity; simplifying the friction, especially the static friction; supposing that all the initial conditions are zero [1,2]. To solve these problems, the block diagram modeling method is presented in this paper.

# II. ARCHITECTURE OF FPVM-EHA

There are several architectures of EHA: EHA with fixed pump displacement and variable motor speed (FPVM), EHA with variable pump displacement and fixed motor speed (VPFM), EHA with variable pump displacement and variable Jean-Charles Mare Department of Mechanical Engineering Institut National des Sciences Appliquees Toulouse, France

motor speed (VPVM). Nowadays, the FPVM-EHA (Fig. 1) is more popular for its simple structure and efficiency. In this system, a bi-directional pump rotates in variable speed and directions given by electric motor. As a result, the oil flow and supply pressure is variable to drive the symmetrical actuator [3].



Fig.1. Architecture of FPVM-EHA

#### III. EHA MODELING

# A. Modeling of DC Motor

A Brushless DC Motor (BLDCM) is used as the driving motor. The maximum speed is 12000r/min, nominal voltage is 270V, and nominal power is 10kW. The mathematic equations of BLDCM are:

$$\begin{cases} U_c = E + L \frac{di}{dt} + Ri \\ E = K_c \omega \\ T_e = K_t i \\ T_e = J \dot{\omega} + k_{fric} \omega + T_l \end{cases}$$
(1)

Where *E* is the back electromotive force, *L* is the motor winding inductance, *R* is the motor winding resistance,  $T_e$  is the electromagnetism torque, *J* is the sum of inertia of motor and pump,  $K_{frie}$  is the sum of viscous coefficient of motor and pump,  $T_i$  is the load torque acting on the motor shaft, and  $\omega$  is the rotational speed of motor.

According to (1), a block diagram model of motor is gotten in Fig. 2 by Simulink. The part in dashed is current protection that could be realized by software.



Fig.2. Block diagram model of motor

A schematic diagram of bi-directional pump considering

#### Modeling of Pump В.

Q1, P1 D۰ω Qel, Pac Oil D•0 в O2, P2

Fig.3. Schematic diagram of pump The flow equation of node A is obtained as:

the internal and external leakage is shown in Fig. 3.

$$Q_{l} = D \cdot \omega - Q_{l} - Q_{ell} = D \cdot \omega - K_{ilp}(P_{1} - P_{2}) - K_{elp}(P_{1} - P_{ac}) \quad (2)$$
  
The flow equation of node P is obtained as:

The flow equation of node B is obtained as:

$$Q_2 = D \cdot \omega - Q_{il} + Q_{el2} = D \cdot \omega - K_{ilp}(P_1 - P_2) + K_{elp}(P_2 - P_{ac}) \quad (3)$$

Where D is the displacement of pump,  $K_{ilp}$  is the internal leakage coefficient of pump,  $K_{elp}$  is the external leakage coefficient of pump,  $P_{ac}$  is the pressure of accumulator. The diagram model of pump is shown in Fig. 4.



Fig.4. Block diagram of pump

# C. Modeling of Refeeding Circuit

In order to keep the closed-circuit of EHA, a refeeding circuit composed of accumulator and check valves is necessary. The schematic diagram is shown in Fig. 5.



Fig.5. Schematic diagram of refeeding circuit The flow equations of refeeding circuit are:

$$\begin{cases} Q_{ac} = Q_{el} - Q_{c1} - Q_{c2} \\ Q_{1f} = Q_1 + Q_{c1} \\ Q_{2f} = Q_2 - Q_{c2} \end{cases}$$
(4)

Where  $Q_{el}$  is the external leakage of pump,  $Q_{c1}$ ,  $Q_{c2}$  are the flow of check values depending on  $(P_1 - P_{ac})$  and  $(P_2 - P_{ac})$ .

The relationship between  $Q_{ac}$  and  $P_{ac}$  could be described as follows:

$$P_{ac} = P_{aci} V_{gasi}^{k} / (V_{gasi} - \int Q_{ac} dt)^{k}$$
<sup>(5)</sup>

Where  $P_{aci}$  is the initial pressure of accumulator,  $V_{gasi}$  is the initial volume of gas, k is the polytropic exponent of gas within the range from 1.0 to 1.4.

The block diagram model of refeeding circuit and accumulator are shown in Fig. 6 and Fig. 7 respectively. The look-up tables are used to describe the flow characteristic of check valves.  $P_{aci}V_{gasi}^k$  is defined as *GasCons* in Fig. 7.



Fig.6. Block diagram of refeeding circuit



Fig.7. Block diagram of accumulator

#### D. Modeling of Hydraulic Actuator

EHA requires a symmetrical actuator in order to ensure flow balance between the actuator and the pump. The hydraulic jack shown in Fig. 8 is divided into two working chambers by the piston [5].



Fig.8. Schematic diagram of hydraulic jack

The flow in Chamber 1 could be described by the following equation:

$$Q_{1f} = A\dot{x}_t + \frac{V_{10} + Ax_t}{B}\dot{P}_1 + K_{ilj}(P_1 - P_2)$$
(6)

The flow in Chamber 2 could be described by the following equation:

$$Q_{2f} = A\dot{x}_t - \frac{V_{20} - Ax_t}{B}\dot{P}_2 + K_{ilj}(P_1 - P_2)$$
(7)

Where A is the active area of piston, B is the bulk modulus of oil,  $K_{aj}$  is the internal leakage coefficient of hydraulic jack,  $V_{10}$  and  $V_{20}$  are the initial volume of chamber 1 and chamber 2 respectively, which are the same in this symmetrical actuator.

The load force balance equation of the piston is:

$$A(P_1 - P_2) = M\ddot{x}_t + F_{ex} + F_{fric}$$
(8)

Where *M* is the total mass of piston and load,  $F_{ex}$  is the external load force, and  $F_{fric}$  is the friction which will be described in Section E.

The diagram model of hydraulic jack and piston are respectively shown in Fig. 9 and Fig. 10:



Fig.9. Block diagram of hydraulic jack



# E. Modeling of Friction

There are several solutions to describe the friction as a function of velocity. The simplest one is:  $F_{fric} = K_{vis} \dot{x}_t$ , where  $K_{vis}$  is the viscous coefficient of piston. For getting high accuracy, the friction model is given in the form:

 $F_{fric}(\dot{x}_{t}) = [F_{c} + (F_{s} - F_{c}) \cdot \exp(-|\dot{x}_{t}|/\alpha) + K_{vis} |\dot{x}_{t}|] \cdot sign(\dot{x}_{t}) \quad (9)$ Where  $F_{c}$  is the Coulomb friction,  $(F_{s} - F_{c}) \cdot \exp(-|\dot{x}_{t}|/\alpha)$  is the Stribeck friction, and  $K_{vis} |\dot{x}_{t}|$ is the viscous friction [6]. For improvement,  $sign(\dot{x}_{t})$  could be replaced by  $\tanh(\dot{x}_{t}/\beta)$  to make the model continuous:

 $F_{fic}(\dot{x}_{t}) = [F_{c} + (F_{s} - F_{c}) \cdot \exp(-|\dot{x}_{t}|/\alpha) + K_{vis} |\dot{x}_{t}|] \cdot \tanh(\dot{x}_{t}/\beta)$ (10)

Where  $\alpha$ ,  $\beta$  are the reference speeds approximately within the range from 0.001m/s to 0.01m/s. The diagram model of the friction is shown in Fig. 11. The relation between  $\dot{x}_t$  and friction is shown in Fig.12.





### F. Block Diagram of EHA Assembly

Based on the above sub-models, a FPVM-EHA without controller could be gotten. All the sub-models are encapsulated and connected with each other by the defined input and output ports. The block diagram of the whole system is shown in Fig. 13.



IV. SIMULATION AND ANALYSIS

# A. Close-loop Simulation

A PID controller is introduced to the EHA model for closeloop simulation. The system parameters are given in Table 1. The step input is 10mm, and the external force of 10000N is loaded at 2.5s. The response shown in Fig. 14 proves the correctness of this model, indicates that the system has good characteristic in rapidity and loading.



Fig.14. Position response

Table 1 EHA Parameters			
Symbol	Value	Symbol	Value
<i>L</i> [H]	2.5e-3	$F_{s}[N]$	50
R [Ohm]	1.5	$F_c$ [N]	15
$K_{t}$ [N*m/A]	0.2	$V_{10}, V_{20}$ [ml]	152
$K_{c}$ [V/(rad/s)]	0.2	<i>B</i> [N/m^2]	6.5e8
J [Kg*m^2]	1.2e-3	A [cm^2]	19
$K_{fric}$ [N*m/(rad/s)]	0.0004	<i>M</i> [Kg]	2000
$K_{ilp}$ [(m^3/s)/Pa]	1e-13	$V_{gasi}$ [ml]	150
$K_{elp}$ [(m^3/s)/Pa]	1e-13	P <sub>aci</sub> [MPa]	2.5
D [ml/r]	1.2	$V_{oili}$ [ml]	150
$K_{ilj}$ [(m^3/s)/Pa]	1e-13	k	1.3
$K_{vis}$ [N/(m/s)]	150		

B. Effects of Refeeding Circuit

General speaking, the roles of EHA refeeding circuit are as follows:

- To make up the closed-circuit
- ◆ To prevent cavitation
- To prevent the excessive pressure build up in pump
- To compensate the external leakage

In the conventional linear model, the refeeding circuit is always neglected due to some nonlinearity such as: the flow characteristic of check valves and the gas in accumulator. For improvement, the refeeding circuit is considered in this nonlinear accuracy model. Fig. 15 and Fig. 16 show the system pressure derived from these two models respectively. An external force of 10000N is loaded at 2.5s.

The simulation result indicates that the refeeding circuit has another effect on reducing the pressure ripple in EHA system.





Fig.16. Pressure response with refeeding circuit

# C. Friction Problem

For the EHA, friction is an unavoidable force that exits in motor, pump and piston, which always lead to tracking errors and creeping in low speed.

Fig. 17 shows the obvious oscillations in EHA velocity response of very low value (0.075mm/s). Because of the negative slope due to Stribeck friction (Fig.12), the damping will decrease while the velocity increases, which leads to the

instability of this system. Moreover, there is friction-induced stick when the velocity passes through zero.

A lot of nonlinear and adaptive control strategies have been developed in the past for friction compensation [7]. Fig. 18 describes a gain-variable PID controller. The gain  $K_v$  is changed according to the velocity of piston:

$$K_{v} = 1.0 + K_{1} \cdot e^{-T|\dot{x}_{t}|} \tag{11}$$

Where  $K_1$  is the variable part of  $K_v$ , T is the time constant to adjust the rate of decay.

Fig. 19 shows the tracking performance of system without compensation, for comparison, Fig. 20 shows the improved tracking performance with this gain-variable PID controller.



Fig.17. Friction-induced oscillations in velocity



Fig.18. Gain-variable PID controller



Fig.19. Position tracking and the error (without compensation)



Fig.20. Position tracking and the error (with gain-variable controller)

# V. CONCLUSION

The nonlinear accuracy model of FPVM-EHA is established by block diagrams in this paper. It contains more information than conventional linear model. The comparison analysis indicates the effect of EHA refeeding circuit on reducing pressure ripple. A gain-variable PID controller is introduced to this model and efficiently compensates the friction. All the simulation results prove the correctness and reliability of this model which is helpful for the further study on EHA.

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