Research Advances and Prospects of Mathematical Morphology in Image Processing☆

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***Abstract*—Mathematical morphology, due to its basic concept of set theory, has an inherent advantage for image processing. Morphological framework shows mighty vitality not only because it can perform tasks from the simplest to the most demanding: noise reduction, edge detection, segmentation, texture and shape analysis, etc, but also because it can be applied to almost all application fields dealing with digital image processing. This paper discusses the extension of set operations, structuring element conformation and combination with other image processing methods, which are three main aspects to research the development of morphology. Besides, an objective forecast for future tendency about mathematical morphology is proposed.**

***Keywords*—Mathematical morphology, image processing, structuring element, set operation**

I. Introduction

Mathematical morphology, initially founded from 1960s by Metheron and Serra, has been extensively used for image processing, pattern recognition and machine vision since the publishing of *Image Analysis and Mathematical Morphology* of Serra in 1982. Mathematical morphology as a theory and methodology for nonlinear image and signal processing is more systematic and rigorous than traditional linear image and signal processing methods and other nonlinear methods like Gradient operator and Laplacian operator which have been restricted in algorithmic level and fail to establish a systematic theory. Its popularity in the image processing community is mainly due to its rigorous mathematical foundation as well as its inherent ability to exploit the spatial relationships of pixels.

Mathematical morphology regards an image as a set and uses another smaller set which is called as structuring element to probe image. This apparent geometric description of set theory makes mathematical morphology more suitable for visual information processing.

Mathematical morphology is originally proposed for binary images, and its basic theory is developed in this application. Accordingly, basic morphological methods in binary image processing is firstly presented in section Ⅱ, and then extension of set operation, structuring element conformation and combination with other image processing methods are analyzed in the following three sections respectively. Section Ⅵ represents prospect of mathematical morphology in image processing. Section Ⅶ is the conclusion.

II. Binary Mathematical Morphology

Morphology is an approach to image analysis which is based on the assumption that an image consists of structures which may be handled by set theory. A binary image is consisted of "0" set which representing background and "1" set representing object. Structuring element is a smaller set compared to the image. The most basic operators of binary morphology are dilation and erosion. The dilation of image *A* by structuring element *B*, denoted by $A \oplus B$, is defined as

$$A \oplus B = \{x \mid x = a + b, \text{for some } a \in A \text{ and } b \in B\}. \quad (1)$$

Accordingly, the erosion of *A* by *B*, denoted by $A \Theta B$, is defined as

$$A \Theta B = \{x \mid b + x \in A, \ \forall\, b \in B\}. \quad (2)$$

Many other morphological operations are based on the two basic operations. The morphological opening of *A* by *B*, denoted by $A \circ B$, is simply erosion of *A* by *B*, followed by dilation of the result by *B*

$$A \circ B = (A \Theta B) \oplus B. \quad (3)$$

The morphological closing of *A* by *B*, denoted by $A \bullet B$, is a dilation followed by erosion

$$A \bullet B = (A \oplus B) \Theta B. \quad (4)$$

And the hit-or-miss transformation of *A* by *B* is denoted $A \otimes B$

$$A \otimes B = (A \Theta B_1) \bigcap (A^c \Theta B_2). \quad (5)$$

where *B* is a structuring element pair, $B = (B_1, B_2)$, rather than a single element as before, and A^c is the complement of *A*.

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The operations of erosion, dilation, opening, closing and hit-or-miss can extract many types of information about a binary image. Openings can be used to remove small objects, protrusions from objects, and thin connections between objects, while closing eliminates small holes, smoothes concaves and fills gaps in the contour. Opening and closing operations can be alternately applied as bilateral filter to eliminate noise and restore images. Hit-or-miss transformation is a basic tool for shape detection and object recognition.

Reconstruction is a morphological transformation involving two images and a structuring element. One image, the marker, is the starting point for the transformation. The other image, the mask, constrains the transformation. If g is the mask and f, which must be a subset of g, is the marker, the reconstruction of g from f, denoted by $R_g(f)$, is defined by the following iterative procedure:

$$h_1 = f,\ h_{k+1} = (h_k \oplus B) \cap g \quad until\ \ h_{k+1} = h_k. \qquad (6)$$

Morphological reconstruction can be applied for restoring the lost image information and for segmenting object image as shown in Fig.1. There are four letters a, b,c and d in the mask image as shown in Fig.1(a). The marker image is the two red points in letter b and d as shown in Fig.1(b). The reconstruction process finds the region of letter b and d as shown in Fig.1(c).

In addition, all kinds of morphology operations can be combined together to carry out different missions. A morphological representation algorithm was designed for 2-D binary shapes using rectangular components[1].

III. Extensions of Set Operations

Binary mathematical morphology is based on set theory such as union, intersection and complementation while Grey-level mathematical morphology is based on the notion of minimum/infimum and maximum/supremum. The gray-scale dilation of f by structuring element b, denoted by $f \oplus b$, is defined as

$$(f \oplus b)(x,y) = \max\{f(x-x', y-y') \mid (x',y') \in D_b\}. \qquad (7)$$

Similarly, the gray-scale erosion of f by structuring element b, denoted by $f \Theta b$, is defined as

$$(f \Theta b)(x,y) = \min\{f(x+x', y+y') \mid (x',y') \in D_b\} \qquad (8)$$

where D_b is the domain of b, and $f(x,y)$ is assumed to equal $-\infty$ outside the domain of f.

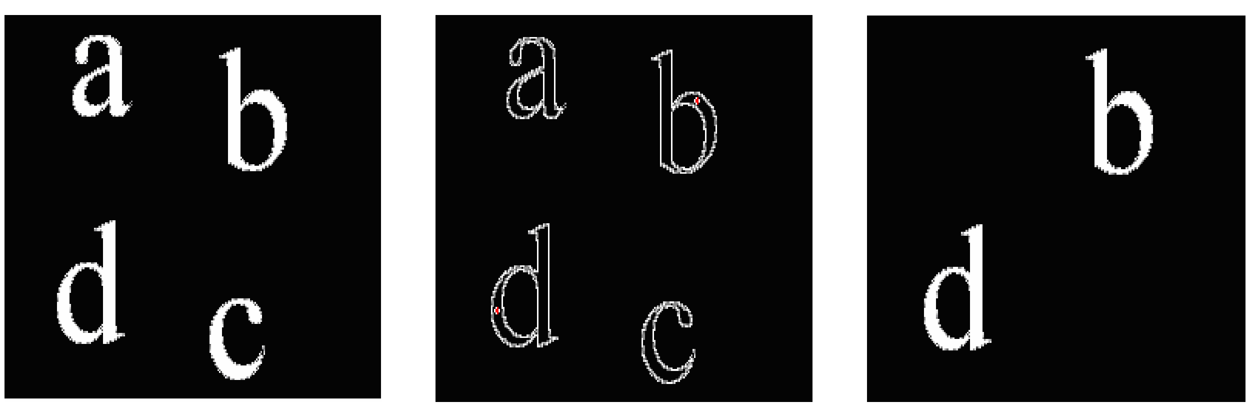
A morphological edge detector algorithm was proposed to compute the image morphological gradient, and then object boundary points were selected from the resultant image. The threshold for segmentation was finally determined by the gray values of object boundary points[2]. The hit-or-miss transforms was extended to gray-level images and applied to angiographic image for vessel segmentation[3].

In the early years of 1980s, Sinha and Doutherty introduced the conception of fuzzy mathematics into mathematical morphology, leading to fuzzy mathematical morphology. In the methodology of fuzzy mathematical morphology, image is regarded as not binary set but fuzzy set. Fuzzy intersection and union operation respectively replaces operation of image intersection and union, and forms fuzzy erosion and dilation. The operations of erosion and dilation of a fuzzy image by a fuzzy structuring element having a bounded support are defined in terms of their membership functions:

$$\begin{aligned}\mu_{A\Theta B}(x) &= \min_{y\in B}\left[\min\left[1, 1+\mu_A(x+y)-\mu_B(y)\right]\right] \\ &= \min\left[1, \min_{y\in B}\left[1+\mu_A(x+y)-\mu_B(y)\right]\right]\end{aligned} \qquad (9)$$

$$\begin{aligned}\mu_{A\oplus B}(x) &= \max_{y\in B}\left[\max\left[0, \mu_A(x-y)+\mu_B(y)-1\right]\right] \\ &= \max\left[0, \max_{y\in B}\left[\mu_A(x-y)+\mu_B(y)-1\right]\right]\end{aligned} \qquad (10)$$

where $x, y \in z^2$ are the spatial coordinates and μ_A, μ_B are the membership functions of the image and the structuring element, respectively.



(a) (b) (c)

Figure 1. Morphological reconstruction. (a) Original image (the mask). (b) Marker image, (c) Reconstruction result

Fuzzy set theory, mathematical morphology and expert system was combined together to build an fuzzy expert systems for image classification [4]. Another approach to mathematical morphology is soft mathematical morphology (soft MM)[5] Instead of applying local maximum and minimum operations, soft mathematical morphology uses a more general weighted order statistics, whose weighted value is related with structuring element. Besides, the structuring element used in soft mathematical morphology is divided into two subsets: the hard center B_1 and the soft boundary B_2. The basic definitions of the binary soft erosion and dilation [6] are as follows:

$$\left(A\Theta[B_1,B_2,k]\right)(x)=\{x\in A|\left(k\times Card\left[A\cap(B_1)_x\right]+ Card\left[A\cap(B_2)_x\right]\right)\geq k\times Card[B_1]+Card[B_2]-k+1\} \tag{11}$$

$$\left(A\oplus[B_1,B_2,k]\right)(x)=\left\{x\in A\,|\left(k\times Card\left[A\cap\left(B_1^{\,s}\right)_x\right] + Card\left[A\cap\left(B_2^{\,s}\right)_x\right]\right)\geq k\right\} \tag{12}$$

where k is called the order index, which determines the number of times that the elements of core participate into the result, and Card [X] denotes the cardinality of set X, i.e., the number of the elements of X.

Accordingly, Soft morphological erosion and dilation of a gray-scale image f : F → Z by a soft grayscale structuring element [α, β, k]: B → Z is[7].

$$f\Theta[\alpha,\beta,k](x)=\min_{\substack{y\in B_1\\ z\in B_2}}{}^{(k)}\left(\{k\Diamond(f(x+y)-\alpha(y))\} \cup\{f(x+z)-\beta(z)\}\right) \tag{13}$$

$$f\oplus[\alpha,\beta,k](x)=\min_{\substack{(x-y),(x-z)\in F\\ y\in B_1\\ z\in B_2}}{}^{(k)}\left(\{k\Diamond(f(x-y)+ \alpha(y))\}\cup\{f(x-z)-\beta(z)\}\right) \tag{14}$$

where $x,y,z\in Z^2$ are the spatial coordinates, $\alpha:B_1\rightarrow Z$ is the core of the gray-scale structuring element, $\beta:B_2\rightarrow Z$ is the soft boundary of the grayscale structuring element, and $F,B_1,B_2\in Z^2$ are the domains of the gray-scale image, the core of the gray-scale structuring element, and the soft boundary of the gray-scale structuring element, respectively. The symbol ◊ denotes the repetition, i.e. $\{k\Diamond f(x)\}=\{f(x),f(x),...f(x)\}$ (k times).

Compared with standard mathematical morphology, soft morphological operations are less sensitive to additive noise and small variations in object shape. A threshold decomposition method of Gray-scale Soft Morphology into Binary soft morphology was proposed in [7].

Fuzzy soft mathematical morphology was proposed by introducing fuzzy-set theory into soft mathematical morphology[8]. Fuzzy soft morphology preserves both the notion of fuzzy fitting and the structuring element of soft MM. The definitions for fuzzy soft erosion and fuzzy soft dilation are as follows:

$$\mu_{A\Theta[B_1,B_2,k]}(x)=\min\left[1,\min_{y\in B_1,z\in B_2}{}^{(k)}\left(\{k\Diamond(\mu_A(x+y) -\mu_{B_1}(y)+1)\}\cup\{\mu_A(x+z)-\mu_{B_2}(z)+1\}\right)\right] \tag{15}$$

$$\mu_{A\oplus[B_1,B_2,k]}(x)=\max\left[0,\max_{(x-y)\in B_1,(x-z)\in B_2}{}^{(k)}\left(\{k\Diamond(\mu_A(x-y) -\mu_{B_1}(y)-1)\}\cup\{\mu_A(x-z)+\mu_{B_2}(z)-1\}\right)\right] \tag{16}$$

where $x,y,z\in Z^2$, are the spatial coordinates and $\mu_A,\mu_{B_1},\mu_{B_2}$ are the membership functions of the image, the core of the structuring element and the soft boundary of the structuring element. A Soft Morphological Filtering using a fuzzy model was developed in [9] and applied to color image processing.

The concepts of scalar-valued morphology cannot be transferred directly to the vector-valued cases such as color images by component-wise performance of traditional morphological operations which might result in corruption of information in the image due to the strong correlation of components. Hence, it seems essential to establish an ordering of colors or vectors. Different types of orderings tactics such as marginal ordering, conditional ordering, partial ordering and reduced ordering are introduced to extended mathematical morphology to vector-valued images. The vector-valued erosion of an image f at pixel x by the structuring element B of size n is designed as follows[10]:

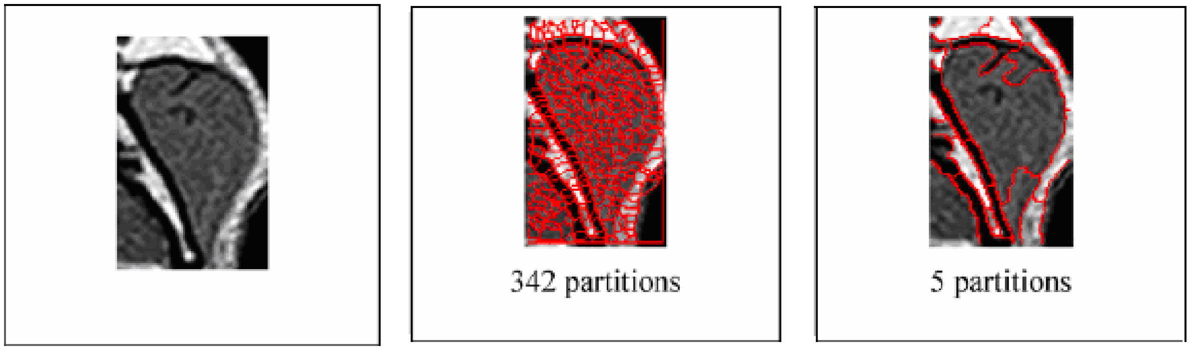
$$\varepsilon_{\Omega,nB}(f)(x)=\{f(y):f(y)=\wedge_\Omega[f(z)],z\in n(B_x)\} \tag{17}$$

where $\wedge_\Omega$ is the infimum according to the ordering Ω. The corresponding color dilation is obtained as follows:

$$\delta_{\Omega,nB}(f)(x)=\{f(y):f(y)=\vee_\Omega[f(z)],z\in n(B_x)\}. \tag{18}$$

A new multi-channel filtering approach based on learning-based color morphological operations was proposed for impulsive noise[11]. A new shape descriptor with morphology method was designed for color-based tracking[12].

Watershed algorithm is an efficient morphological method for image segmentation. It needs not to probe image using structuring element. Instead, image is divided into different sets based on spatial relationships of pixels directly. Over-segmentation is the main demerit of watershed algorithm.



(a) (b) (c)

Figure 2. Compare of traditional watershed and improved watershed. (a) The MRI image. (b) Traditional watershed segmentation result. (c) Improved watershed segmentation result

In order to overcome this problem, watershed transformation is usually combined with other methods. A new automated and fast procedure was proposed to segment the left ventricular myocardium in 4D (3D+t) cardiac MR images based on watershed algorithm[13]. A method which is capable of segmenting yeast cells was proposed based on watershed transform and space-scale analysis of the Tree of Critical Lakes[14]. An improved watershed algorithm was proposed by combinding with k-means clustering method [15] and Fig. 2 shows the result.

IV. Conformation of Structuring Element

Traditional mathematical morphology has limitations in application, because structuring element remains constant during processing course, which may results in over-processing or less-processing. The theory of spatially varient (SV) mathematical morphology is developed to overcome this difficulty. The basic definitions of SV mathematical morphology erosion and dilation in [16] are as follows:

$$\varepsilon_{\theta}(X)=\{z\in\xi:\theta(z)\subseteq X\}=\bigcap_{y\in X^c}\theta'^c(y) \tag{19}$$

$$\delta_{\theta}(X)=\{z\in\xi:\theta(z)\cap X\neq\phi\}=\bigcup_{y\in X}\theta'(y) \tag{20}$$

where ξ is a set, $\mathrm{P}(\xi)$ denotes the set of all subsets of ξ, $X\in\mathrm{P}(\xi)$ the SV structuring element θ is a mapping from ξ to $\mathrm{P}(\xi)$ and the transposed SV structuring element θ' is a mapping from $\mathrm{P}(\xi)$ to ξ:

$$\theta'(y)=\{z\in\xi:y\in\theta(z)\}\quad(y\in\xi). \tag{21}$$

A great many scholars propose series of improved algorithms depending on their own application. Multiscale morphology method was proposed to detect the edge of brain Magnetic Resonance Image[17]. General Sweep mathematical morphology which was proposed in [18] is a kind of self-adaptive mathematical morphology actually. It varies the shapes and orientations of ellipse structuring element according to the slope direction of curve, and proposed a General Sweep dilation and erosion transformation for image enhancement and edge connection. A variable structuring element based on fuzzy morphology operations was designed for single viewpoint omnidirectional images[19]. A Top-hat morphology filter whose structuring element is adjusted according to two-layer feed-forward network was designed to detect infrared target [20].

Furthermore, multi-structuring elements mathematical morphology makes up the limitations of single-structuring element mathematical morphology[21]. A double structuring elements mathematical morphology method was proposed for ultrasound image sharpening[22], in which two different structuring elements *SE1* and *SE2* were designed and applied to opening and closing translation as follows:

$$O(SE1,SE2)=(f(x)\Theta SE1)\oplus SE2 \tag{22}$$

$$C(SE1,SE2)=(f(x)\oplus SE1)\Theta SE2\,. \tag{23}$$

Mathematical operations are convolution operations. So most image processing methods use structuring elements of a limited size to reduce time consumption. Therefore, difficulties arise when dealt with large-sized structuring elements. Many scholars have dedicated to structuring element decomposition. Structuring element decomposition methods were proposed for binary morphological and gray-scale morphological respectively in [23] and [24].

V. Combination With Other Image Processing Methods

Morphological wavelet, which combines the advantages of wavelet decomposition with mathematical morphology, has better denoising function than traditional morphological methods. Two examples are given to reveal the advantages and applicability of morphological wavelets. A kind of morphological wavelet-based stereo image coders was proposed in [25]. An algorithm was proposed for multi-focus image fusion using morphological wavelets[26], and Fig. 3 shows the result.

Genetic algorithm is a nonlinear optimizing method with the character of global optimization. It has been successfully

Figure 3. Original multi-focus images and the fused image with morphological wavelets: (a) Near focused image; (b) middle focused image; (c) far focused image; (d) fused image with morphological wavelets

applied to many fields such as optimal solution of complex function, structure optimal design, adaptive control, system control and pattern recognition, etc. Genetic algorithm is introduced into mathematical morphology to address complicated works. A hierarchical procedures of mathematical morphology using genetic algorithms was constructed in [27]. A simple genetic algorithms was applied to the search for optimum morphological filters for specific signal/image processing tasks [28].

Mathematical morphology operations can be combined with many other methods such as neural network, active contour models and so on. A simple and efficient algorithm was proposed to eliminate the redundant pixels of damaged or smeared areas based on radial basis function network and mathematical[29]. A method of characterization and synthesis of object using the neural network and mathematical morphology was proposed in [30].

VI. Prospect of Mathematical Morphology in Image Processing

In view of the above three clues for advancement of morphology, one of the most appealing aspects of morphological image processing is the extensive set-theoretical foundation from which many general morphological techniques have evolved. Watershed algorithm, a new form of morphological operation, is simple in principle and easy to realize in that there is no need to analyze the characteristic of images or construct structuring element. Thus more universal operation methods like watershed algorithm should be proposed in the future. And Watershed operation should keeps on developing by means of combining with other methods to overcome the over-segmentation and should be developed for more digital image formats such as 3D image or color image.

Structuring element is still the key factor of morphology operations. Applying different structuring elements leads to diverse results of analyzing and processing of geometric characteristic. Therefore, structuring element determines the effect and performance of morphological transformation. Besides bigger size of structuring element will lead to rapid growth of time consumption because structuring element determines not only the data-distribution form but also the amount of data for use. As a result, self-adaptability and decomposability of structuring element should be the key point in the future research.

Furthermore, methodology of mathematical morphology should be organically combined with other image processing methods to compensate self limitations. And more effort should be dedicated to introducing other image processing conception to reinforce morphological theory.

VII. CONCLUSIONS

In the paper, we investigate the advancement and prospect of morphology in image processing in three aspects which are extension of set operations, structuring element conformation and combination with other image processing methods. Comparing with these three aspects, we can come to the following conclusions. The morphological concepts and techniques constitute a powerful set of tools for extracting features of interest in an image. The maturity theory and novelty thought provide a wide academic platform for its advancement. Extension of set operations which leads to wide application of mathematical morphology has been developed from union or intersection of sets to vector-valued morphology. However, structuring element conformation and combination with other image processing methods still need more effort to satisfy the requirement of complicated application. For example, more efficient methods are needed for the decomposition of structuring element to speed up algorithm. Besides mathematical morphology based on structuring element contour which can preserve edge information and smoothing image should be taken into account when dealing with digital image denoising.

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