An Example of Balanced Truncation Method and Its Surprising Time Domain Performance

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***Abstract* — This paper presents an example of using the balanced truncation method (BTM) for a distributed RLC interconnect model reduction. The example shows a surprising result and some possible hidden phenomenon in model reduction methods. We find that a high order BTM model has much worse performance than a low order BTM model, in view of the original model and their step responses in the time domain. This unexpected finding contradicts to our common expectation from the well-known approximation error upper bound results of the BTM in frequency domain. It reveals a fact that though the high order system keeps more states than the low order system, the approximation error behavior in the time domain may be totally different from its counterpart in the frequency domain. In addition, the paper also presents a closed form of the state space model for general evenly distributed RLC interconnect line circuits. The closed form has a very low computation complexity of O(1), which may be extended for use not only in circuits, but also in various complex systems with transmission lines, networks, or delay lines. Since the BTM is a popular practical model reduction method in various areas, thus this example may bring some further insight thinking for development.**

***Keywords*—complex system, balanced truncation method, example, time domain performance, model reduction error**

I. Introduction

The Balanced Truncation Method (BTM) [1, 3, 19] is a very popular and practical model reduction method in various areas [19]. There is a growing of publications of the BTM for the VLSI interconnects model reduction in the literature such as [4, 8, 16, 17]. It may also be used for model reduction of various high order plants in other complex or intelligent systems. The BTM has made success and progress to fix some requirements in applications, e.g., to preserve the passivity of the reduced model by solving additional Riccati equations [8], and improve the accuracy in the low frequency range by a cost of the accuracy in the high frequency range. One well-known major advantage of the BTM is that it provides upper-bound of approximation error, as its H_∞-norm of the transfer function difference of the reduced model from the original model.

On the other hand, while we apply it and make progress, we also find an unexpected result on a model reduction example. This paper mainly presents and shares this example of using the BTM. This example reveals and shows a very surprising result and some possible hidden phenomenon in model reduction methods. We find that a high order BTM model has much worse performance than a low order BTM model in view of the original model and their step responses in the time domain! In this example, the comparison is its 2nd (even 4th or 70th) order system with its 1st order system. It contradicts to our expectation because the high order BTM model guarantees to have a smaller approximation error bound than the low order BTM model in a sense of the H_∞-norm of the transfer function model error in the frequency domain, with the strict theoretical proof. This finding means that a reduced order model with a smaller error in the frequency domain may have a dramatically, radically worse difference from the original model in the time domain. Moreover, it is a surprise because a high order BTM system keeps more states than a low order BTM system. Furthermore, the time responses performance is more important and concerned for the VLSI interconnect analysis and design. Thus, this example shows that some special attention to model reduction methods, including the BTM, may be needed when the time domain performance is important for the system.

The presented example is a distributed RLC interconnect line circuit. The advancement of high-speed deep-submicron VLSI technology makes the interconnects to be a complex system of distributed RLC circuits, and a major factor of signal propagation delay, as well as a key factor of modeling difficulty [10, 11, 18]. In some standard ASIC with 90nm nanotechnology, the ratio of the interconnect delay to the gate delay may approach to 4:1. Furthermore, its distribution and complex structure make the system order of millions. Such a detailed modeling level eventually results in large-scale linear RLC circuits to be analyzed. Hence, an effort of providing high quality model reduction is then necessary in order to evaluate the circuit performance and characteristics in a reasonable time period as required by real design practice. Many methods have been developed for this issue, such as the asymptotic waveform evaluation (AWE) [9], the Padé approximation via Lanczos process (PVL) in the Krylov space [2], the projection-based algorithms with generalized orthonormal basis functions in Hilbert and Hardy space [13], the even-length-order model simplification method (ELO) [11, 12], the BTM as mentioned above, and others. Due to the large size of the systems, the computation cost is a significant challenge to many methods.

For the BTM, it is noticed that a frequency-weighted error norm may be more appropriate [7, 19], and a balanced stochastic truncation (BST) model reduction may have a tighter bound [15]. It is known that the BTM may have DC-mismatching in its low order reduced models. Some efforts have been developed by considering DC matching with a cost of mismatching in high frequency range. However, the presented example further shows some interesting surprise for model reductions as mentioned above, e.g., the unexpected behavior shown in high-order vs. low-order BTM models.

In addition, this paper also presents a closed form of the state space model for the general evenly distributed RLC interconnect line circuits with the external source and loads. A key feature is that the computation complexity of our state space closed form is only $O(1)$ for any evenly distributed RLC interconnect line. Here, the computation complexity is defined as the number of total scalar multiplications and divisions, in a more detailed level than the conventional definition as the times which the method traverses nodes or components.

It is noticed that the topic of the RLC network and interconnect is among the interesting topics which Prof. R. Kalman [6] addressed in his plenary speech at 2005 IFAC.

The paper is organized as follows. Section 2 addresses the problem formulation. Section 3 presents a general closed form of the state space model for the general evenly distributed RLC interconnect line circuits, and the state space model of the example for applying the BTM. The BTM algorithm is summarized in Section 4. Section 5 presents the simulation and analysis of the BTM reduction models, including their step responses in the time domain, the Bode plots in the frequency domain, and the error upper bounds of the reduced models. Section 6 further provides some insight discussion and remarks. Finally, Section 7 concludes the paper. Due to the page limit, the theoretical derivations are omitted here.

II. Problem Formulation

Consider an evenly distributed RLC interconnect circuit of 0.01cm long with the distribution characteristic data of resistor 5.5kΩ/m and capacitor 94.2pF/m, while the inductor value l is calculated from the light speed in the material and the capacitor value c.

In this paper, we also present a closed form of the state space model for the general evenly distributed RLC interconnect line circuits with the external source and load parts as shown in Fig.1 [14]. The system order is assumed as $2n$ as general. The distribution parameters are resistors $R_i = R = r$, inductors $L_i = L = l$, and capacitors $C_i = C = c$, $i = 1, \cdots, n$. The external source resistor is R_s, the load resistor is R_0, and the load capacitor is C_0. The index is ordered from the output/sink terminal to the input/source terminal. Denote the circuit node voltages as $v_i, i = 1, \cdots, n$, respectively.

For a simple demonstration of the example, we use a 200th-order model as its original model. Thus, we have

$$r = 5.5 \cdot 10^{-3}\Omega,\ c = 9.42 \cdot 10^{-5}pF,\ l = 0.2831pH \quad (1)$$

$$n = 100,\ N = 200\,. \quad (2)$$

The external parameter data are

$$R_s = 500\Omega,\ R_0 = 1M\Omega,\ C_0 = 0\,. \quad (3)$$

The interconnect line input port is with a voltage $v_{in}(t)$, and the output port then has a voltage v_o. In general, we may set the output as any node voltage, i.e.,

$$v_o(t) = v_i(t),\ i \in \{1, \cdots, n\}. \quad (4)$$

However, as usual, it is the terminal node voltage v_1.

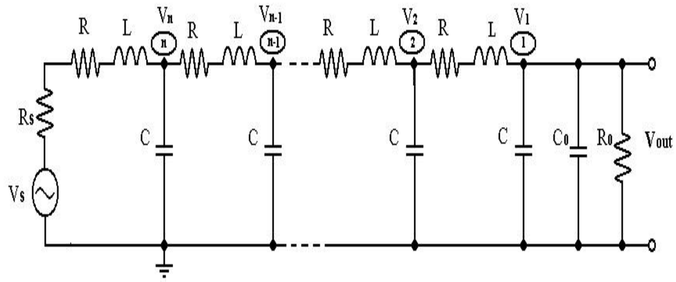

Fig. 1. Evenly distributed RLC interconnect circuit with external parameters

The linear state-space model is $\{A, B, C, D\}$ in

$$\dot{x}(t) = Ax(t) + Bu(t),\ y(t) = Cx(t) + Du(t). \quad (5)$$

The BTM reduced models from the original model $S = \{A, B, C, D\}$ are notated as $S_m = \{A_m, B_m, C_m, \mathrm{D}\}$, where m is the reduced model order, and $m < N$. Comparing with the original high-order model, the evaluation of the reduced model is by the following performance criteria: 1) the step responses; and 2) the approximation error upper bounds for the H_∞-norm of their transfer function error as

$$E_m = \|T(s) - T_m(s)\|_\infty \le \bar{E}_m \quad (6)$$

where $T(s)$ and $T_m(s)$ are the transfer functions of the original model S and the *m-th* order BTM reduced model S_m, respectively. The BTM provides an error upper bound $\bar{E}_m$, so that it is easy to determine a selection of the reduced-order m by (6) as usual, for a guaranteed performance in (6). It is a benefit of the BTM compared with many other model reduction methods.

III. State Space Model of the Circuit

A closed form of the state space model of the general evenly distributed RLC interconnect circuits [14] is presented below.

Theorem 3.1. Consider a general 2n-th order evenly distributed RLC interconnect circuit in Fig. 1 with its distributed parameters: the resistor r, the inductor l and the capacitor c, and its external parameters: the source resistor R_s, the load resistor R_0, and the load capacitor C_0. Then, its state space model $S = \{A, B, C, D\}$ in (5) is as follows:

$$A = \begin{bmatrix} 0 & I \\ A_{21} & A_{22} \end{bmatrix} \quad (7)$$

$$A_{21} = \frac{1}{cl}\begin{bmatrix} -2 & 1 & 0 & \cdots & 0 & \frac{-R_s}{R_0} \\ 1 & -2 & 1 & \ddots & \ddots & 0 \\ 0 & 1 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 1 & -2 & 1 \\ 0 & \cdots & \cdots & 0 & \frac{1}{(1+C_0/c)} & \frac{-(r+R_0)}{R_0(1+C_0/c)} \end{bmatrix} \quad (8)$$

$$A_{22} = -\begin{bmatrix} \frac{r+R_s}{l} & \frac{R_s}{l} & \cdots & \frac{R_s}{l} & \frac{R_s(1+C_0/c)}{l} \\ 0 & \frac{r}{l} & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \frac{r}{l} & 0 \\ 0 & \cdots & \cdots & 0 & \frac{r}{l} + \frac{1}{R_0(c+C_0)} \end{bmatrix}$$

$$= -\frac{r}{l}\begin{bmatrix} 1+R_s/r & R_s/r & \cdots & R_s/r & (R_s/r)(1+C_0/c) \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 & 0 \\ 0 & \cdots & \cdots & 0 & 1+(l/r)/R_0(c+C_0) \end{bmatrix} \quad (9)$$

$$B = (1/cl)[0 \cdots 0 \vdots 1\ 0 \cdots 0]^T, C = [0 \cdots 0\ 1 \vdots 0 \cdots 0], D = 0 \quad (10)$$

where matrices *A, B* and *C* have appropriate dimensions as

$$A \in R^{2n\times 2n},\ A_{21}, A_{22} \in R^{n\times n}, B \in R^{2n\times 1}, C \in R^{1\times 2n}\ . \quad (11)$$

Remark 3.1: The system sub-matrix A_{21} has non-zero elements only in the tri-diagonals and the upper-left corner. The i-th row sum of the tri-diagonals equals to 0 for $i = 2,\cdots,n-2$.

Remark 3.2: The system sub-matrix A_{22} has non-zero entries only in the diagonal and the first row.

Remark 3.3: In general, for any point output $y(t) = v_o(t) = v_i(t),\ i = 1,\cdots,n$, their state space models share the same matrices *A*, *B* and *D*, but the output matrix $C = [e_{n-i+1}^T \vdots 0 \cdots 0]$, where $e_i \in R^n$ is a unit vector with the i-th entry 1 and all other entries 0.

Remark 3.4. Theorem 3.1 presents the state space closed form of the distributed RLC circuits. The closed form does not involve any matrix inverse, LU matrix factorization, or matrix multiplication. Its computation complexity is only $O(1)$, i.e., it involves only constant number of scalar multiplications and divisions for any orders of the models. The closed form not only reduces the computation complexity, but also provides the accurate state space models of any orders for the evenly distributed RLC interconnect in Fig. 1.

By Theorem 3.1, the 200-th order state space model $S = \{A, B, C, D\}$ of the example is:

$$A = \begin{bmatrix} 0 & I \\ A_{21} & A_{22} \end{bmatrix}$$

$$A_{21} = 3.75\cdot 10^{28}\begin{bmatrix} -2 & 1 & 0 & \cdots & 0 & -5\cdot 10^{-4} \\ 1 & -2 & 1 & \ddots & \ddots & 0 \\ 0 & 1 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 1 & -2 & 1 \\ 0 & \cdots & \cdots & 0 & 1 & -1 \end{bmatrix}$$

$$A_{22} = -1.943\cdot 10^{10}\begin{bmatrix} 9.091\cdot 10^4 & \cdots & \cdots & \cdots & 9.091\cdot 10^4 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 & 0 \\ 0 & \cdots & \cdots & 0 & 1.546 \end{bmatrix}$$

$$B = [0 \cdots 0 \vdots 3.75\cdot 10^{28}\ 0 \cdots 0]^T$$

$$C = [0 \cdots 0\ 1 \vdots 0 \cdots 0],\quad D = 0 \quad (12)$$

$$A \in R^{200\times 200},\ \ A_{21}, A_{22} \in R^{100\times 100},\ B \in R^{200\times 1}$$
$$C \in R^{1\times 200}\ . \quad (13)$$

IV. Balanced Truncation Method

This section briefly summarizes the balanced truncation method [1, 3, 19] as follows.

The Balanced Truncation Method (BTM): Suppose an *N-th* order system $S = \{A, B, C, D\}$ is a minimal realization. The model reduction algorithm of the BTM for the system $\{A, B, C, D\}$ is as follows:

1) Calculate its controllability and observability Gramians *P* and *Q* respectively, i.e.,
$$AP + PA^* + BB^* = 0, \text{ and } QA + A^*Q + C^*C = 0. \quad (14)$$
2) Solve *R* such that
$$P = R^*R. \quad (15)$$
3) Do singular value decomposition to get
$$RQR^* = U\Sigma^2 U^* \quad (16)$$
$$\Sigma = diag\{\sigma_i I_{s_i}\}_{i=1,\cdots,k} \quad (17)$$
with $\sigma_i > \sigma_j,\ i < j$, the multiplicity s_i of σ_i has
$$\textstyle\sum_{i=1}^k s_i = n. \quad (18)$$
4) Let $T^{-1} = R^*U\Sigma^{-1/2}$. (19)
5) The balanced realization S_b of the system is
$$S_b = \{TAT^{-1}, TB, CT^{-1}, D\}, \text{ i.e.,}$$
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} TAT^{-1} & TB \\ CT^{-1} & D \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D \end{bmatrix}. \quad (20)$$
6) Then, the truncated *m-th* order reduced system of $\{TAT^{-1}, TB, CT^{-1}, D\}$ is
$$S_m = \{A_m, B_m, C_m, D\}$$
$$= \{(TAT^{-1})_{(1\sim m)\times(1\sim m)},\ (TB)_{1\sim m},\ (CT^{-1})_{1\sim m},\ D\} \quad (21)$$
where A_m, B_m and C_m consist of the first *m* rows and first *m* columns of TAT^{-1}, the first *m* rows of TB and the first *m* columns of CT^{-1}, respectively, and $m = \sum_{i=1}^{\nu} s_i$.
7) The error between the original system *S* and the reduced order system S_m is defined as the H_∞-norm of their transfer functions error, and it is bounded as
$$E_m = \|T(s) - T_m(s)\|_\infty \le 2\textstyle\sum_{i=\nu+1}^k \sigma_i = \bar{E}_m \quad (22)$$
$$\bar{E}_{m2} < \bar{E}_{m1} \text{ if } m_1 < m_2\ . \quad (23)$$

V. Reduced Models and Time Response Performance

This section presents the reduced models of the example described in above sections via the BTM.

Because the distributed interconnect circuit parameters are very small, we develop a scaling skill for the original system, which is with a time scale and a corresponding frequency scale in reverse. Then, we apply the BTM algorithm to the scaled original system, i.e., under a scaled time system. On the other hand, the scaled system is also under a corresponding scaled frequency system. The BTM algorithms that we used for the example are (i) the ready-commands from the MATLAB, and (ii) our m-file. They both give the same results.

As mentioned above, for simulation and computation a time scaling skill is developed and used. Here, a time scale of 10^{-12} is used. Thus, the scaled original system is

$$A = \begin{bmatrix} 0 & I \\ A_{21} & A_{22} \end{bmatrix}$$

$$A_{21} = 3.75 \cdot 10^4 \begin{bmatrix} -2 & 1 & 0 & \cdots & 0 & -5 \cdot 10^{-4} \\ 1 & -2 & 1 & \ddots & \ddots & 0 \\ 0 & 1 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 1 & -2 & 1 \\ 0 & \cdots & \cdots & 0 & 1 & -1 \end{bmatrix}$$

$$A_{22} = -1.943 \cdot 10^{-2} \begin{bmatrix} 9.091 \cdot 10^4 & \cdots & \cdots & \cdots & 9.091 \cdot 10^4 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 & 0 \\ 0 & \cdots & \cdots & 0 & 1.546 \end{bmatrix}$$

$$B = [0 \ \cdots 0 \ \vdots \ 3.75 \cdot 10^4 \ 0 \ \cdots 0]^T$$

$$C = [0 \ \cdots 0 \ 1 \ \vdots \ 0 \ \cdots 0], \quad D = 0 \qquad (24)$$

where their dimensions are in (13).

By using the BTM on (24), we have its 1st, 2nd and 4th orders reduced models as follows, respectively.

The 1st order model of the scaled system is:

$$A = -3.8594 \cdot 10^{-2}, B = C = -2.050 \cdot 10^{-1}, D = 0 \,. \qquad (25)$$

Its approximation error upper bound is:

$$\bar{E}_1 = 157.28. \qquad (26)$$

The 2nd order scaled BTM model is:

$$A = 10^{-4} \cdot \begin{bmatrix} -385.9 & 3243 \\ -3243 & -3.322 \end{bmatrix}, B = -10^{-2} \begin{bmatrix} 20.50 \\ 1.881 \end{bmatrix}$$

$$C = 10^{-2} \cdot [-20.50 \quad 1.881], D = 0 \ \text{ with} \qquad (27)$$

$$\bar{E}_2 = 156.22. \qquad (28)$$

The 4th order scaled BTM model is:

$$A = 10^{-4} \cdot \begin{bmatrix} -385.9 & 3243 & 50.97 & 6376 \\ -3243 & -3.322 & 105400 & -6.608 \\ 50.97 & 105400 & -6.733 & -113400 \\ -6376 & -6.608 & 113400 & 13.14 \end{bmatrix}$$

$$B = -10^{-2} \cdot [20.50 \quad 1.881 \quad -2.678 \quad 3.741]^T$$

$$C = 10^{-2} \cdot [-20.50 \quad 1.881 \quad 2.678 \quad 3.741], D = 0 \qquad (29)$$

$$\bar{E}_4 = 154.09. \qquad (30)$$

The step responses of the original model and its BTM reduced order models are shown in the following figures with the time scale, i.e., all simulation and figures are executed on the scaled original system (24) and its BTM models in (25), (27) and (29). They are exactly equivalent to the original system and its BTM reduced models in the original time system, respectively.

Figures 2 and 3 show the step response and Bode plot of the original model of order 200, respectively.

In Figs. 4 and 5 we clearly show the step responses and Bode plots of the 200th-order original RLC interconnect model (blue), and the 1st (green) and 2nd (red) order BTM reduced models, respectively.

Figures 6 and 7 further show the step responses and Bode plots of the 200th-order original RLC interconnect model (blue), and the 1st (green), 2nd (red), 4th (cyan) and 70th (magenta) order BTM reduced models, respectively.

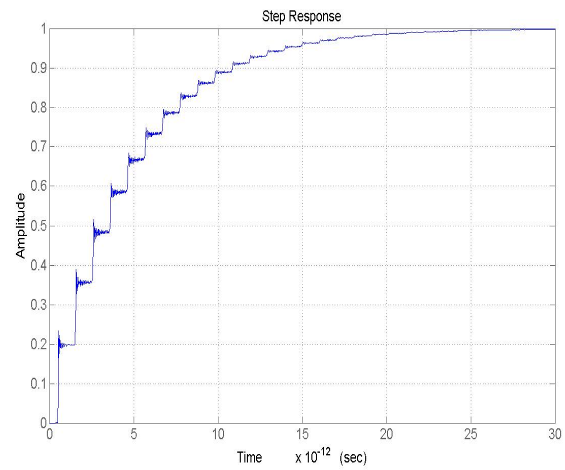

Fig. 2. Step response of evenly distributed RLC interconnect

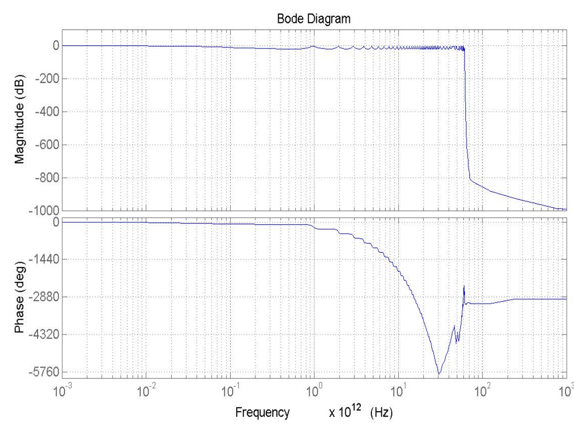

Fig. 3. Bode plot of evenly distributed RLC interconnect

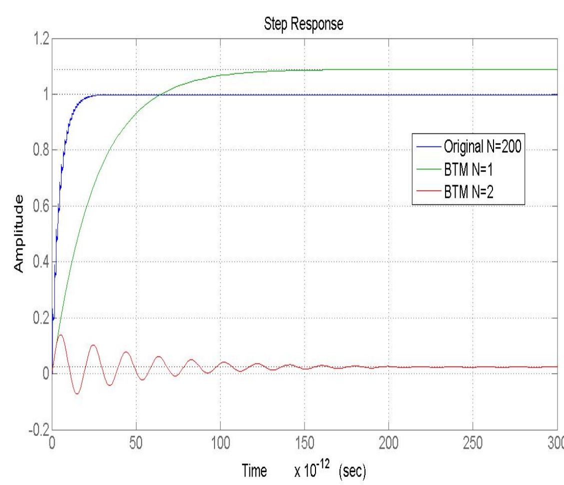

Fig. 4. Step responses of the original model (blue), and its 1st order (green) & 2nd order (red) BTM models

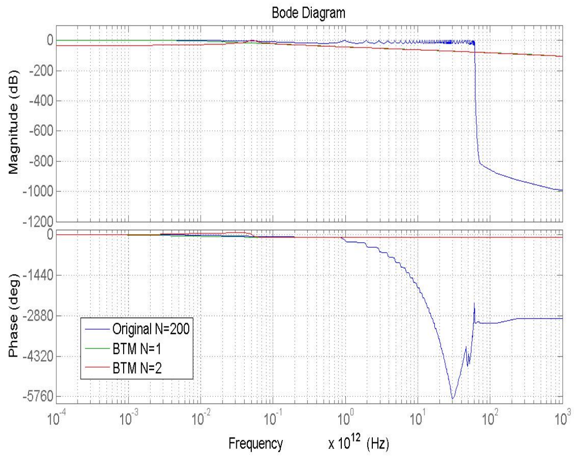

Fig. 5. Bode plots of the original model (blue), and its 1st order (green) & 2nd order (red) BTM models

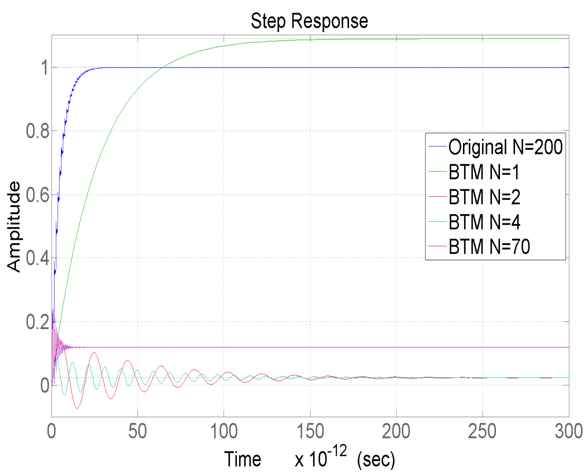

Fig. 6. Step responses of the original model (blue), and its 1st (green), 2nd (red), 4th (cyan) and 70th (magenta) order BTM models

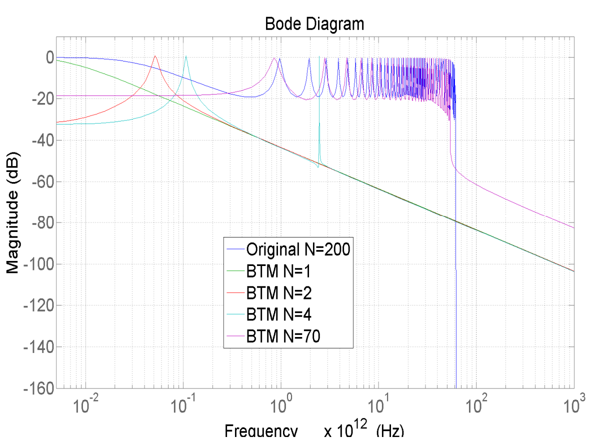

Fig. 7. Bode plots of the original model (blue), and its 1st (green), 2nd (red), 4th (cyan) and 70th (magenta) order BTM models

It clearly shows that the step response of the 2nd order BTM model (red) is far different from the one of the original model (blue), and much worse than the one of the 1st order BTM model (green). Furthermore, the step responses of the 4th order (cyan) and the 70th order (magenta) BTM models are also far different from the one of the original model (blue), and much worse than the one of the 1st order BTM model (green). It is noticed that the error bound $\bar{E}_{70}$ of the 70th order BTM model is much less than $\bar{E}_1$ of the 1st order BTM model:

$$\bar{E}_{70} = 87.28 < \bar{E}_1 = 157.28 \quad (31)$$

The step responses of the 1st, 2nd, 4th and 70th order models have their final steady DC values of 1.09, 0.0238, 0.024 and 0.119, respectively, but the correct value from the original system response is 1. The peak response values of the 2nd, 4th and 70th order models are 0.14, 0.0842 and 0.241, respectively. The settling times of the 1st, 2nd, 4th and 70th BTM models are 101, 201, 195 and 10.3 (ps), respectively. The original model has its settling time of 18.4 ps. For a simple comparison, they are listed in Table 1.

TABLE I. THE PERFORMANCE COMPARISON OF THE ORIGINAL MODEL AND ITS BTM MODELS

Performance	Original N=200	BTM N=1	N=2	N=4	N=70
Error bound	—	157.28	156.22	154.09	87.28
DC value	1	1.09	0.0238	0.024	0.119
Peak value	—	—	0.14	0.0842	0.241
Settling time (ps)	18.4	101	201	195	10.3

Here, the important issue is not only the difference of their DC values, but also is that the whole step response of the higher order BTM model is much worse than the one of its 1st order, i.e., the lower order model.

For completeness, we present the corresponding non-scaled BTM model based on the scale skill as follows.

The 1st order non-scaled BTM model is:

$$A = -3.859 \cdot 10^{10}, B = C = -2.050 \cdot 10^5, D = 0 . \quad (32)$$

The 2nd order non-scaled BTM model is:

$$A = 10^8 \cdot \begin{bmatrix} -385.9 & 3243 \\ -3243 & -3.322 \end{bmatrix}, B = -10^4 \begin{bmatrix} 20.50 \\ 1.881 \end{bmatrix}$$

$$C = 10^4 \cdot [-20.50 \quad 1.881], D = 0 . \quad (33)$$

Then, the 4th order non-scaled BTM model is:

$$A = 10^8 \cdot \begin{bmatrix} -385.9 & 3243 & 50.97 & 6376 \\ -3243 & -3.322 & 105400 & -6.608 \\ 50.97 & 105400 & -6.733 & -113400 \\ -6376 & -6.608 & 113400 & 13.14 \end{bmatrix}$$

$$B = -10^4 \cdot [20.50 \quad 1.881 \quad -2.678 \quad 3.741]^T$$

$$C = 10^4 \cdot [-20.50 \quad 1.881 \quad 2.678 \quad 3.741], D = 0 . \quad (34)$$

They are the BTM reduced models of the original system model (12) under the original time system.

VI. Discussion and Remarks

As mentioned above, a surprising fact is shown in Figs. 4 and 6, i.e., the 2nd-order (even 4th order, or 70th order) BTM model is *worse* than the 1st-order BTM model in view of the original model step response. Figure 6 further shows that the 4th-order BTM model and even the 70th order BTM model do not improve their step responses any more to compare with the 2nd-order BTM model and the 1st-order BTM model.

The feature of the BTM method is to first use a special similarity transformation to the system, which makes the input matrix and output matrix balanced in the similarity transformed state space basis, and then to truncate the system in that state space. The principle of the BTM is based on matching of system Hankel singular values and truncating the small system Hankel singular values. The controllability and observability Gramians of the balanced system are equal. The twice sum of the truncated balanced system Hankel singular values is a measure of the approximation error of the BTM reduced order system in the frequency domain over the whole frequency range.

The BTM model reduction method has been a successful model reduction method for many problems, such as to the high order distributed RC interconnect line [4, 16, 17, 19]. It has been noticed that the BTM method may not keep the DC match well, thus the DC match or frequency-weighted BTM has been developed [4, 7, 15, 19] to compensate a match in the low frequency range, but with a cost of its high frequency performance and computation load. However, it can provide a required DC match performance.

Here, the presented example is a complex system, i.e., an RLC interconnect line. Due to the fast increasing chip operation speed, the interconnect inductance cannot be omitted and it must be modeled, and its behavior is totally different from distributed RC circuits. *The surprising finding is that the higher order BTM model may be much worse than the 1st order BTM model in view of their whole time responses*.

In view of (23) and (31), a high order BTM model has a smaller error upper bound of the transfer function than a low order BTM model. Thus, the simulations of the example make a surprise to us to observe Figs. 4 and 6. Moreover, it is a surprise because any high order (BTM) approximation system holds more state variables than any low order (BTM) approximation system as we know.

The above revealed facts demonstrate that the error criterion in frequency domain may not be enough to represent the time domain behavior of the reduced order system. A high order approximation may not be better than a low order approximation system to represent a complex system. Therefore, we need to consider approximation error criterions in both time and frequency domains, at least for many complex systems.

VII. Conclusions

The example in the paper shows unexpected results of the BTM reduced order models in the time domain. The surprising results tell us that a high order (BTM) model may not be better than a low order (BTM) model if the time response performance is important. Thus, some special attention to verify the time domain performance of the BTM reduced models is needed when the time response is important.

The state space closed form of the evenly distributed RLC interconnect in Theorem 3.1 may be extended to any piece-wise evenly distributed RLC line circuits, and be even further developed for the tree-type distributed RLC circuits modelling. The main feature of the closed-form of state space model is its very low computation complexity $O(1)$. For any unevenly distributed interconnect line consisting of m evenly distributed RLC line sections, the extended closed form also has very low computation complexity of $O(m)$, where m is much lower than the total circuit order N, i.e., $m \ll N$. Also, the results may be applied to control systems, complex systems or intelligent systems with long distributed transmission lines, networks, or delay lines.

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