Optimal Distributed Fusion Update with Same Lag Time Out-of-Sequence Measurements

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***Abstract*— In the multisensor distributed fusion systems, observations produced by sensors typically arrive at local processors out of sequence. The resulting problem at the central processor/fusion center–how to update current estimate using multiple local out-of-sequent-measurement (OOSM) updates–is a nonstandard distributed estimation problem. In this paper, based on three update algorithms with "Out-of-Sequence" measurement (OOSM), we propose three optimal distributed fusion updates with local OOSM updates, which are, under some regularity conditions, equivalent to the centralized updates with all same lag time OOSMs respectively.**

I. Introduction

In distributed target tracking systems, there often exist various propagation times from the sensors to the local processors, it is clearly possible that some measurements will arrive out of sequence, as discussed in [1]. Since the state equations are usually defined in continuous time and then discretized, a delayed measurements from a sensor systems arrive with a lag time stamp τ. Typically, the distributed fusion systems usually keep only track sufficient statistics, such as state estimates and their covariances. The update problem with OOSM is how to find a way to update current estimate with the older OOSM.

There have been some methods for solving the update problem with single OOSM. The early work of [10] and [5] presented an approximate solution to the problem to be called "algorithm B" in [2]. The later extended the previous work to create an algorithm with optimal output, "algorithm A". It was also shown by numerical examples in [2] that the "algorithm B" is nearly optimal. All of these, however, assumed that the lag in the OOSM was less than one time step. Subsequently, the optimal update with OOSM for the l-step-lag case was proposed in [12], [13], which require an iteration back for l steps and considerable amount of storage. Therefore, the generalized solutions of "algorithm A" and "algorithm B" for the l-step-lag case, to be called "algorithm Al1" and "algorithm Bl1", were derived in [4]. The approach originally presented in [3] was also presented independently in [6] and [7]. The algorithms, Al1 and Bl1, have practically the same requirements as those of algorithms A and B, respectively. It was also shown by numerical examples that "algorithm Al1" and "algorithm Bl1" are only slightly suboptimal compared with the optimal procession of the measurements.

It is well known that a distributed fusion system with multiple distributed subsystems has many advantages, including decreases in communication bandwidth and computational burden, as well as, increases in the reliability, robustness, and survivability of the system since all subsystems are also sub-processors and do not need transmit their unprocessed observations to the fusion center (see [14]). In [8], [9], based on Kalman filtering, an optimal distributed Kalman filtering fusion was discussed, which was proved, under some regularity conditions, to be equivalent to the centralized Kalman filtering with all "in sequence" sensor measurements. The result, however, can not be used to fusion update problem with OOSMs (see Figure 1). In this paper, we focus on optimal distributed fusion update problem with local OOSM updates. We discuss two cases of OOSMs. The first case is the locate processors receiving same lag time OOSMs. The corresponding results can be extended to the case of some locate processors without receiving OOSMs.

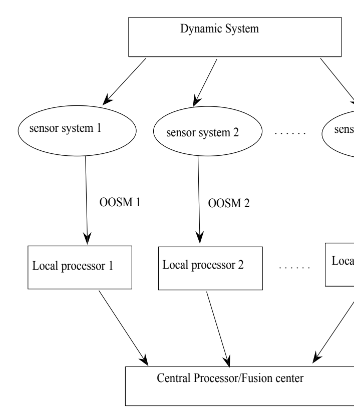

Fig. 1. Distributed Dynamic System with OOSMs.

Based on "Algorithm A", "Algorithm Al1" and "Algorithm Bl1", we present three optimal distributed fusion updates respectively, which can be proved, under some regularity conditions, to be equivalent to the centralized updates with all same lag time OOSMs respectively. An advantage of these optimal distributed fusion update algorithms is that we need not calculate the inverse of the high dimensional matrix in the centralized update algorithms. Note that if the received multiple OOSMs at central processor have same lag times, we stack the multiple OOSMs as a single OOSM, then the "Algorithm A", "Algorithm Al1" and "Algorithm Bl1" are still valid as the centralized update algorithms (in [11]).

This paper is organized as follows. In Section 2, problem formulation is given. Based on "Algorithm A" "Algorithm Al1" and "Algorithm Bl1", three optimal distributed fusion algorithms are proposed in Sections 3-5 respectively. Section 6 concludes. All proofs are omitted in the conference version.

II. Problem Formulation

A. Problem Formulation of m-sensor Distributed Linear Dynamic System with OOSMs

The m-sensor distributed linear dynamic system is given as follows. The state equation is

$$\mathbf{x}(k) = F(k,k-1)\mathbf{x}(k-1) + v(k,k-1), \tag{1}$$

where $F(k,k-1)$ is the state transition matrix to t_k from t_{k-1} and $v(k,k-1)$ is the cumulative effect of the process noise for this interval, with the weak assumptions about the noises as zero-mean, white with covariances $E(v(k,j)v(k,j)') = Q(k,j)$. The measurement equations of local processors are

$$\mathbf{z}^i(k) = H^i(k)\mathbf{x}(k) + w^i(k),\ i = 1, \ldots, m, \tag{2}$$

The stacked equation is written as

$$\mathbf{z}(k) = H(k)\mathbf{x}(k) + w(k), \tag{3}$$

where

$$\mathbf{z}(k) = (\mathbf{z}^1(k)', \ldots, \mathbf{z}^m(k)')', \tag{4}$$

$$H(k) = (H^1(k)', \ldots, H^m(k)')', \tag{5}$$

$$w(k) = (w^1(k)', \ldots, w^m(k)')', \tag{6}$$

and the covariance of the noise $w(\kappa)$ is given by

$$\text{cov}(w(k)) = R(k), \tag{7}$$

$$\text{cov}(w^i(k)) = R^i(k), \tag{8}$$

$$\text{cov}(w^i(k), w^j(k))' = 0, i, j = 1, \ldots, m, \tag{9}$$

where $R(k)$ and $R^i(k)$ are both invertible for $i, = 1, \ldots, m$.

Similarly to (1)

$$\mathbf{x}(k) = F(k,\kappa)\mathbf{x}(\kappa) + v(k,\kappa), \tag{10}$$

where κ is the discrete time notation for τ, which is the "time stamp" assumed to such that $t_{k-l} < \tau < t_{k-l+1}$. The above can be rewritten backward as

$$\mathbf{x}(\kappa) = F(\kappa,k)[\mathbf{x}(k) - v(k,\kappa)], \tag{11}$$

where $F(\kappa,k) = F(k,\kappa)^{-1}$ is the backward transition matrix.

Similarly to (2)[1]

$$\mathbf{z}^i(\kappa) = H^i(\kappa)\mathbf{x}(\kappa) + w^i(\kappa),\ i = 1, \ldots, m. \tag{12}$$

To compare performances between the centralized update with OOSMs and the distributed fusion update with the local OOSM updates, the stacked measurement equation is written as

$$\mathbf{z}(\kappa) = H(\kappa)\mathbf{x}(\kappa) + w(\kappa), \tag{13}$$

where

$$\mathbf{z}(\kappa) = (\mathbf{z}^1(\kappa)', \ldots, \mathbf{z}^m(\kappa)')', \tag{14}$$

$$H(\kappa) = (H^1(\kappa)', \ldots, H^m(\kappa)')', \tag{15}$$

$$w(\kappa) = (w^1(\kappa)', \ldots, w^m(\kappa)')', \tag{16}$$

and the covariance of the noise $w(\kappa)$ is given by

$$\text{cov}(w(\kappa)) = R(\kappa), \tag{17}$$

$$\text{cov}(w^i(\kappa)) = R^i(\kappa), \tag{18}$$

$$\text{cov}(w^i(\kappa), w^j(\kappa))' = 0, i, j = 1, \ldots, m, \tag{19}$$

where $R(\kappa)$ and $R^i(\kappa)$ are both invertible for all i.

Define the centralized OOSM update $\hat{\mathbf{x}}(k|\kappa)$, the local OOSM updates $\hat{\mathbf{x}}^i(k|\kappa), i = 1, ..., m$ and corresponding covariances as

$$\hat{\mathbf{x}}(k|\kappa) \triangleq E[\mathbf{x}(k)|\mathbf{Z}^\kappa], \quad P(k|\kappa) \triangleq \text{cov}[\mathbf{x}(k)|\mathbf{Z}^\kappa], \tag{20}$$

where

$$\mathbf{Z}^\kappa \triangleq \{\mathbf{Z}^k, \mathbf{z}(\kappa)\},\ \mathbf{Z}^k \triangleq \{\mathbf{z}(j)\}_{j=1}^k; \tag{21}$$

and

$$\hat{\mathbf{x}}^i(k|\kappa) \triangleq E[\mathbf{x}(k)|\mathbf{Z}^{\kappa,i}], \quad P^i(k|\kappa) \triangleq \text{cov}[\mathbf{x}(k)|\mathbf{Z}^{\kappa,i}], \tag{22}$$

where

$$\mathbf{Z}^{\kappa,i} \triangleq \{\mathbf{Z}^{k,i}, \mathbf{z}^i(\kappa)\},\ \mathbf{Z}^{k,i} \triangleq \{\mathbf{z}^i(j)\}_{j=1}^k. \tag{23}$$

The problem is similar to the problem in [8], [9], that is, can we still express the the centralized OOSM update $\hat{\mathbf{x}}(k|\kappa)$ in terms of the local OOSM updates $\hat{\mathbf{x}}^i(k|\kappa), i = 1, ..., m$.

B. Summary of the Optimal Update Algorithm with 1-step-lag OOSM

To deduce the optimal distributed fusion update with two same 1-step-lag OOSM updates, we firstly summarize the optimal update "algorithm A" with a 1-step-lag OOSM (see [2]). To be easy to read and express simply, we use the same notation of [2].

Summary of the optimal solution for 1-step-lag OOSM is as follows (see also [2]).

[1] If the i-th local processor does not receive a out-of-sequence measurement, we let $H^i(\kappa) = 0$, $F^i(k,\kappa) = F^i(\kappa,k) = 0$, $Q(k,\kappa) = 0$, then the last three fusion algorithms are unified.

The retrodiction of the state to κ from k is

$$\begin{aligned}\hat{\mathbf{x}}(\kappa|k) &= F(\kappa,k)[\hat{\mathbf{x}}(k|k)\\ &- Q(k,\kappa)H(k)'S(k)^{-1}v(k)],\end{aligned} \tag{24}$$

where

$$\begin{aligned}S(k) &= \operatorname{cov}[\mathbf{z}(k)|\mathbf{Z}^{k-1}]\\ &= H(k)P(k|k-1)H(k)'+R(k),\end{aligned} \tag{25}$$

$$\begin{aligned}v(k) &= \mathbf{z}(k)-\hat{\mathbf{z}}(k|k-1)\\ &= \mathbf{z}(k)-H(k)\hat{\mathbf{x}}(k|k-1).\end{aligned} \tag{26}$$

The covariances associated with the state retrodiction are

$$\begin{aligned}P_{vv}(k,\kappa|k) &= Q(k,\kappa)-Q(k,\kappa)\\ &\quad\times H(k)'S(k)^{-1}H(k)Q(k,\kappa),\end{aligned} \tag{27}$$

$$\begin{aligned}P_{xv}(k,\kappa|k) &= Q(k,\kappa)-P(k|k-1)\\ &\quad\times H(k)'S(k)^{-1}H(k)Q(k,\kappa),\end{aligned} \tag{28}$$

$$\begin{aligned}P(k|k) &= P(k|k-1)-P(k|k-1)\\ &\quad\times H(k)'S(k)^{-1}H(k)P(k|k-1),\end{aligned} \tag{29}$$

The covariance of the state retordiction is

$$\begin{aligned}P(\kappa|k) &= F(\kappa,k)[P(k|k)+P_{vv}(k,\kappa|k)\\ &- P_{xv}(k,\kappa|k)-P_{xv}(k,\kappa|k)']F(\kappa,k)',\end{aligned} \tag{30}$$

The covariance of the retrodicted measurement is

$$S(\kappa)=H(\kappa)P(\kappa|k)H(\kappa)'+R(\kappa), \tag{31}$$

The covariance between the state at k and this measurement is

$$P_{xz}(k,\kappa|k)=[P(k|k)-P_{xv}(k,\kappa|k)]F(\kappa,k)'H(\kappa)'. \tag{32}$$

The gain used for the update is

$$W(k,\kappa)=P_{xz}(k,\kappa|k)S(\kappa)^{-1}. \tag{33}$$

The update with the OOSM $\mathbf{z}(\kappa)$ of the most recent state estimate $\hat{\mathbf{x}}(k|k)$ is

$$\hat{\mathbf{x}}(k|\kappa)=\hat{\mathbf{x}}(k|k)+W(k,\kappa)[\mathbf{z}(\kappa)-H(\kappa)\hat{\mathbf{x}}(\kappa|k)]. \tag{34}$$

The covariance of the updated state estimate is

$$P(k|\kappa)=P(k|k)-P_{xz}(k,\kappa|k)S(\kappa)^{-1}P_{xz}(k,\kappa|k)'. \tag{35}$$

III. Optimal Distributed fusion Update Algorithm FA

In this section, based on algorithm A, we derive the optimal distributed fusion algorithm with two local 1-step-lag OOSM updates.

From (34), the centralized update with OOSMs $\mathbf{z}(\kappa)$ of the most recent state estimate $\hat{\mathbf{x}}(k|k)$ is

$$\begin{aligned}\hat{\mathbf{x}}(k|\kappa) &= \hat{\mathbf{x}}(k|k)+W(k,\kappa)[\mathbf{z}(\kappa)-H(\kappa)\hat{\mathbf{x}}(\kappa|k)]\\ &= [I-W(k,\kappa)H(\kappa)F(\kappa,k)]\hat{\mathbf{x}}(k|k)\\ &+ W(k,\kappa)H(\kappa)F(\kappa,k)Q(k,\kappa)H(k)'S(k)^{-1}v(k)\\ &+ W(k,\kappa)\mathbf{z}(\kappa),\end{aligned} \tag{36}$$

the last equation is from the definition of $\hat{\mathbf{x}}(\kappa|k)$ (24).

The terms in (36) are equivalent to

$$\begin{aligned}&W(k,\kappa)H(\kappa)F(\kappa,k)\\ &=[P(k|\kappa)-P(k|k)][P(k|k)\\ &-Q(k,\kappa)P(k|k-1)^{-1}P(k|k)]^{-1},\end{aligned} \tag{37}$$

$$\begin{aligned}&H(k)'S(k)^{-1}v(k)\\ &= P(k|k-1)^{-1}[\hat{\mathbf{x}}(k|k)-\hat{\mathbf{x}}(k|k-1)].\end{aligned} \tag{38}$$

To avoid calculating the inverse of high dimensional matrix $S(\kappa)$ (31), $P(k|\kappa)$ can be calculated as follows

$$\begin{aligned}&P(k|\kappa)^{-1}\\ &= \{P(k|k)^{-1}+[I+[P(k|k)^{-1}-P(k|k-1)^{-1}]\\ &\cdot Q(k,\kappa)]P_{FHR}[I-Q(k,\kappa)P(k|k-1)^{-1}]\}\\ &\cdot\{I+Q(k,\kappa)P_{FHR}[I-Q(k,\kappa)P(k|k-1)^{-1}]\}^{-1},\end{aligned} \tag{39}$$

where

$$\begin{aligned}P_{FHR} &\triangleq F(\kappa,k)'H(\kappa)'R(\kappa)^{-1}H(\kappa)F(\kappa,k)\\ &= \sum_{i=1}^{m}F(\kappa,k)'H^i(\kappa)'R^i(\kappa)^{-1}H^i(\kappa)F(\kappa,k)\\ &= \sum_{i=1}^{m}{P_W^i}^{-1}[P^i(k|k)-P^i(k|\kappa)]P^i(k|k)^{-1}\\ &\quad\cdot P^i(k|k-1)[P^i(k|k-1)-Q(k,\kappa)]^{-1},\end{aligned} \tag{40}$$

$$\begin{aligned}P_W^i &\triangleq P^i(k|\kappa)[P^i(k|k)^{-1}-P^i(k|k-1)^{-1}]Q(k,\kappa)\\ &\quad+P^i(k|\kappa)-Q(k,\kappa).\end{aligned} \tag{41}$$

Similarly, the centralized can be defined as

$$P_W \triangleq P(k|\kappa)[P(k|k)^{-1} - P(k|k-1)^{-1}]Q(k,\kappa) + P(k|\kappa) - Q(k,\kappa). \quad (42)$$

To express the centralized update $\hat{\mathbf{x}}(k|\kappa)$ in terms of the local updates $\hat{\mathbf{x}}^i(k|\kappa)$, $i = 1, \ldots, m$, $W(k,\kappa)\mathbf{z}(\kappa)$ can be equivalent to

$$\begin{aligned} & W(k,\kappa)\mathbf{z}(\kappa) \\ = & P_W F(\kappa,k)' H(\kappa)' R(\kappa)^{-1}\mathbf{z}(\kappa) \\ = & P_W \sum_{i=1}^{m} F(\kappa,k)' H^i(\kappa)' R^i(k)^{-1}\mathbf{z}^i(\kappa) \\ = & P_W \sum_{i=1}^{m} {P_W^i}^{-1} \{\hat{\mathbf{x}}^i(k|\kappa) \\ & - P^i(k|\kappa)P^i(k|k)^{-1}\hat{\mathbf{x}}^i(k|k) \\ & + P^i(k|\kappa)[P^i(k|\kappa)^{-1} - P^i(k|k)^{-1}] \\ & \cdot[Q(k,\kappa)^{-1} - P^i(k|k-1)^{-1}]^{-1} \\ & \cdot P^i(k|k-1)^{-1}\hat{\mathbf{x}}^i(k|k-1)\}. \end{aligned} \quad (43)$$

Substituting (37) (38) (43) into (36), we have the centralized update $\hat{\mathbf{x}}(k|\kappa)$ in terms of the $\hat{\mathbf{x}}(k|k)$, $\hat{\mathbf{x}}(k|k-1)$ and the local estimates $\hat{\mathbf{x}}^i(k|\kappa)$, $\hat{\mathbf{x}}^i(k|k)$, $\hat{\mathbf{x}}^i(k|k-1)$, that is,

$$\begin{aligned} & \hat{\mathbf{x}}(k|\kappa) \\ = & P(k|\kappa)P(k|k)^{-1}\hat{\mathbf{x}}(k|k) \\ - & P(k|\kappa)[P(k|\kappa)^{-1} - P(k|k)^{-1}] \\ & \cdot[Q(k,\kappa)^{-1} - P(k|k-1)^{-1}]^{-1} \\ & \cdot P(k|k-1)^{-1}\hat{\mathbf{x}}(k|k-1) \\ + & P_W \sum_{i=1}^{m} {P_W^i}^{-1} \{\hat{\mathbf{x}}^i(k|\kappa) \\ & - P^i(k|\kappa)P^i(k|k)^{-1}\hat{\mathbf{x}}^i(k|k) \\ & + P^i(k|\kappa)[P^i(k|\kappa)^{-1} - P^i(k|k)^{-1}] \\ & \cdot[Q(k,\kappa)^{-1} - P^i(k|k-1)^{-1}]^{-1} \\ & \cdot P^i(k|k-1)^{-1}\hat{\mathbf{x}}^i(k|k-1)\}. \end{aligned} \quad (44)$$

where the weighting matrices are the combination of $P(k|k)$, $P(k|k-1)$, $Q(k,\kappa)$ and the local $P^i(k|\kappa)$, $P^i(k|k)$, $P^i(k|k-1)$. If we let $W_i, i = 1, \ldots, 5$ are the weighting matrices of the $\hat{\mathbf{x}}(k|k)$, $\hat{\mathbf{x}}(k|k-1)$ and the local estimates $\hat{\mathbf{x}}^i(k|\kappa)$, $\hat{\mathbf{x}}^i(k|k)$, $\hat{\mathbf{x}}^i(k|k-1)$ respectively, then $\sum_{i=1}^{5} W_i = I$.

IV. Optimal Distributed Fusion Update Algorithm FAℓ

The work of [4] generalizes the 1-step-lag algorithms to l-step-lag algorithms while preserving their main feature of solving the update problem without iterating.

Similarly, the optimal distributed fusion algorithm FA with 1-step-lag OOSMs can be generalized to the optimal distributed fusion algorithm FAl with l-step-lag OOSMs while preserving their main feature.

The difference between FA and FAl is that the $\hat{\mathbf{x}}(k|k-1)$, $P(k|k-1)$, $P^i(k|k-1)$ in FA are replaced by $\hat{\mathbf{x}}(k|k-l)$, $P(k|k-l)$, $P^i(k|k-l)$ respectively.

The optimal distributed fusion algorithm FAl based on suboptimal update algorithm Al1 is as follows.

The centralized estimate $\hat{\mathbf{x}}(k|\kappa)$ can be obtained in terms of the $\hat{\mathbf{x}}(k|k)$, $\hat{\mathbf{x}}(k|k-1)$ and the local estimates $\hat{\mathbf{x}}^i(k|\kappa)$, $\hat{\mathbf{x}}^i(k|k)$, $\hat{\mathbf{x}}^i(k|k-l)$, that is,

$$\begin{aligned} & \hat{\mathbf{x}}(k|\kappa) \\ = & P(k|\kappa)P(k|k)^{-1}\hat{\mathbf{x}}(k|k) \\ - & P(k|\kappa)[P(k|\kappa)^{-1} - P(k|k)^{-1}] \\ & \cdot[Q(k,\kappa)^{-1} - P(k|k-l)^{-1}]^{-1} \\ & \cdot P(k|k-l)^{-1}\hat{\mathbf{x}}(k|k-l) \\ + & P_W \sum_{i=1}^{m} {P_W^i}^{-1} \{\hat{\mathbf{x}}^i(k|\kappa) \\ & - P^i(k|\kappa)P^i(k|k)^{-1}\hat{\mathbf{x}}^i(k|k) \\ & + P^i(k|\kappa)[P^i(k|\kappa)^{-1} - P^i(k|k)^{-1}] \\ & \cdot[Q(k,\kappa)^{-1} - P^i(k|k-l)^{-1}]^{-1} \\ & \cdot P^i(k|k-l)^{-1}\hat{\mathbf{x}}^i(k|k-l)\}. \end{aligned} \quad (45)$$

where the weighting matrices are the combination of $P(k|k)$, $P(k|k-l)$, $Q(k,\kappa)$ and the local $P^i(k|\kappa)$, $P^i(k|k)$, $P^i(k|k-l)$,

$$P_W \triangleq P(k|\kappa)[P(k|k)^{-1} - P(k|k-l)^{-1}]Q(k,\kappa) + P(k|\kappa) - Q(k,\kappa). \quad (46)$$

$$P_W^i \triangleq P^i(k|\kappa)[P^i(k|k)^{-1} - P^i(k|k-l)^{-1}]Q(k,\kappa) + P^i(k|\kappa) - Q(k,\kappa). \quad (47)$$

$$\begin{aligned} & P(k|\kappa)^{-1} \\ = & \{P(k|k)^{-1} + [I + [P(k|k)^{-1} - P(k|k-l)^{-1}] \\ & \cdot Q(k,\kappa)]P_{FHR}[I - Q(k,\kappa)P(k|k-l)^{-1}]\} \end{aligned}$$

$$\cdot\{I+Q(k,\kappa)P_{FHR}[I-Q(k,\kappa)P(k|k-l)^{-1}]\}^{-1} \quad (48)$$

where

$$\begin{aligned} P_{FHR} &\triangleq F(\kappa,k)'H(\kappa)'R(\kappa)^{-1}H(\kappa)F(\kappa,k) \\ &= \sum_{i=1}^{m} F(\kappa,k)'H^i(\kappa)'R^i(\kappa)^{-1}H^i(\kappa)F(\kappa,k) \\ &= \sum_{i=1}^{m} {P_W^i}^{-1}[P^i(k|k)-P^i(k|\kappa)]P^i(k|k)^{-1} \\ &\quad \cdot P^i(k|k-l)[P^i(k|k-l)-Q(k,\kappa)]^{-1}. \end{aligned} \quad (49)$$

V. Optimal Distributed Fusion Update Algorithm FBℓ

The algorithm Bl1 is simpler than the algorithm Al1 while also being nearly optimal, it was recommended for practical applications in [4]. Thus, in this section, we generalize the optimal distributed fusion algorithm FAl based on algorithm Al1 to the optimal distributed fusion algorithm FBl based on algorithm Bl1.

The optimal distributed fusion algorithm FBl based on suboptimal update algorithm Bl1 is as follows.

The centralized estimate $\hat{\mathbf{x}}^B(k|\kappa)$ can be obtained in terms of the $\hat{\mathbf{x}}(k|k)$ and the local estimates $\hat{\mathbf{x}}^{Bi}(k|\kappa)$, $\hat{\mathbf{x}}^i(k|k)$, that is,

$$\begin{aligned} &\hat{\mathbf{x}}^B(k|\kappa) \\ =\ &[P^B(k|\kappa)-P_{xv}^B(k,\kappa|k)']{P_{kxv}^B}'^{-1}\hat{\mathbf{x}}(k|k) \\ +\ &P_W^B\sum_{i=1}^{m}{P_W^{Bi}}^{-1}\{\hat{\mathbf{x}}^{Bi}(k|\kappa) \\ &-[P^{Bi}(k|\kappa)-P_{xv}^{Bi}(k,\kappa|k)']{P_{kxv}^{Bi}}'^{-1}\hat{\mathbf{x}}^i(k|k)\} \end{aligned} \quad (50)$$

where

$$P_W^B = P_{kxv}^B + [P^B(k|\kappa)-P(k|k)]{P_{kxv}^B}'^{-1}P_{kxvv}^B, \quad (51)$$

$$P^B(k|\kappa) = P(k|k) - P_{kxv}^B P_{FHR} {P_{kxv}^B}' {P_U^B}^{-1}, \quad (52)$$

$$P_{xv}^B(k,\kappa|k) = P(k|k)P(k|k-l)^{-1}Q(k,\kappa), \quad (53)$$

$$P_{kxv}^B = P(k|k) - P_{xv}^B(k,\kappa|k), \quad (54)$$

$$\begin{aligned} P_{kxvv}^B &= P(k|k)+Q(k,\kappa) \\ &\quad - P_{xv}^B(k,\kappa|k) - P_{xv}^B(k,\kappa|k)', \end{aligned} \quad (55)$$

$$P_{FHR} = \sum_{i=1}^{m}{P_W^{Bi}}^{-1}[P^{Bi}(k|k)-P^{Bi}(k|\kappa)]{P_{kxv}^{Bi}}'^{-1}, \quad (56)$$

$$P_U^B = I + {P_{kxv}^B}'^{-1}P_{kxvv}^B P_{FHR}{P_{kxv}^B}', \quad (57)$$

and

$$P_W^{Bi} = P_{kxv}^{Bi} + [P^{Bi}(k|\kappa)-P^i(k|k)]{P_{kxv}^{Bi}}'^{-1}P_{kxvv}^{Bi}, \quad (58)$$

$$P_{xv}^{Bi}(k,\kappa|k) = P^i(k|k)P^i(k|k-l)^{-1}Q(k,\kappa), \quad (59)$$

$$P_{kxv}^{Bi} = P^i(k|k) - P_{xv}^{Bi}(k,\kappa|k), \quad (60)$$

$$\begin{aligned} P_{kxvv}^{Bi} &= P^i(k|k)+Q(k,\kappa) \\ &\quad - P_{xv}^{Bi}(k,\kappa|k) - P_{xv}^{Bi}(k,\kappa|k)'. \end{aligned} \quad (61)$$

Note that $P^B(k|\kappa)$, P_W^B can be expressed by $P(k|k)$, $P(k|k-l)$, $Q(k,\kappa)$, $P^{Bi}(k|\kappa)$, $P^i(k|k)$, $P^i(k|k-l)$, and P_W^{Bi} can be expressed by $P^{Bi}(k|\kappa)$, $P^i(k|k)$, $P^i(k|k-l)$, $Q(k,\kappa)$. Thus, the weighting matrices are the combination of $P(k|k)$, $P(k|k-l)$, $Q(k,\kappa)$ and the local $P^{Bi}(k|\kappa)$, $P^i(k|k)$, $P^i(k|k-l)$.

VI. Conclusion

We have presented the optimal distributed fusion algorithm FA with the local 1-step-lag OOSM updates. Then, it is generalized to the optimal distributed fusion algorithm FAl1 and algorithm FBl1 with the local l-step-lag OOSM updates based on algorithm Al1 and algorithm Bl1 respectively. Although both the local update algorithms Al1 and Bl1 with l-step lag OOSMs are suboptimal in comparison with the optimal algorithm with l-step lag OOSMs proposed in [13], the distributed fusion algorithms based on algorithm Al1 and algorithm Bl1 are optimal in the sense that the distributed fusion updates based on algorithm Al1 and algorithm Bl1 are equivalent to the centralized updates based on algorithm Al1 and algorithm Bl1 respectively. Note that the three fusion algorithms are suitable for the cases that there are same lag time OOSMs or some local processors without OOSMs. Thus, a more interesting problem is how to optimally fuse the local updates with different lag time OOSMs, which is a new challenge in the future.

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