Affine Transformation of Multiple Mobile Robot Formation by Generalized Ant Colony Optimization*

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***Abstract*—Transformation of a multi-robot formation is one of basic problems of coordination. A transformation scheme for the multi-robot formation is proposed in this paper by imposing algorithms of affine transformation and generalized ant colony optimization (GACO). The affine transformation pre-determine target positions of each robot. Then, the GACO algorithm can help the robots find the shortest paths to their target positions. The coordination between robots is obtained by the sense and communication technology, which is supposed to be equipped in the robots. The sense can make the robot perceive obstacles as well as its neighbors in the natural environment, especially, in a blind area. Therefore, the multi-robot system can change its formation to the new one without any collision. The proposed transformation is effectively used in a simulation of a seven-robot system by Star-Logo.**

***Keywords*—affine transformation, generalized ant colony optimization, multi-robot system, formation**

I. INTRODUCTION

Transformation of a swarm formation is one of swarm behaviors, by which a swarm forages for their food or try to avoid their predators [11]-[12], [14]. The transformation of a swarm formation happen to be seen in the natural world, such as a flock of flying birds dividing two symmetrical parts in the sky. Recently, the researchers in many fields have paid more attention to the man-made swarm systems, multi-agent systems and multi-robot systems. These man-made systems are as a multi-robot systems in automated highways, air traffic control, satellite formations, control involved in search and rescue operations, control of playing games, and formation flying of autonomous unmanned aerial vehicles or robots (UAV), *etc.* [1], [13]. In all the above cases, the transformation of a formation displays a characteristic of coordination between the robots. Coordinated behavior of robots may avoid collision with others and obstacles. Therefore, when a multi-robot formation executes a transformation, the robots must have the information about their target position, as well as the environment around themselves, *e.g.*, the relative position of other robots and obstacles [5], [7], [17]-[18]. Sometimes, the robot in a system may not acquire any information from outside by its sensor and communication facility, especially, in the case of sense blind area. To overcome this obstacle, the coordinated relation of robots has to be established by the proposed algorithm with pre-processing and advanced communication means.

The purpose of this paper is to present a new transformation scheme for a multi-robot formation. This scheme is of three cooperative actions. First, the algorithm pre-specifies the target position of each robot by using affine transformation, where, the original position of each robot is necessary for calculating the new one. Usually, the robot number $N \geq 3$, otherwise, the system may reduce to a single or a pair of robots with only one formation. Then, the algorithm can find for each robot the shortest path from the original position to the target position by using the generalized ant colony optimization. Based on the ant colony optimization (ACO) [6], [10], we present a generalized ant colony optimization (GACO) in this paper, which can solve the problems of complicated combinational optimization as in the transformation of a multi-robot formation. Finally, we suppose that: (i) All robots of the system are the same in the kinematic, dynamic and physical functions. Each robot is of an omni-directional sensor in theory, or is equipped by some sonar, laser and visual system, etc., in practice. (ii) There is a commander of the system, who may be outside the system, *e.g.* supervisor, upper layer, or human being, or may be inside the system, *e.g.* a leading robot or some edge one. (iii) A two-dimension space is considered for the transformation of a multi-robot formation, in which the obstacles may be static objects, other moving neighbors and the pre-specified shortest paths for the neighbors.

II. TARGET POSITION PRE-DETERMINET

Consider an affine transformation in a two-dimension space π, described by Cartesian coordinates with both X-axis and Y-axis. Then, an affine transformation is a linear transformation, from original positions to target positions on π, is defined by

$$\begin{gathered} x_i = a_{11}x_{i0} + a_{12}y_{i0} + a_{13}, \\ y_i = a_{21}x_{i0} + a_{22}y_{i0} + a_{23}, \\ \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0, \\ i = 0, 1, \ldots, N\text{-}1. \end{gathered} \tag{1}$$

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where, (x_{i0}, y_{i0}) is a original coordinate of robot i in the system, (x_i, y_i) is transformed one of robot i, that is, target coordinate; N is a number of robots; a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, and a_{23} are transform parameters.

To determine the parameters above, we need three pairs of original and transformed coordinates in robots. Then, by equation (1) we can calculate and obtain the values of these parameters. It is known that the number of robot member $N \geq 3$, otherwise, the system may reduce to a single or a pair of robots with only one formation. After acquiring the parameters above, we can calculate each transformed coordinate (x_i, y_i) of robot i, i= 0, 1,…, N-1, according to its original coordinate by equation (1). The transformed coordinate (x_i, y_i) indicates the target position of robot i. Now a new linear formation is formed. It is obvious that after the affine transformation, the transformed coordinates are still in a line if their original ones are in a line. Therefore, in the case of a string multi-robot formation, the transformation for any multi-member linear formation can be realized by the affine transformation. In this way, we can extend transformation for a linear formation to the one for any form of formation, *e.g.* curve, round, ellipse, as well as array formation [2], [15].

It is the fact that the affine transformation can only pre-determine the target position of each robot for transformation of a multi-robot formation. It cannot give the information of the shortest path to robots for them to accomplish this formation transform. Usually, the path seeking should be in the framework of optimization algorithm for each robot to avoid collision with others and obstacles.

III. SHORTEST PATH SELECTION

Ant colony algorithm [6], [10] is an effective method in solving the complicated combinational optimization problems. In order to deal with a transformation for the multi-robot formation, we extend the ACO to a generalized ant colony optimization (GACO).

Suppose there are N colonies of ants corresponding to the N robots of the system. Each ant colony has its own nest and will find its food. Also, it sends out its special pheromone. If there is a conflict between ant colonies i and j for food, i, j=0, 1, …, N-1, the victor of shorter time or stronger force can possess food as its own. So all nest and food positions of ant colonies will be different. In the seeking process for food, ant colony i leaves more pheromone on the shorter path to food [6], [10]. The shortest path has probably been formed when a higher level of pheromone left on the right path, more and more ants of colony i turn to the right by the stronger stimulus. At the same time, the intensity of the pheromone trails on other attempted paths of ant colony i will reduce to zero step by step. Once a shortest path from nest to food for ant colony i is determined, it will be an obstacle for other ant colonies. Then, the other ant colonies have to avoid this path to find their own food. Therefore, no collision happens between ant colonies. This is one of characteristics of the GACO proposed here.

A. Path Selection Rules of GACO

We define a training time t_i as the time spent by ant colony i, i =0, 1, …, N-1, when it completes its shortest path seeking between its nest and food. The total seeking time is a sum of N ant colonies, which is described by

$$T_{\max}= \Sigma_{i=0}^{N-1} t_{\mathrm{i}}. \tag{2}$$

Therefore, each ant colony seeking its shortest path between its nest and food is always in the period of time [0, $T_{\max}$].

Suppose that there are m_i paths of ant colony i, i =0, 1, …, N-1, between its nest position (x_{i0}, y_{i0}) and food position (x_i, y_i). In [0, t_i], ant colony i attempts the j-th path, j=1, 2, …, m_i, which length is expressed by d_{ji}. The pheromone on the j-th path in [0, t_i] of ant colony i is defined by $f_{ji}(t_i)$. If the initial pheromone intensity of ant colony i is C on the j-th path, then the selection probability for the j-th path of ant colony i from its nest to its food is represented by p_{ji},

$$p_{ji} = \begin{cases} [f_{ji}(t_i)]^{\alpha}[1/d_{ji}]^{\beta} / \sum_{s=1}^{m_i}[f_{si}(t_i)]^{\alpha}[1/d_{si}]^{\beta}, & j=1,2,\ldots,m_i, \\ 0, & \text{others}, \end{cases} \tag{3}$$

where, α and β are the parameters to be adjusted.

Once the seeking time t_i arrives, ant colony i will renew its pheromone on all the j-th path, j=1, 2, …, m_i, and the process will go on.

B. Renewal Rules of Pheromone

Taking j-th path, j=1, 2, …, m_i, for an example, for ant colony i, i=0, 1,…, N-1, in [0, t_i], we renew the pheromone of the j-th path by the following:

$$f_{ji}(t_i) = (1\text{-}Y)(1\text{-}\rho)\Delta f_{ji},$$

$$Y = \begin{cases} 1, & \Delta f_{jk} \neq 0, \\ 0, & \Delta f_{jk} = 0, \end{cases}$$

$$\Delta f_{jk} = \Sigma_{k=0,k\neq i}^{N-1} f_{jk},$$

$$\Delta f_{ji} = Q/\, d_{ji} \tag{4}$$

where, Δf_{ji} indicates the pheromone intensity of ant colony i remaining on the j-th path, j=1, 2, …, m_i, in [0, t_i], which is inversely proportional to the length d_{ji} between nest to food; Q is a positive constant; (1-ρ) represents a volatility ratio of pheromone with $|\rho|\leq 1$; Δf_{jk} denotes the sum of pheromone intensity left on the j-th path by other ant colonies.

When the pheromone renewal of the j-th path is finished, the selection probability of ant colony i for each path is calculated by (1). The maximum of selection probability is corresponding to the shortest path for ant colony i, that is,

$$p_{\max i}=\max_j(p_{ji}), \qquad j=1, 2, \ldots, m_i. \tag{5}$$

Once the shortest path is found for ant colony i, then, we let the parameter ρ= -1 for this path to strengthen the pheromone intensity of the path and determine the selection. At the same time, we let ρ=1 to reduce the pheromone intensity of other paths. Moreover, from (4), we can see that if there is pheromone of other ant colony left on the j-th path, then Y=1, which makes $f_{ji}(t_i)$ =0. It means that the pheromone intensity of other ant colony on the j-th path becomes zero, after the pheromone renewal is completed. In this case, from (3), the selection probability of ant colony i for selecting the j-th path is not disturbed by other ant colonies. Therefore, the collision between ant colony i and other ant colonies, *e.g.* ant colony k, $k \neq i = 0, 1, \ldots, N$-1, will not happen.

By using the same method, other ant colonies can seek their shortest path from nest positions to food positions. The GACO algorithm is now established and can be used for the transformation of a swarm formation.

IV. SYSTEM MODELS

There are two kinds of system models for the multi-robot formation, the one in theory and the other in practice. Both of these we will consider in this section. In fact, the robots in any system have their own size. For simplicity of simulations, in theory, the size of the robot is normalized to a grid with the same unit size on a plan. The plan or environment, in which the robots move, is defined on a two-dimension space in the Cartesian coordinates. Figure 1 is displayed some robots and an obstacle in a plan sketch map. Black grids in the plan indicate both size and position of robots, while the hollow grid in the plan represents that there is no robot. All the robots can move to anywhere in the plan, except the grid occupied by the other robots and the obstacle. Usually, the obstacle may possess more than one grid (grey ones) as also shown in Figure 1. The determined shortest path for a robot can be considered as a string of obstacles in front of the other robots.

In practice, we equip each robot with software modules and hardware modules [4]. Each robot is composed of three hardware modules, that is, network system, sensor system, and walking system. Also it is of three software modules, *i.e.*, communication, self-orientation, and motion control.

The hardware modules are designed and fitted into robots. In hardware modules, the network system includes wireless network card or wireless modem, in which the wireless network channels carry out the data exchange between the robot; the sensor system is composed of distance meter, sonar, laser range finder, vision system and GPS system, the function of which is to determine self-orientation and perceive the environment including other members, obstacles, and specified paths; the walking system consists of wheel-drive motor and motion controller. The hardware structure of a robot is shown in Figure 2.

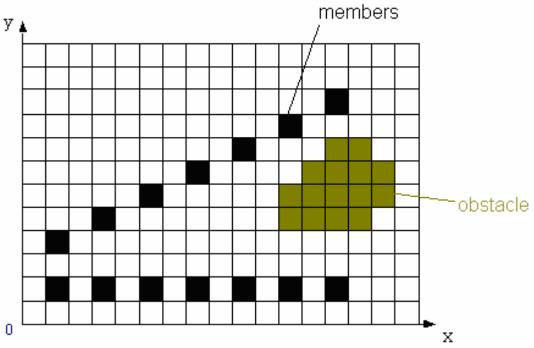


Figure 1. The Cartesian coordinates system with unit grid for the environment of a multi-robot formation.

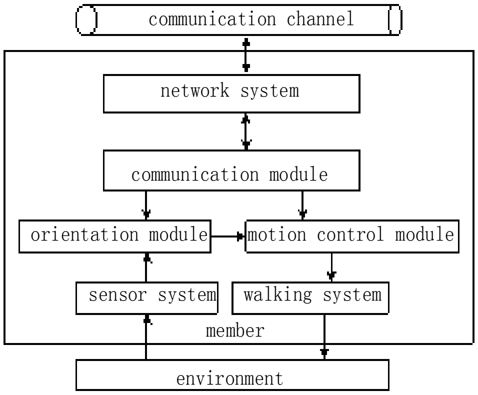


Figure 2. The hardware structure of a robot.

In order to communicate, robots must transmit and receive information by reliable software. Two protocols of UDP/TCP are adopted for robots. In software, the communication module is divided into four parts, *i.e.*, UDP receiving module, UDP transmitting module, TCP receiving module, and TCP transmitting module. The UDP module is used for transmitting and receiving state information of the robots, TCP module for conversation information, *e.g.*, when there is something wrong with a robot, the robots will re-transform their formation through TCP. It also receives control commands. The software structure of a robots is shown in Figure 3.

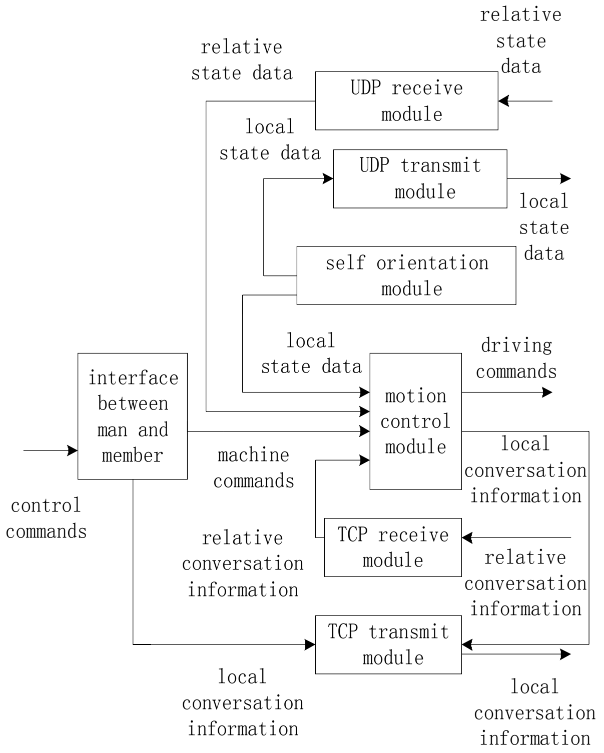


Figure 3. The software structure inside a robot.

V. A TRANSFORMATION OF FORMATION

The transformation of the multi-robot formation proposed in this paper includes the following procedures.

First of all, the self-orientation module of each robot takes samples from its GPS orientation module system and obtains the current information of all robot positions. Then, the robot can exchange the position information with others. This work is done by sending out the position information to others using its UDP transmitting module and by receiving the position information of others using its own UDP receiving module. Simultaneously, the robot also sends out its position information to its motion control module. When a machine command for the transformation of the multi-robot formation comes, in the motion control module of each robot, the target positions of all robots can be calculated by the affine transformation proposed in Section II. Then, the shortest paths of all robots without any collision with others and obstacles can be determined by the GACO proposed in Section III.

Then, the motion control module of the robot sends out the local conversation information including the target positions, the shortest paths, moving speed and direction angle, etc., to its TCP transmitting module. While, the TCP transmitting module transmits all the information to others, and other robots receive the information by their TCP receiving modules, which also gives the information to their motion control modules.

In the end, the motion module of the robot sends out driving command to its executing motors. The robot starts to move to the target position along its shortest paths obtained by GACO. The specified formation then is formed.

Using Star-Logo [3], the transformation is simulated for a seven-robot formation. Suppose the multi-robot formation consists of seven members, N=7. In the shortest path seeking, the seven robots are expressed by seven ant colonies, which are coded by 0, 1, 2, …, 6. The original formation displays a shape "—"with the numbers coded for the robots. A machine command makes the seven robots to be a target formation in shape "/".

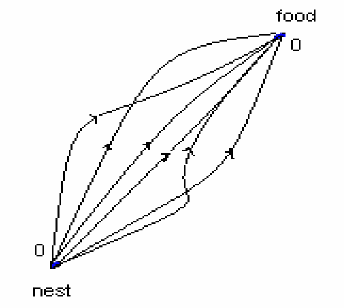
Through GPS in practice and by the affine transformation in theory, both the original and target positions of the robots can be obtained. The original positions can be regarded as the nest positions of the ant colonies, and the target positions as food positions, when we want to seek the N shortest paths for the N robots. Then, by the GACO, the shortest paths of the transformation for the formation without any collision can be selected with the coordination between the robots.

The simulation procedures are divided into training process and realizing process. The training process is to seek the shortest paths for the seven ant colonies corresponding to seven robots, respectively. This process needs the period of time [0, $T_{\max}$], where the total seeking time $T_{\max}$ is defined by the GACO. In this simulation, the values of the parameters by the GACO are given by the following [16]: α=1, β=2, ρ=0.7, Q=1 and $T_{\max}$ is set to be 70 seconds. The training time of each ant colony is 10 seconds, and motion speed v = 2 cm/sec. Each ant colony is composed of 20 ants.

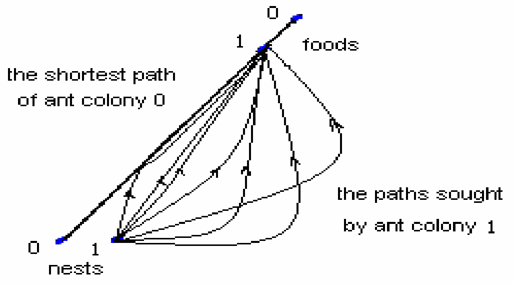
Starting with ant colony 0, we show that, in Figure 4 (a), the possible paths of ant colony 0 from its nest to food with proper pheromone on each path. Figure 4 (b) displays that the shortest path of ant colony 0 is a straight line (the thicker one), on which the pheromone on the shortest path is strengthened, and the pheromone on other paths sought by ant colony 0 are set to be zero. This implies that the pheromone intensity on other paths is thinner and reduces to zero.

In the sequence, ant colony 1 then seeks the shortest path from its nest and food. All the paths of ant colony 1 are beyond the thicker one specified for ant colony 0, which are also shown as in Figure 4(b). Among these paths, ant colony 1 will select the shortest one. Figure 4(c) demonstrates the shortest paths of both ant colony 0 and ant colony 1. The other ant colonies seek their shortest paths as the method above. The possible result of the shortest path is illustrated in Figure 5.

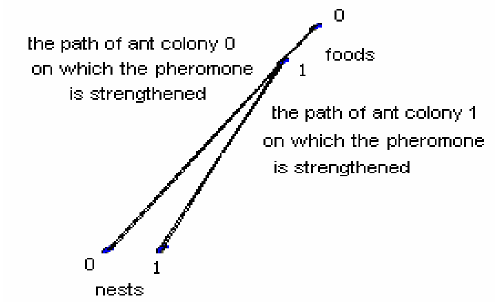
For the transformation of other multi-robot formation, the way of seeking the shortest path of each robot without collision with others is the same above.



(a) Possible paths sought by ant colony 0



(b) The shortest path of ant colony 0 and the paths of ant colony 1



(c) The strengthened pheromone paths of both ant colony 0 and 1

Figure 4. Illustrations of ant colonies seeking their shortest paths without collision with other ant colonies.

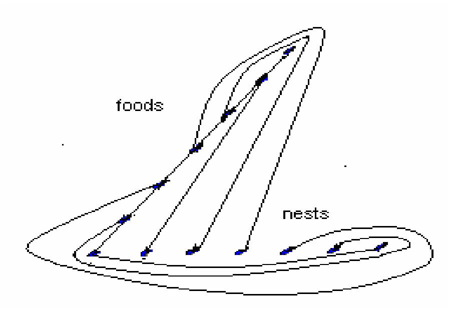


Fig. 5. The Illustration of each ant colony sending out pheromone on their shortest paths

After the training process, the realization process will take place. Suppose in this simulation as the mentioned above, the original formation is displayed in a shape "—". When a machine command of transformation comes, for a formation from the original to the target shape "⁄ ", each robot can obtains its position information from GPS and transmits and receives the information through UDP transmitting module, UDP receiving module, TCP transmitting module and TCP receiving module. The motion control module will seek the shortest paths of each robot by the GACO without collision after affine transformation. When all the preparative work is completed, the motion control module sends out driving command. The executing motors drive all robots to move to the target position along their shortest paths. The new formation is formed.

If a new transformation command of another formation comes, the present positions of all robots are again regarded as the nests and the following procedure goes on.

By the transformation method proposed in this paper for the multi-robot formation, each robot moves along its own shortest path to its target position without collision with others and obstacles. Therefore, the waiting time is the total seeking time $T_{\max}$, which can be calculated by (2). Comparing with $T_{\max}$, the motion speed of the robots may be ignored. It is interesting to see that there are a lot of transformations of the swarm formation in the references [8]-[9], [16]-[18], where the waiting time seems evident since considering that collisions happen between the robots or between the robot and the obstacle.

VI. CONCLUSIONS

In order to avoid the collision when multi-robot formation is transformed, this paper has proposed an algorithm based on the affine transformation and the general ant colony optimization (GACO). Under the combination algorithm, the transformation for the multi-robot formation can be done exactly and quickly. In the meantime, to avoid the blind area between robots, we have introduced a communication mode. By using these communication modules, the information can be exchanged among the robots. The formation can be transformed smoothly even there are a blind area between the robots. The simulation results have illustrated the algorithm proposed in this paper is effective for the formation transformation. The scheme is promising for applications in practice. It also has a prospect of considering the relationship and coordination of a pair-wise member in a swarm.

REFERENCES

[1] T. Arai, E. Pagello, and L. E. Parker, "Guest editorial advances in multi-robot systems," IEEE Transaction on Robotics and Automation, vol. 18, pp. 655-661, 2002.

[2] G. Bebis, M. Georgiopoulos, N. V. Lobo, and M. Shah, "Learning affine transformations," Pattern recognition, vol. 32, pp. 1783-1799, 1999.

[3] J. P. Bigus and J. Bigus, Constructing intelligent agents with java. John Wiley & Sons, Inc., New York, 1998.

[4] R. A. Brooks, "A Robust Layered Control System for A Robot," IEEE Journal of Robotics & Automation, vol. RA-2, pp. 14-23, 1986.

[5] S. Carpin and L. E. Parker, "Cooperative leader following in a distributed multi-robot system," in Proceedings of IEEE International Conference on Robotics and Automation, vol. 4, pp. 2994-3001, 2002.

[6] A. Colorni, M. Dorigo, V. Maniezzo, et al, "Distributed optimization by ant colonies," in Proc of European Conf on Artificial Life, pp.134-142, 1991.

[7] A. K. Das, R. Fierro, V. Kumar, J. P. Ostrowski, J. Spletzer, and C. J. Taylor, "A vision-based formation control framework," IEEE Trans. Robot. And Automat., vol. 18, pp.813-825, 2002.

[8] J. P. Desai, J. Ostrowski, and V. Kumar, "Modeling and control of formations of nonholonomic mobile robots," IEEE Transactions on Robotics and Automation, vol. 17, pp. 905-908, 2001.

[9] J. P. Desai, J. Ostrowski, and V. Kumar, "Controlling formations of multiple mobile robots," in IEEE International Conference on Robotics and Automation, pp. 2864-2869, 1998.

[10] M. Dorigo, V. Maniezzo, and A. Colorni, "The ant system: optimization by a colony of cooperating agents," IEEE Transactions on Systems, Man, and Cybernetics–Part B, vol. 26, pp. 1-13, 1996.

[11] V. Gazi and K. M. Passino, "Stability analysis of swarms," IEEE Transactions on Automatic Control, vol. 48, pp.692-697, 2003.

[12] Y. Liu, K. M. Passino, and M. M. Polycarpou, "Stability analysis of m-dimensional asynchronous swarms with a fixed communication topology," IEEE Transactions on Automatic Control, vol. 48, pp.76-95, 2003.

[13] R. Olfati-Saber and R. M. Murray, "Distributed cooperative control of multiple vehicle formation using structural potential functions," in Proc. of the 15th IFAC world congress, 2002.

[14] C. W. Reynolds, "Flocks, herds, and schools: A distributed behavioral model," Computer Graphics, vol. 21, pp. 25-34, 1987.

[15] J. Sprinzak and M. Werman, "Affine point matching," Pattern Recognition Letters, vol. 15, pp. 337-339, 1994.

[16] S.-C. Zhan, J. Xu, and J. Wu, "The Optimal Selection on the Parameters of the Ant Colony Algorithm," Bulletin of Science and Technology, vol. 5, pp. 381-386, 2003.

[17] X. Chen and Y. Li, "Smooth formation navigation of multiple mobile robots for avoiding moving obstacles", International Journal of Control, Automation, and Systems, vol. 4, no.4, pp.466-479, 2006.

[18] Y.. Li and X. Chen, "Leader-formation navigation using dynamic formation pattern", in IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM05), pp.1494-1499, 2005.