Formation Control of Multiple Mobile Robots Based on Orientation Bias*

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***Abstract*—The state coordination between robots in a multi-robot system is a crucial problem. In order to have a proper relative position and orientation of robots in a moving multi-robot formation, a coordination method is presented by using discrete orientation bias control. A moving structure graph for the formation is pre-specified. Based on the graph, a model of the formation is defined by two new kinds of artificial potential functions acting on each robot. The adjacent pair-wise robots in the formation are described in the cooperative leader-following framework without orientation constraints. The proposed control makes the global potential function maximum when the orientation bias between robots is zero. Therefore, the formation will follow its graph. In this case, the stability of the formation is proofed in the sense of Lyapunov. A simulation result of a marching square array with 9 robots is illustrated to validate the control with orientation bias.**

***Keywords*—multi-robot system, pair-wise, formation control, coordination**

I. INTRODUCTION

Both cooperation and coordination are important problems between subsystems in a complex system. A system may be broken down when its coordinated control is failed. An intuitive example of cooperation or coordination is swarm behavior between members in a swarm of the natural world, such as in a flock of birds, a herd of land animals, and a school of fish [5], [7], [12]. In many man-made systems, the coordinated control is also investigated for a broad range of applications, such as multi-area power systems [1]-[2], complex networks [9], multi-agent systems [10] in automated highways, air traffic control, control of a cluster of telescopes, satellite formations, vehicle control involved in search and rescue operations, control of mobile robots capable of playing games, and formation flying of autonomous unmanned aerial vehicles (UAV), mentioned a few. Multiple mobile robots formation control is essentially a coordinated control for the position and orientation of multi-robots. Generally, there are a lot of approaches to modeling and solving multi-robot formation problems, which are based on swarm behaviors [5], [7]-[8], leader-follower [3], [11], virtual structures [6], graphs [4] and artificial potential functions [10]. Most of the mentioned methods usually associate with each other as a combinational one.

In order to develop a novel approach to controlling multi-robot formation, a coordination method is presented using a discrete orientation bias control. First, the specification for the structure and motion of a multi-robot formation is described in the framework of graphs. In this graph, a vertex indicates a robot and its position and orientation. The edge is regarded as a relation between each pair of leaders and followers. Relative positions and orientations of each pairs are also defined based on a global heading direction of the multi-robot formation. Then, two artificial potential functions are defined to establish a model of multi-robot systems. The potential function acting on a robot is a function of orientation bias between the leader and the follower in the process of the formation following its graph. In the sense of orientation bias, the coordinated control is to make the global potential function of the formation modal maximum as the bias or error comes to zero. This implies that the multi-robot formation follows its pre-specified moving graph. Generally speaking, the formation structure in a two-dimension space are composed of triangular localizations, only when in the narrow space its composition may switch to pair-wise localizations [3]. However, in this paper, it is noted that the orientation between leaders and followers are not constrained. This idea allows us have a tool to deal with a three-robot group in a line marching on its vertical direction [4]. Therefore, the adjacent pair-wise robots in the formation can be described in the cooperative leader-following framework with all orientations. Finally, the stability of multi-robot formation is proofed in the sense of Lyapunov, witch guarantees convergence and collision-free stabilization of the formation, since all robots may be modeled as points [5], [7]. A marching square array of 9 robots, as an application, illustrates validity and feasibility of the proposed method for multi-robot formation control.

The multi-robot formation of this paper is based on the following constraints: (i) All robots of the formation are the same in kinematic, dynamic and physical functions. Especially, each robot is as a point physically and is of a physical sensor, *e.g.*, an omni-directional camera. Therefore, the robot can obtain the positions and orientations of its leaders and followers, and collisions between robots may not be happen. (ii) There is a virtual robot as the leader of the system, which not only plans a graph of moving structure and direction for the system, but also organizes robots into a formation by

* This research reported herein was supported by the NSF of China under grant No. 60574010, by the Programs for LNET No. 2006R31 and for LNIG No. 2007T082.

communication. (iii) Every robot except the virtual one in the system has at least one leader. (iv) The space for the multiple mobile robots is planar, that is, $SE(2)$. Moreover, for simplicity, no static or dynamic obstacle is considered in the two-dimension space. So the robot can always successfully step to its next desired position smoothly without any time delays.

II. Graphs and Models of a Formation

Consider that a multi-robot system with a virtual member is composed of n+1 robots in a two-dimension space, i.e. $SE(2)$, which is described by a set $R = \{R_i\}$, i= 0, 1, 2, …, n, where, R_0 is the virtual robot with its heading direction O as a leader of R, subjected to constraint (ii). While each real robot R_i, $i\neq 0$, has the same heading direction as the one of R_0 all the time. With R, we associate a vertex set $G=\{G_i\}$ in a pre-specified moving digraph, where, G_i is corresponding to R_i, i= 0, 1, 2, …, n, respectively. Each vertex G_i in the digraph represents a desired position and orientation of R_i. Between vertexes G_i and G_j, a bidirectional edge with a direction l_{ij} or l_{ji} indicates a potential (a kind of attraction) acting on R_i or R_j. It is noted that this potential function is established by an orientation bias between robots R_i, i= 0, 1, 2, …, n, in the process of R_i following G_i. Therefore, all robots of R will form a formation and follow its moving digraph G.

Assume that α_i and α_j are angles in polar coordinates for G_i and G_j, respectively. Let $\Delta\alpha_{ij} = \alpha_i - \alpha_j$, then, α_{ij} is defined as a relative angle between α_i and α_j by

$$\alpha_{ij} = \begin{cases} \Delta\alpha_{ij}, & |\Delta\alpha_{ij}| \leq \pi, \\ \Delta\alpha_{ij} - 2\pi\,\mathrm{sign}(\Delta\alpha_{ij}), & |\Delta\alpha_{ij}| > \pi, \end{cases} \tag{1}$$

Definition 1. A graph is said to be a discrete time dynamic digraph $G(k)$ with a set $G(k) = \{G_i(k)\}$ and a heading direction $O(k)$ at time k, if its vertexes $G_i(k)$ can be described by a triple $(\beta_i^*(k), x_i^*(k), y_i^*(k))$ in a plan, i= 0, 1, 2, …, n, where, $\beta_i^*(k)$ is a directed angle clockwise from $O(k)$ to the X-axis in Cartesian coordinates, while, $x_i^*(k) = G_i(k)\cos(\alpha_i(k))$, $y_i^*(k) = G_i(k)\sin(\alpha_i(k))$, both of which indicate two coordinates of $G_i(k)$ in Cartesian coordinates, respectively; and if its directed edge between vertexes $G_i(k)$ and $G_j(k)$ is represented by $l_{ij}(k)$ when the edge goes from $G_i(k)$ to $G_j(k)$ and by $l_{ji}(k)$ when from $G_j(k)$ to $G_i(k)$.

The above definition is commonly one to describe the position of robots in a formation, which can be found in the literatures (*e.g.* [3], [8]). However, it is evidently that a directed angle $\beta_i^*(k)$ of $G_i(k)$ is different from its angle $\alpha_i(k)$. This idea brings a new potential function acting on $R_i(k)$ by an orientation bias between $R_i(k)$, its leaders and followers when $R_i(k)$ is following $G_i(k)$. This orientation bias depends on the relative angle of each pair of leaders and followers in the plan.

According to Definition 1, a digraph for the multi-robot formation can be constructed by the cooperative action between the vertexes of $G(k)$ as the desired position and orientation of a leader and/or follower in $R(k)$. Because of constraint (ii), the virtual leader R_0, a supervisor or a moving target, plays an important role in the formation. R_0 arranges a pattern of the formation and pre-specifies a path or trajectory for the multiple mobile robots. It implies that $R_0(k)$ organizes original and future positions and orientations for $R_i(k)$ as $G_i(k) = (\beta_i^*(k), x_i^*(k), y_i^*(k))$, i=0, 1, 2, …, n, k = 1, 2, …. Therefore, its diversified heading direction $O(k)$ is always as a mobile direction for the formation, although the rest of $R(k)$ may not be moving in the same direction instantly. Furthermore, $R_0(k)$ definitely choose at least one follower in $R(k)$, which allows the robots to form a specified formation and moving in the determined direction properly. The real robots $R_i(k)$, i=1, 2, …, n, may be as a follower of $R_0(k)$, and a follower and/or a leader of other members, which depends on both its adjacent robots and the decision of $R_0(k)$. All decisions and actions of $R_i(k)$, i=1, 2, …, n, come from the interaction and communication between each other, especially the command of $R_0(k)$.

In the framework of digraph $G(k)$ corresponding to $R(k)$, if $G_i(k)$, i= 1, 2, …, n, has m followers, then, there are m outgoing edges named leader-attraction from it to its followers. While, if $G_i(k)$ follows l leaders, then, there are l incoming edges from its leaders to itself. The opposite ones, from $G_i(k)$ to its leaders, are regarded as follower-attraction. Therefore, the sum of all outgoing (or incoming) edge numbers of $G_i(k)$ is $m+l$, where, $m \geq 0$, $l \geq 1$, $1 \leq m+l \leq n$. Distinguishingly, for $G_0(k)$, $l = 0$.

Moreover, suppose that $G_i(k)$ is a leader of $G_j(k)$, i, j= 1, 2, …, n. From Definition 1 and (1), $L_{ij}(k)$ is defined as a leader-attraction edge with direction $l_{ij}(k)$, attraction point L_{ij} along $l_{ij}(k)$, and leader-attraction angle $\theta_{ij}(k)$. On the other hand, $F_{ji}(k)$ is defined a follower-attraction edge with direction $l_{ji}(k)$, attraction point F_{ji} along $l_{ji}(k)$, and follower-attraction angle $\varphi_{ji}(k)$. The relations between leader-attraction and follower-attraction is as following:

$$|L_{ij}(k)| = |F_{ji}(k)|, \quad |\theta_{ij}(k) - \varphi_{ji}(k)| = \pi. \tag{2}$$

A part of digraph for $G(k)$ is shown in Figure 1. The first part of Figure 1 shows that the relations of leader-attraction and follower-attractions for a pair of vertexes $G_i(k)$ and $G_j(k)$. That the vertex $G_i(k)$ has one leader $G_1(k)$ and two followers $G_2(k)$ and $G_3(k)$ are also illustrated as a second part of Figure 1.

For convenience, suppose that the formation is dynamic and time discrete in $SE(2)$ at time k, short for t_k. Corresponding to desired $G(k) = \{G_i(k)\}$ of Definition 1, the set $R=\{R_i\}$ can also describe real positions and orientations of member R_i, i= 0, 1, 2, …, n, by $R(k) = \{R_i(k)\}$ with a triple $(\beta_i(k), x_i(k), y_i(k))$ of its robot $R_i(k)$. If $R_i(k)$ is a leader of $R_j(k)$, then, the position coordinates of attraction points L_{ij} or F_{ji} are given by $(x_{ij}(k), y_{ij}(k))$ and $(x_{ji}(k), y_{ji}(k))$.

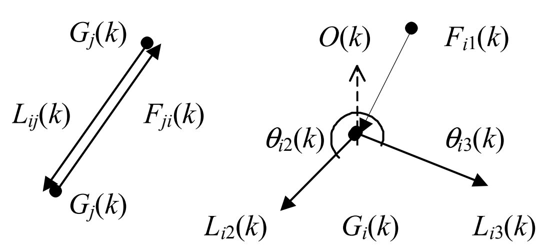

Figure 1. A part of digraph for $G(k)$

Now, let us introduce two potential functions based on the orientation bias.

Definition 2. Let $\delta_{\theta1}$, $\delta_{\theta2}$, $\delta_{\theta3}$, λ and μ are positive constants, $\delta_{\theta1} \leqslant \delta_{\theta2} << \pi/2$, $\delta_{\theta3} > \pi/2$, $\lambda<1$, $\mu<1$, then the two potential functions $P_L(\Delta\alpha)$ and $P_F(\Delta\alpha)$ for leader-attraction and follower-attraction are, respectively, defined by

$$P_L(\Delta\alpha) = \begin{cases} \cos(\delta_{\theta1}) + (\delta_{\theta1}{}^2 - \Delta\alpha^2)/2\lambda, & |\Delta\alpha| \le \delta_{\theta1}, \\ \cos(\Delta\alpha), & \delta_{\theta1} < |\Delta\alpha| \le \pi/2, \\ \pi/2 - |\Delta\alpha|, & |\Delta\alpha| > \pi/2, \end{cases} \tag{3}$$

$$P_F(\Delta\alpha) = \begin{cases} \mu, & |\Delta\alpha| \le \delta_{\theta2}, \\ \mu\cos(\Delta\alpha - \mathrm{sign}(\Delta\alpha)\delta_{\theta2}), & \delta_{\theta2} < |\Delta\alpha|,\ |\Delta\alpha - \mathrm{sign}(\Delta\alpha)\delta_{\theta2}| \le \pi/2, \\ \mu(\pi/2 - |\Delta\alpha - \mathrm{sign}(\Delta\alpha)\delta_{\theta2}|), & \pi/2 < |\Delta\alpha - \mathrm{sign}(\Delta\alpha)\delta_{\theta2}| \le \delta_{\theta3}, \\ \mu(\pi/2 - \delta_{\theta3}), & \delta_{\theta3} < |\Delta\alpha - \mathrm{sign}(\Delta\alpha)\delta_{\theta2}|. \end{cases} \tag{4}$$

The curves of potential functions $P_L(\Delta\alpha)$ and $P_F(\Delta\alpha)$ are shown in Figure 2.

To impose these two potential functions for the model of a multiple mobile robot system, a transformation has also to be taken into account, that is,

$$\Delta\alpha = \pi\Delta l/D \tag{5}$$

where, Δl represents a distance bias, D is a positive constant, a transformation coefficient.

By Definition 2 and (5), if $R_i(k)$ is a leader of $R_j(k)$ and a follower of $R_h(k)$, then, the leader-attraction and follower-attraction potential functions acting on $R_i(k)$ are represented by, respectively,

$$\begin{aligned} P_L(R_{ij}(k)) &= P_L(\beta_j(k) - \beta_i(k)) + \\ &\quad + P_L(\pi\Delta l(x_j(k) - x_{ij}(k))/D) + P_L(\pi\Delta l(y_j(k) - y_{ij}(k))/D), \\ P_F(R_{ih}(k)) &= P_F(\beta_h(k) - \beta_i(k)) + \\ &\quad + P_F(\pi\Delta l(x_h(k) - x_{ih}(k))/D) + P_L(\pi\Delta l(y_h(k) - y_{ih}(k))/D). \end{aligned} \tag{6}$$

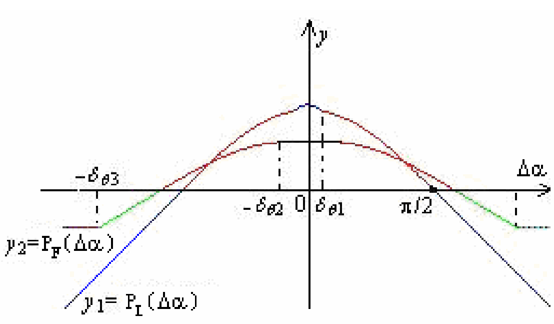

Figure 2. The curves of potential functions $P_L(\Delta\alpha)$ and $P_F(\Delta\alpha)$

If $R_i(k)$, i= 1, 2, …, n, has m followers and l leaders, where, $m \ge 0$, $l \ge 1$, $1 \le m+l \le n$, then the global potential function acting on $R_i(k)$ is given by

$$\begin{aligned} P(R_i(k)) &= P_L(R_i(k)) + P_F(R_i(k)) \\ &= \Sigma_{j=1}^{m} P_L(R_{ij}(k)) + \Sigma_{h=1}^{l} P_F(R_{ih}(k)). \end{aligned} \tag{7}$$

From (7), it is easy to see that $P(R_i(k))$ acting on $R_i(k)$ will obtain maximum value when all biases of relative angles from $R_i(k)$ to its leader and/or follower are zero. When all the global potential functions acting on every robot of $R(k) = \{R_i(k)\}$ with a triple ($\beta_i(k)$, $x_i(k)$, $y_i(k)$), i= 0, 1, 2, …, n, have reached their maximum value, the robots comes to its specified formation. As a discrete time dynamic model of the multi-robot formation, (7) characterizes the positive attraction aspect of robots in the system. This model combined with the digraph $G(k)$ will give a basis for us to coordinate and control the formation.

III. Formation Control

The formation control for the multi-robot system is based on the model established in Section 2 and under the constraints given in Section 1. Consider $G(k)=\{G_i(k)\}$ in Definition 1, i= 0, 1, 2, …, n, which specifies the robots $R(k)=\{R_i(k)\}$ with a triple ($\beta_i(k)$, $x_i(k)$, $y_i(k)$), i= 0, 1, 2, …, n, respectively, at time k= 0, 1,…. For each robot $R_i(k)$, i= 0, 1, 2, …, n, its heading direction $O(k)$ will not change at instant time k, unless it is in a period of time (k, k+1). The distance and velocity of $R_i(k)$ moving a step $\Delta S_i(k)$ may be variant in a period (k, k+1).

The motion of the virtual robot $R_0(k)$ of $R(k)$ with a triple ($\beta_0(k)$, $x_0(k)$, $y_0(k)$) is decided by the vertex $G_0(k)$ of the digraph $G(k)$ with a triple ($\beta_0{}^*(k)$, $x_0{}^*(k)$, $y_0{}^*(k)$). The model of $R_0(k)$ is given by Definition 2, then its control can be constructed by

$$\begin{aligned} S_0(k+1) &= S_0(k) + \Delta S_0(k), \\ \beta_0(k+1) &= \beta_0(k) + \Delta\beta_0(k), \\ x_0(k+1) &= x_0(k) + S_0(k+1)\cos(\beta_0(k+1)), \\ y_0(k+1) &= y_0(\mathrm{k}) + S_0(k+1)\sin(\beta_0(k+1)), \end{aligned} \tag{8}$$

where, $\Delta S_0(k)$ and $\Delta\beta_0(k)$ are a moving step and a deviation of directed angle of $R_0(k)$ at time k, inputted by the digraph $G_0(k)$ or an external controller. The initial moving step $S_0(0)\ge 0$.

The robot $R_i(k)$ of $R(k)$ with a triple ($\beta_i(k)$, $x_i(k)$, $y_i(k)$) is also driven by the vertex $G_i(k)$ of the digraph $G(k)$ with a triple ($\beta_i{}^*(k)$, $x_i{}^*(k)$, $y_i{}^*(k)$). To increase the value of the global potential function for $R_i(k)$, its control is designed by

$$\begin{aligned} S_i(k+1) &= S_0(k), \\ \beta_i(k+1) &= \beta_i(k) + \lambda\Delta\beta_i(k), \\ x_i(k+1) &= x_i(k) + S_i(k+1)\cos(\beta_i(k+1)) + \lambda\Delta x_i(k), \\ y_i(k+1) &= y_i(\mathrm{k}) + S_i(k+1)\sin(\beta_i(k+1)) + \lambda\Delta y_i(k),\ i=1,2,\dots,n, \end{aligned} \tag{9}$$

where, λ is a step coefficient defined in Definition 2, ($\Delta\beta_i(k)$, $\Delta x_i(k)$, $\Delta y_i(k)$) is an increment of the triple ($\beta_i(k)$, $x_i(k)$, $y_i(k)$), a orientation bias, which can be obtained by solving the gradient of $P(R_i(k))$ in (6), i.e., $\nabla P(R_i(k))$.

Definition 3. Let $\delta_{\theta 1}$, $\delta_{\theta 2}$, $\delta_{\theta 3}$, λ and μ are positive constants, $\delta_{\theta 1} \leqslant \delta_{\theta 2} << \pi/2$, $\delta_{\theta 3} > \pi/2$, $\lambda<1$, $\mu<1$, then, the derivatives of two potential functions $P_L(\Delta\alpha)$ and $P_F(\Delta\alpha)$ for leader-attraction and follower-attraction are defined by, respectively,

$$\frac{\partial P_L(\Delta\alpha)}{\partial\Delta\alpha}=\begin{cases}-\Delta\alpha/\lambda, & |\Delta\alpha|\leq\delta_{\theta 1},\\ -\sin(\Delta\alpha), & \delta_{\theta 1}<|\Delta\alpha|\leq\pi/2,\\ -\text{sign}\,|\Delta\alpha|, & |\Delta\alpha|>\pi/2,\end{cases} \quad (10)$$

$$\frac{\partial P_F(\Delta\alpha)}{\partial\Delta\alpha}=\begin{cases}-\mu\sin(\Delta\alpha-\text{sign}(\Delta\alpha)\delta_{\theta 2}), & \delta_{\theta 2}<|\Delta\alpha|,\ |\Delta\alpha-\text{sign}(\Delta\alpha)\delta_{\theta 2}|\leq\pi/2,\\ -\mu\,\text{sign}(\Delta\alpha), & \pi/2<|\Delta\alpha-\text{sign}(\Delta\alpha)\delta_{\theta 2}|\leq\delta_{\theta 3},\\ 0, & \text{other cases}.\end{cases} \quad (11)$$

From Definition 3 and (6) and (7), three parts of the gradient $\nabla P(R_i(k))$, i= 1, 2, …, n, is given by

$$\Delta\beta_i(k)=\partial P(R_i(k))/\partial\beta_i(k),\ \ \Delta x_i(k)=\partial P(R_i(k))/\partial x_i(k),$$
$$\Delta y_i(k)=\partial P(R_i(k))/\partial y_i(k). \quad (12)$$

To guarantee stability of the formation, the constraint

$$\lambda(l+\mu m)\leqslant 1 \quad (13)$$

is required, where, l and m are the leader and follower numbers of $R_i(k)$.

When $R_i(k)$ of $R(k)$ is driven by the controller of (9), it implies that constraint (i) must be satisfied. If all the real robots of $R(k)$ have reach their desired positions in a period of time (k, k+1), the next step will be considered.

IV. Stability of Formation Control

The problem of stability for the multi-robot formation is described as following: with the conditions (13) satisfied, the robots are driven by two controllers (8) and (9) defined for the virtual robot $R_0(k)$ and the real robots $R_i(k)$, i= 1, 2, …, n, respectively. If there is a finite positive integer K such that when the time $k\geqslant K$, the robots forms a pre-specified formation according to the digraph $G(k)$ with $\Delta S_0(k)=0$ and $\Delta\beta_0(k)=0$, then the system is stable, otherwise unstable.

For simplicity, assume $\Delta S_0(k)=0$ and $\Delta\beta_0(k)=0$ when $k\geqslant K$, $|\beta_i(k)-\beta_j(k)|<\pi$ for any robots $R_i(k)$ and $R_j(k)$ of $R(k)$. Therefore, the orientation error of the triple ($\beta(k)$, $x(k)$, $y(k)$) between $R(k)$ and $G(k)$ is represented by

$$e_{1i}(k)=\beta_i(k)-\beta_0(k),\quad e_{2i}(k)=x_i(k)-x_i^*(k),\quad e_{3i}(k)=y_i(k)-y_i^*(k),$$
$$i=0, 1, 2, \ldots, n, \quad (14)$$

and the formation system can be rewritten as

$$e_{j0}(k+1)=0,$$
$$e_{1i}(k+1)=e_{1i}(k+1)+\lambda\Delta\beta_i(k),$$
$$e_{2i}(k+1)=e_{2i}(k)+\lambda\Delta x_i(k)+S(\cos\beta_i(k)-\cos\beta_0(k)),$$
$$e_{3i}(k+1)=e_{3i}(k)+\lambda\Delta y_i(k)+S(\sin\beta_i(k)-\sin\beta_0(k)),$$
$$i=1, 2, \ldots, n,\ j=1, 2, 3. \quad (15)$$

Theorem 1. There exists a finite positive integer K such that a vector $E_1(k)=(e_{1i}(k))=0$, i= 0, 1,…, n, for any time $k\geqslant K$.

Proof: Consider a real robot $R_i(k)$. From (12) and (15), we get $E_1(k)=(e_{1i}(k))=0$, i= 0, 1,…, n, as a discrete, time-invariant, nonlinear system, with its state equilibrium at the origin of the space. Construct a Liapunov function $V(E_1(k))=\max(|e_{1i}(k)|, i=1, 2, \cdots, n)$ and its differential $\nabla V(E_1(k))=V(E_1(k+1))-V(E_1(k))$. Let $E_{Ri}(k)=(e_{1i}(k), e_{1h}(k), e_{1j}(k))$, h=1, 2,…, l, j=1, 2,…, m, $e_{\max}=\max(E_{Ri}(k))$, $e_{\min}=\min(E_{Ri}(k))$. From (1), (10)-(13), $\nabla P_F(R_i(k))\leq 0 \Rightarrow e_{1i}(k+1)\leq e_{1i}(k)+\lambda\nabla P_L(R_i(k))\leq e_{1i}(k)+e_{\max}-e_{1i}(k)=e_{\max}$. While, $\nabla P_F(R_i(k))>0$, there exist at least one $e_{1j}(k)$, j= 1, 2,…, m, such that $e_{\max}-e_{1i}(k)-\delta_{\theta 2}\geqslant e_{1j}(k)-e_{1i}(k)-\delta_{\theta 2}>0 \Rightarrow e_{\max}>e_{1i}(k)+\delta_{\theta 2}\Rightarrow e_{\max}>e_{1i}(k)+\delta_{\theta 1}$.

Therefore, in the case $e_{1h}(k)-e_{1i}(k)\leqslant\delta_{\theta 1}$, we have $e_{1i}(k+1)<e_{1i}(k)+\delta_{\theta 1}+\lambda m\mu(e_{\max}-e_{1i}(k)-\delta_{\theta 2})=(e_{1h}(k)+\delta_{\theta 1})+\lambda m\mu e_{\max}-\lambda m\mu(e_{1i}(k)+\delta_{\theta 2})<e_{\max}+\lambda m\mu(e_{1i}(k)+\delta_{\theta 1})-\lambda m\mu(e_{1i}(k)+\delta_{\theta 2})<e_{\max}$. In the case $e_{1h}(k)-e_{1i}(k)>\delta_{\theta 1}$, then, $e_{1i}(k+1)<e_{1i}(k)+\lambda(e_{\max}-e_{1i}(k))+\lambda m\mu(e_{\max}-e_{1i}(k)-\delta_{\theta 2})\leqslant e_{\max}-\lambda m\mu\delta_{\theta 2}<e_{\max}$. The two cases above imply that $e_{1i}(k+1)\leqslant e_{\max}$, h= 1, 2,…, l.

In the same way, we also get $e_{\min}\leqslant e_{1i}(k+1)$, then $e_{\min}\leqslant e_{1i}(k+1)\leqslant e_{\max}$ implies $|e_{1i}(k+1)|\leqslant\max(|e_{\min}|, |e_{\max}|)\leqslant V(E_1(k))$.

As $R_i(k)$ is an arbitrary real robot, we obtain $V(E_1(k+1))\leqslant V(E_1(k))$, that is, $\nabla V(E_1(k))=V(E_1(k+1))-V(E_1(k))\leqslant 0$.

Obviously, $\nabla V(E_1(k))$ will be not always equal to zero when $V(E_1(k))\neq 0$. By using Liapunov stability theorem, we can derive $lim_{k\to\infty}E_1(k)=0$. There must be a finite time K_1 such that for any real $R_i(k)$ and any $k\geqslant K_1$, $|e_{1i}(k)|\leqslant\delta_{\theta 1}/2$ implies $e_{1i}(K_1+1)=e_{1i}(K_1)$. So if $e_{1h}(K_1)=0$, then $e_{1i}(K_1+1)=0$; and if $e_{1h}(K_1)\neq 0$, then at K_1+n, $e_{1i}(K_1+n)=0$. Let $K=K_1+n$, we can derive $E_1(k)=0$ when $k\geqslant K$.

Theorem 2. There exists a finite positive integer K such that a vector $E(k)=(e_{ji}(k))=0$, i= 0, 1, 2, …, n, j= 1, 2, 3, for any time $k\geqslant K$.

Proof: Construct two Liapunov functions based on (15), $V(E_j(k))=\max(|e_{ji}(k)\pi/D|, i=1,2, \cdots, n, j=2, 3)$, by the way of proofs for Theorem 1, then the proof is straight forward.

V. Simulations

In this section, a formation of marching square array with 9 robots is taken as an example to show the validity and feasibility of the proposed method for the multiple mobile robot formation control. Let R_0 be a moving target, which will turns $\pi/90$ clockwise at a step, and the moving direction of which will not change after 45 steps. The specified digraph $G(k)$ and the parameters of the swarm formation are the following: D=2.4, λ=0.19, $\delta_{\theta 1}$=π/90, $\delta_{\theta 2}$=π/30, $\delta_{\theta 3}$=4, $S_0(k)$=0.22; vertexes $G_i(k)$ associated with robots $R_i(k)$, i=1, 2, …, 9, have their leaders $G_i(k)$, i=0, 1, …, 8, interval 3 and leader-attraction angle π, except that the interval between $G_0(k)$ and $G_1(k)$ is 2; $G_1(k)$ and $G_4(k)$ are the leaders of $G_4(k)$ and $G_7(k)$ with interval 2 and leader-attraction angle -π/2, respectively.

By using the orientation bias control, two cases are considered. μ=0 is in Case one and μ=0.3 in Case two. Figure 3 and Figure 4 show the both simulation results of two cases, when the formation comes to 60 steps and 80 steps.

From Figures 3 and 4, a formation of marching square array is obtained under pre-specified digraph and constraints. The robots move along the specified path stably. Furthermore, it is clearly to see that the robot moving trajectories in Case one are converged a little bit fast than, but are not as smooth as those in Case two. The reason is that the coordination between the leader and follower is not considered in Case one when μ=0. In Case two, when μ=0.3, the potential functions of both leader-attraction and follower-attraction are taken into account as in [11]. Each robot $R_i(k)$ by this orientation bias control acts as a coordinator negotiating between its leaders and followers. The collaboration of the robots makes the formation harmony as a result of tradeoff in Figure 4.

VI. Conclusions

There are three main contributions in this paper. First, a pre-specified discrete time dynamic digraph $G(k)$ with a virtual leader has been introduced for a basis of orientation bias formation control, where the real robots of the system may be of more than one leader and/or follower. The potential functions of both leader-attraction and follower-attraction then have been defined for modeling of the multi-robot formation. The considered follower-attraction can make the multi-robot formation harmony in its moving process. In the framework of coordination, the global potential function comes to maximum when the orientation bias reduced to zero, and the formation control algorithm is convergent and stable. Second, the stability of the formation control has been proofed in the sense of Lyapunov. The gradient of the global potential function has been calculated and helped for the proof. Finally, a simulation result of a formation of marching square array with 9 robots has been obtained in order to validate the proposed method for multi-robot formation control. The future research work may be introducing the leader-repulsion and follower-repulsion into our artificial potential functions. The action of leader- repulsion and follower-repulsion may help robots to prevent collision each other and to avoid barriers in the space. Although the repulsion is not considered in this paper as in the most literatures, the proposed coordinated control is promising for a swarm formation in a three-dimension space.

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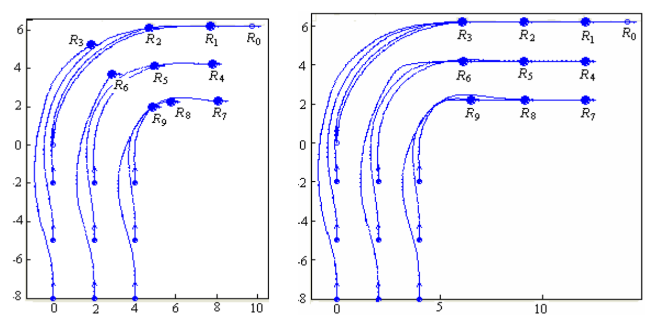
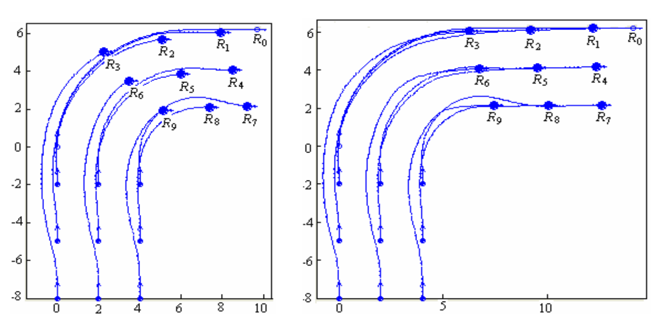


(a) 60 steps (b) 80 steps

Figure 3. A formation turns π/2 clockwise (μ=0).



(a) 60 steps (b) 80 steps

Figure 4. A formation turns π/2 clockwise (μ=0.3).