Validating Multiattribute Decision Making Methods for Supporting Group Decisions

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***Abstract*—Different multiattribute decision making (MADM) methods often produce inconsistent ranking outcomes for the same problem. In group decision settings, individual ranking outcomes made by individual decision makers are often inconsistent with the group ranking outcome. To address the inconsistency problem of ranking outcomes, this paper develops a new validation approach for selecting the most valid ranking outcome among all feasible outcomes. Based on four normalization procedures and three aggregation procedures, nine MADM methods are developed to solve the general group MADM problem that requires cardinal ranking of the decision alternatives. The validation approach selects the group ranking outcome of an MADM method which has the highest consistency degree with its corresponding individual ranking outcomes. A scholarship student selection problem is used to illustrate how the approach works. The approach is applicable to large-scale multiattribute group decision problems where inconsistent ranking outcomes often exist between different MADM methods and between different decision makers.**

***Keywords*—MADM, group decision making, validation**

I. INTRODUCTION

Multiattribute decision making (MADM) has been widely used in supporting decisions in ranking or selecting one or more alternatives from a finite number of alternatives with respect to multiple criteria or attributes. Quite a few MADM methods have been developed for a wide variety of decision problems [1][2]. These decision problems can be solved by various MADM methods. However, there is no best method for the general MADM problem, due to the multiplicity and complexity of multiattribute decisions. In decision situations where cardinal ranking of all or a subset of the alternatives is required, different methods often produce inconsistent ranking outcomes for the same problem [3][4]. As such there is a need to apply all the methods available to solve an MADM problem and select the most valid method that best reflects the values of the decision makers [5][6].

Validation in MADM research have been conducted to address three main issues including the problem formulation in relation to the definition and scoring of attributes, the quantification of weights, and the selection of the MADM methods [7]. The results of MADM research suggest that the validation of MADM methods remains a major challenging issue [8]. In this paper, we focus on the validation of decision outcomes produced by various MADM methods for a given group MADM problem. In this group MADM problem, a collective or group decision is to be made for evaluation or selection of alternatives based on the opinions of all individual decision makers involved with respect to a set of conflicting criteria or attributes.

The issue of the MADM method selection has been addressed from various decision contexts along two lines of development: (a) experimental comparisons of MADM methods for examining their appropriateness of use and/or theoretical validity, and (b) method selection procedures for specific characteristics of the decision problem and distinct features of available methods in the form of decision support systems or as general selection principles [3]. These studies cannot result in a set of guidelines or practical decision support systems that enable decision makers to select a proper MADM method for a specific problem in practice. Due to their implicit and explicit assumptions, these studies do not normally examine the validity of the decision outcome. To address this validation issue in MADM research for supporting group multiattribute decisions, we present a new validation approach. The approach can select the most valid ranking outcome from a number of ranking outcomes produced by available MADM methods in the context of group decision making.

The most widely used theory in solving MADM problems probably is the multiattribute utility theory or multiattribute value theory (MAVT) [9]. With simplicity in both concept and computation, MAVT-based MADM methods are intuitively appealing to the decision makers in practical applications. These methods are particularly suited to decision problems where a cardinal preference or ranking of the decision alternatives is required. In addition, these methods are the most appropriate quantitative tools for group decision support systems [10][11]. As such, this paper considers three main MAVT-based MADM methods which are applicable to large-scale decision problems where the ranking outcomes produced by different methods are most likely to be significantly different.

Despite their diversity, MAVT-based MADM problems often share the following common characteristics: (a) a finite number of comparable alternatives, (b) multiple attributes (evaluation criteria) for evaluating the alternatives, (c) non-commensurable units for measuring the performance rating of the alternatives on each attribute, and (d) attribute weights for representing the relative importance of each attribute. The performance ratings of the alternatives on all attributes are to be aggregated with the attribute weights using an MADM method in order to obtain an overall preference value for each alternative. The resultant overall preference values provide a cardinal ranking of the alternatives.

In subsequent sections, we first discuss the general MAVT-based group MADM problem together with available methods. We then present the new approach for validating the ranking outcomes produced by available methods. Finally we conduct an empirical study of a scholarship student selection problem to demonstrate the effectiveness of the approach.

II. THE GROUP MADM PROBLEM

The group MADM problem involves a finite set of m decision alternatives A_i ($i = 1, 2, ..., m$), which are to be evaluated by a group of p decision makers DM_k ($k = 1, 2, \ldots, p$) with respect to a set of n attributes or criteria C_j ($j = 1, 2, ...,n$). These evaluation criteria are measurable quantitatively or assessable qualitatively, and are independent of each other. Assessments are to be made by each decision maker DM_k ($k = 1, 2, \ldots, p$) to determine (a) the weight vector $W^k = (w^k_1, w^k_2, \ldots, w^k_n)$ and (d) the decision matrix $X^k = \{x^k_{ij}, i = 1, 2, ..., m; j = 1, 2, ..., n\}$.

The weight vector W^k represents the weights (relative importance) of the attributes C_j ($j = 1, 2, ...,n$) given by the decision maker DM_k using a cardinal scale. Cardinal weights are usually normalized to sum to 1, in order to allow the weight value to be interpreted as the percentage of the total importance weight. The decision matrix X^k represents the performance ratings (x_{ij}) of alternative A_i ($i = 1, 2, ..., m$) with respect to attributes C_j ($j = 1, 2, ..., n$), which are either objectively measured (for quantitative attributes) or subjectively assessed by the decision maker DM_k (for qualitative attributes) using cardinal values.

The cardinal values given in the weight vector W^k and the decision matrix X^k represent the absolute preferences of the decision maker DM_k. These individual weight vectors and the decision matrices are averaged to represent the group weight vector W and group decision matrix X. As such, the group weight vector is given by

$$W = (w_1, w_2, \ldots, w_n) \tag{1}$$

where $w_j = \sum_{k=1}^{p} w_j^{\,k} \Big/ p\,;\;\; j = 1, 2, ..., n.$

The group decision matrix X is given by

$$X = \begin{bmatrix} x_{11} & x_{12} & \ldots & x_{1n} \\ x_{21} & x_{22} & \ldots & x_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ x_{m1} & x_{m2} & \ldots & x_{mn} \end{bmatrix} \tag{2}$$

where $x_{ij} = \sum_{k=1}^{p} x_{ij}^{\,k} \Big/ p\,;\;\; i = 1, 2, ..., m;\; j = 1, 2, ..., n.$

The W in (1) and X in (2) indicate that the opinions of all p decision makers on attribute weights and alternatives' performance ratings are weighted equally.

Given the group weight vector W and the group decision matrix X, the objective of the problem is to rank all the alternatives by giving each of them an overall preference value with respect to all attributes.

III. MADM METHODS

The general group MADM problem presented above can be solved by MAVT-based methods, such as (a) the simple additive weighting (SAW) method, and (b) the technique for order preference by similarity to ideal solution (TOPSIS) and (c) the weighted product (WP) method. The main differences between these methods lie in (a) the normalization procedure for comparing all performance ratings measured using non-commensurable units on a common scale, and (b) the aggregation procedure for combining the normalized decision matrix and weight vector for obtaining an overall preference value for each alternative [3]. Due to these structural differences, the ranking outcome produced by these methods may not always be consistent for a given decision matrix and weight vector. In fact, the empirical study presented in this paper shows that the individual rankings are so different from the group rankings that the relative effectiveness of the MADM methods used needs to be examined to help make rational group decisions.

An MADM method in essence involves two key procedures: normalization and aggregation. In general applications, quantitative performance ratings of the alternatives are often assessed by different measurement units. MADM methods thus use a normalization procedure in order to make the comparison across performance ratings under different units in a decision matrix compatible. For example, SAW uses the normalization procedure of linear scale transformation (max), and TOPSIS uses the vector normalization procedure. However, there are other normalization procedures that can be used with MADM methods, independent of the aggregation procedure used. The following lists four available normalization procedures for MADM methods [12].

A. Normalization Procedures

1) Vector Normalization (N1)

This procedure divides the performance ratings of each attribute in the decision matrix by its norm. The normalized performance ratings (r_{ij}) of x_{ij} in the decision matrix are calculated as

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \tag{3}$$

The vector normalization procedure implies that all attributes have the same unit length of vector. The main advantage of this procedure is that every attribute is measured in dimensionless units, thus making it easier for inter-attribute comparisons. The main disadvantage is that it does not lead to a measurement scale of equal length because the minimum and maximum values of the scales are not equal to each attribute.

Due to a non-linear scale transformation, a straightforward comparison is hard to make [2].

2) Linear Scale Transformation, Max-Min Method (N2)

This procedure uses the following formulas to normalize the decision matrix (x_{ij}) for benefit (the larger x_j, the greater the preference) attributes and cost attributes (the smaller x_j, the greater the preference) respectively.

$$r_{ij} = \begin{cases} \dfrac{x_{ij} - x_j^{\min}}{x_j^{\max} - x_j^{\min}}, \text{ if } j \text{ is a benefit attribute} \\ \dfrac{x_j^{\max} - x_{ij}}{x_j^{\max} - x_j^{\min}}, \text{ if } j \text{ is a cost attribute} \end{cases} \quad (4)$$

where $x_j^{\max}$ and $x_j^{\min}$ are the maximum and minimum values of the j^{th} attribute respectively. The advantage of this normalization procedure is that the scale of measurement ranges precisely from 0 to 1. The lowest normalized performance rating of a attribute is 0, while the highest normalized performance rating is 1. A possible drawback of this procedure is that the scale transformation does not lead to a proportional change in performance ratings [2].

3) Linear Scale Transformation - Max Method (N3)

This procedure divides the performance ratings of each attribute by its maximum value. The normalized value of x_{ij} for benefit and cost attributes is given respectively as

$$r_{ij} = \begin{cases} \dfrac{x_{ij}}{x_j^{\max}}, \text{ if } j \text{ is a benefit attribute} \\ 1 - \dfrac{x_{ij}}{x_j^{\max}}, \text{ if } j \text{ is a cost attribute} \end{cases} \quad (5)$$

where $x_j^{\max}$ is the maximum value of the j^{th} attribute. The value of the normalized r_{ij} ranges from 0 to 1, and the attribute is more favorable as r_{ij} approaches 1. The significance of the scale transformation is that all performance ratings are transformed in a linear (proportional) way, so that the relative order of magnitude of the performance ratings remains equal [2].

4) Linear Scale Transformation - Sum Method (N4)

This procedure divides the performance ratings of each attribute C_j $(j = 1, 2, ..., n)$ by the sum of performance ratings for that attribute, as follows:

$$r_{ij} = \frac{x_{ij}}{\sum_{j=1}^{n} x_j} \quad (6)$$

where x_j is performance rating for each alternative A_i ($i = 1, 2, ..., m$) with respect to attribute C_j $(j = 1, 2, ..., n)$. With the normalized decision matrix, an aggregation procedure can be applied to obtain an overall preference value for each alternative, on which the alternative ranking can be based. The following lists three aggregation procedures available in MAVT-based MADM methods [3][6].

B. Aggregration Procedures

1) The Simple Additive Weighting (SAW) Method

The SAW method, also known as the weighted sum method, is probably the best known and most widely used MADM method [2]. The basic logic of the SAW method is to obtain a weighted sum of the performance ratings of each alternative over all attributes. With a normalized decision matrix (r_{ij}) and a weight vector (w_j), the overall preference value of each alternative (V_i) is obtained by

$$V_i = \sum_{j=1}^{n} w_j r_{ij}; \ i = 1, 2, ..., m. \quad (7)$$

The greater the value (V_i), the more preferred the alternative (A_i). Research results have shown that the linear form of trade-offs between attributes used by the SAW method produces extremely close approximations to complicated nonlinear forms, while maintaining far easier to use and understand [2].

2) The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)

The TOPSIS method is based on the concept that the most preferred alternative should not only have the shortest distance from the positive ideal solution, but also have the longest distance from the negative ideal solution [2]. With a normalized decision matrix (r_{ij}) and a weight vector (w_j), the positive ideal solution A^+ and the negative ideal solution A^- can be determined based on the weighted normalized performance ratings (y_{ij}) by

$$y_{ij} = w_j r_{ij}; \ i = 1, 2, ..., m; \ j = 1, 2, ..., n. \quad (8)$$

$$A^+ = (y_1^+, y_2^+, ..., y_n^+); \quad A^- = (y_1^-, y_2^-, ..., y_n^-) \quad (9)$$

$$y_j^+ = \begin{cases} \max\limits_i y_{ij}, \text{ if } j \text{ is a benefit attribute} \\ \min\limits_i y_{ij}, \text{ if } j \text{ is a cost attribute} \end{cases}$$

$$y_j^- = \begin{cases} \min\limits_i y_{ij}, \text{ if } j \text{ is a benefit attribute} \\ \max\limits_i y_{ij}, \text{ if } j \text{ is a cost attribute} \end{cases}$$

The distance (D_i^+) between alternatives A_i and the positive ideal solution, and the distance (D_i^-) between alternatives A_i and the negative ideal solution can be calculated respectively by

$$D_i^+ = \sqrt{\sum_{j=1}^{n} (y_i^+ - y_{ij})^2}; \quad D_i^- = \sqrt{\sum_{j=1}^{n} (y_{ij} - y_i^-)^2};$$

$$i = 1, 2, ..., m. \quad (10)$$

The overall preference value of each alternative (V_i) is given by

$$V_i = \frac{D_i^-}{D_i^+ + D_i^-}; \ i = 1, 2, ..., m. \quad (11)$$

The greater the value (V_i), the more preferred the alternative (A_i).

The advantages of using TOPSIS have been highlighted by (a) its intuitively appealing logic, (b) its simplicity and comprehensibility, (c) its computational efficiency, (d) its ability to measure the relative performance of the alternatives with respect to individual or all attributes in a simple mathematical form, and (e) its applicability in solving various practical MAVT-based MADM problems [13]. Despite its merits in comparison with other MADM methods, the TOPSIS method does not consider the relative importance (weight) of the distances from the positive and the negative ideal solutions [14]. This issue has been addressed by modifying TOPSIS to incorporate attribute weights in the distance measurement [13][15]. In the modified TOPSIS procedure, (8) and (10) in the aggregation procedure are replaced with (12) and (13) respectively, given as follows:

$$y_{ij} = r_{ij}\,; \qquad i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n. \tag{12}$$

$$D_i^+ = \sqrt{\sum_{j=1}^{n} w_j \left(y_i^+ - y_{ij}\right)^2}\,; \quad D_i^- = \sqrt{\sum_{j=1}^{n} w_j \left(y_{ij} - y_i^-\right)^2}\,; \quad i = 1, 2, \ldots, m. \tag{13}$$

In this study we use the modified TOPSIS procedure to weight the distance between the alternative and the positive (or negative) ideal solution. This modified TOPSIS procedure reflects the interrelationship between the distance and the corresponding attribute weight as implied by the concept of the degree of optimality used [13].

3) *The Weighted Product (WP) Method*

The WP method uses multiplication for connecting attribute ratings, each of which is raised to the power of the corresponding attribute weight. This multiplication process has the same effect as the normalization process for handling different measurement units, as such it requires no normalization procedure presented in the previous section. The logic of WP is to penalize alternatives with poor attribute values more heavily. With a decision matrix (x_{ij}) and a weight vector (w_j), the overall preference score of each alternative (S_i) is given by

$$S_i = \prod_{j=1}^{n} x_{ij}^{w_j}\,; \quad i = 1, 2, \ldots, m. \tag{14}$$

where $\sum_{j=1}^{n} w_j = 1$. w_j is a positive power for benefit attributes and a negative power for cost attributes. In this study, for easy comparison with other methods, the relative overall preference value of each alternative (V_i) is given by

$$V_i = \frac{\prod_{j=1}^{n} x_{ij}^{w_j}}{\prod_{j=1}^{n} (x_j^*)^{w_j}}\,; \quad i = 1, 2, \ldots, m. \tag{15}$$

where $x_j^* = \max_i x_{ij}$ and $0 \le V_i \le 1$. The greater the value (V_i), the more preferred the alternative (A_i).

C. *MADM Methods*

Combining the four normalization procedures (N1, N2, N3, and N4) with the first two aggregation procedures (SAW and modified TOPSIS) will result in 8 different methods, named as SAW-N1, SAW-N2, SAW-N3, SAW-N4, TOPSIS-N1, TOPSIS-N2, TOPSIS-N3, and TOPSIS-N4 respectively. These 8 methods and the WP method can be used to solve the same general group MADM problem that requires cardinal ranking of all the alternatives.

IV. Validation of Group Decision Outcomes

Often there are quite a few methods available for solving a specific MADM problem. Due to their structural difference, these methods may produce inconsistent ranking outcomes for a given weight vector and decision matrix [16]. Despite significant developments in MADM method selection research, the validation of decision outcomes remains an open issue. This is mainly due to the fact that the "true" cardinal ranking of alternatives is not known [3]. To address this issue for the 9 available MADM methods presented, we develop a new validation approach for selecting the most valid ranking outcome among all feasible outcomes in the context of group decision making.

In group decision making, the stakeholders or decision makes often have different views of the attribute weights and the alternatives' performance ratings. To reach a compromised solution, the values given by individual decision makers for the weight vector and decision matrix are often averaged as the group values. As such, the individual ranking outcomes based on the values given by individual decision makers may not be consistent with the final group ranking outcome derived from the averaged group values. As suggested by existing studies [3][17] and evidenced in the empirical study of this paper, different MADM methods often produce different final group ranking outcomes for the same group weight vector W and group decision matrix X. Among all feasible group ranking outcomes, individual decision makers would prefer the outcome which is most consistent with their own ranking outcome. This is the notion on which the new validation approach is based.

With m decision alternatives A_i ($i = 1, 2, \ldots, m$) and p decision makers DM_k ($k = 1, 2, \ldots, p$) using an MADM method, one group ranking outcome V_i ($i = 1, 2, \ldots, m$) and p individual ranking outcomes V_i^k ($k = 1, 2, \ldots, p$) will be produced. The consistency degree (or the correlation) between V_i and each V_i^k can be measured by Pearson's correlation coefficients or Spearman's rank correlation coefficients, resulting in p correlation coefficients. The consistency degree of the method between the group and individual ranking outcomes for the given problem can be obtained as the average of the p correlation coefficients. The approach will select the group ranking outcome of an MADM method which has the highest consistency degree, as compared to that of other methods. That is, the ranking outcome selected will have the greatest consistency with all individual ranking outcomes produced by the same method. This implies that the method used is the most valid one, as the ranking outcome it produces is most acceptable by the decision makers as a whole.

V. Empirical Study

To illustrate the inconsistent ranking outcomes by different MADM methods and different decision makers, we present a scholarship student selection problem. To show how the new validation approach can be used to select the most valid ranking outcome for a given problem, we apply the 9 MADM methods presented in the previous section to solve the scholarship student selection problem.

The objective of the scholarship student selection problem is to select applicants for industry sponsored scholarships based on their performance on non-academic, qualitative selection criteria via an interview process. The reason for excluding the academic criteria is that all the applicants have overcome a considerable academic hurdle to become eligible for study. Therefore they are expected to be capable of meeting all the specified academic requirements of the scholarship. During the scholarship duration of three years, scholarship students are required to work with industry sponsors for a total period of one year under an industry-based learning program. Based on comprehensive discussions with industry sponsors, a set of eight attributes (selection criteria) relevant to the industry-based learning program is determined, including community services, sports and hobbies, work experience, energy, communication skills, business attribute, maturity, and leadership.

In this scholarship student selection problem, the eight attributes are weighted equally, because the three interviewers (decision makers) or the industry sponsors (stakeholders) cannot determine other acceptable weights in a fair, convincing manner. This is in line with the principle of insufficient reason [18], which suggests the use of equal weights if the decision makers have no reason to prefer one attribute to another. In addition, no single attribute weighting method can guarantee a more accurate result, and the same decision makers may elicit different weights using different methods [19][20]. This suggests that there is no easy way for determining attribute weights and there are no criteria for determining what the true weight is.

In a recent scholarship run, 61 applicants attended the interview. Their performance on the eight attributes was assessed by three interviewers (decision makers) on a 6-point Likert-type scale, ranging from 5 (extremely high) to 0 (extremely low). The result of this interview process constituted three individual decision matrices (X^1, X^2 and X^3) and one group decision matrix X with m = 61 and n = 8. The group decision matrix X was generated from X^1, X^2 and X^3 using (2). The group weight vector W used was (0.125, 0.125, 0.125, 0.125, 0.125, 0.125, 0.125, 0.125) which satisfies $\sum_{j=1}^{n} wj = 1$.

By applying the nine MADM methods to the group decision matrix X and the group weight vector W, nine group ranking outcomes are obtained. These nine group ranking outcomes are not consistent. As a simple illustration, Table I shows the group rankings (out of 61) of the first 10 applicants ($A_1, A_2, \ldots, A_{10}$) (based on the SAW-N1 method) using three of the nine methods. For easy comparison, applicants A_i (i = 1, 2, …, 61) are denoted in order of their overall preference value V_i (i = 1, 2, …, 61) by the SAW-N1 method. If there were only 10 applicants to be selected, A_{10} would not be selected using TOPSIS-N1 or WP. For most decision situations where the number of applicants to be selected varies, there will be some applicants being included using some methods and being excluded with other methods.

TABLE I. Group Ranking Comparison of First 10 Applicants Between Three Representative Methods

	SAW-N1		**TOPSIS-N1**		**WP**	
A_i	V_i	*Ranking*	V'_i	*Ranking*	V_i	*Ranking*
A_1	0.972	1	0.924	1	0.968	1
A_2	0.897	2	0.842	4	0.886	2
A_3	0.886	3	0.847	3	0.880	3
A_4	0.881	4	0.853	2	0.861	4
A_5	0.877	5	0.817	8	0.845	7
A_6	0.865	6	0.823	6	0.853	6
A_7	0.857	7	0.787	10	0.846	8
A_8	0.855	8	0.819	7	0.845	9
A_9	0.845	9	0.827	5	0.853	5
A_{10}	0.829	10	0.746	15	0.804	12

By applying each of nine MADM methods to the three individual decision matrices (X^1, X^2 and X^3) and the weight vector W separately, three individual ranking outcomes are obtained. These individual ranking outcomes by individual decision makers using each method are not consistent with the group ranking outcome using the same method. As a simple illustration, Table II shows the group ranking (out of 61) of the first 10 applicants (A_1, A_2 , …, A_{10}) and three individual rankings made by the three decision makers (DM_1, DM_2, and DM_3) using the SAW-N1 method. Each of the three decision makers will select a different set of top 10 applicants and indeed a different set of scholarship students when the number of applicants to be selected is more than one.

TABLE II. Ranking Comparison of First 10 Applicants between Three Decision Makers Using the SAW-N1 Method

	Group		**DM_1**		**DM_2**		**DM_3**	
A_i	V_i	*Ranking*	V_i	*Ranking*	V_i	*Ranking*	V_i	*Ranking*
A_1	0.972	1	0.983	1	0.981	1	0.966	1
A_2	0.897	2	0.935	2	0.894	3	0.880	2
A_3	0.886	3	0.896	3	0.856	4	0.883	3
A_4	0.881	4	0.876	5	0.913	2	0.850	6
A_5	0.877	5	0.882	4	0.842	8	0.845	7
A_6	0.865	6	0.852	7	0.873	5	0.866	5
A_7	0.857	7	0.872	6	0.837	9	0.845	8
A_8	0.855	8	0.849	9	0.859	7	0.842	9
A_9	0.845	9	0.801	12	0.866	6	0.880	4
A_{10}	0.829	10	0.857	8	0.788	14	0.814	13

To select among the inconsistent group ranking outcomes produced by nine methods, we apply the validation approach to each of nine MADM methods individually. The consistency degrees of SAW-N1, SAW-N2, SAW-N3, SAW-N4, TOPSIS-N1, TOPSIS-N2, TOPSIS-N3, TOPSIS-N4, and WP using Pearson's correlation coefficients (Spearman's rank correlation coefficients) are 0.81 (0.66), 0.77 (0.59), 0.79 (0.61), 0.80 (0.62), 0.67 (0.51), 0.65 (0.52), 0.59 (0.45), 0.52 (0.46) and 0.73 (0.41) respectively. This result suggests that the group ranking outcome produced by the SAW-N1 method should be used, as it is most consistent with the views of individual decision makers, thus most acceptable by them.

It is noteworthy that the selection of the group ranking outcome produced by the SAW-N1 method is justifiable only for the problem data set used in the empirical study. Different problem data sets may result in a different method being selected. This suggests that no single best method can be assumed for the general group cardinal ranking problem. In solving a given group decision problem with many methods available, the validation approach developed in this paper can be applied to all available methods for identifying the most valid ranking outcome from the perspective of all decision makers as a whole.

VI. CONCLUSION

There are normally a number of methods available for solving group MADM problems, defined by a given weight vector and decision matrix. Inconsistent ranking outcomes are often produced by different decision makers and MADM methods. Despite the importance of validating decision outcomes produced by different MADM methods, very few studies have been conducted to help the decision makers deal with the inconsistency problem of ranking outcomes and make valid decisions. In this paper, we have developed a new empirical validation approach for selecting the most valid group ranking outcome for a given problem data set. The most valid group ranking outcome is the one that is most consistent with individual rankings made by individual decision makers using a particular MADM method. We have presented a scholarship student selection problem to illustrate how the approach can be used to help select the most valid group ranking outcome for a given data set. With its simplicity in both concept and computation, the approach can be applied in general group decision problems solvable by compensatory MADM methods. It is particularly suited to large-scale MADM problems where the decision outcomes produced by different methods differ significantly.

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