A Modified Direct Data Domain Method in Space-time Adaptive Processing

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***Abstract*—Traditionally space-time adaptive processing(STAP) approaches have been implemented to detect target in airborne radar systems. These approaches typically assume a wide sense stationary environment and utilize data in neighbor range cells to estimate the statistics of cell under test. Unfortunately the airborne radar environment can be highly non-stationary, which could drastically impact statistical STAP performance. Various approaches have been investigated to address this problem. One approach is direct data domain (D^3LS) method. This paper does research an modified method based on D^3LS approach in space-time adaptive processing. On a snapshot-by-snapshot basis, this algorithm can restrain jamming and clutter effectively while keep the signal of interest. Real weights connected with the space-time data reduce the computational complexity. By implementing multiple space-time constraints, system gain is maintained on the signal of interest when the signal arrivals slightly offset in angle, Doppler, or both. Simulation results show the effectiveness of this method.**

***Keywords*—multiple constraints, real weights, direct data domain approach, space-time adaptive processing**

I. INTRODUCTION

Recently, a direct data domain least squares method(D^3LS)[1,2] is widely used for adaptively enhancing signals in space-time adaptive processing(STAP). Compared with traditional STAP approaches, this method operates on a snapshot by snapshot basis to determine the adaptive weights instead of training data from neighbor range cells and is proved to be an effective way in non-stationary environment.

Generally, D^3LS algorithm performs by changing both amplitude and phase associated with an array at each of the antenna elements, which offer the system complexity and the burden of computation. Furthermore, the performance of D^3LS degrades greatly as the signal arrivals slightly offset in angle, Doppler, or both, which is known a prior. Chio presented an amplitude-only weigh control approach in space adaptive signal processing[3]. In this paper, this technology is extended to space-time adaptive processing for simpler hardware and software design and faster response. By implementing multiple space-time constraints, system gain is maintained on the signal of interest (SOI) when it deviates its original value[4]. Simulation results show the effectiveness of this method.

II. DIRECT DATA DOMAIN LEAST SQUARE APPROACH

Consider a uniform liner array composed of N antennas separated by a distance d and the radar transmits a burst of K pulses in a coherent process interval. For nth antenna ant the kth pulse, the received signal can be described as[5]

$$\mathbf{X}_{n,k} = \exp(j2\pi(n-1)d\sin\theta_s/\lambda + j2\pi(k-1)f_d/f_r) + \\ Clutter + Interference + Noise \tag{1}$$

Where α is the amplitude of signal of interest(SOI), θ_s is angle of arrival and f_d is Doppler frequency of signal of interest, λ is the wavelength, f_r is pulse repetition frequency.

For a given snapshot, the received data can be arranged in a $N \times K$ matrix as

$$\mathbf{X} = \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1K} \\ X_{21} & X_{22} & \cdots & X_{2k} \\ \vdots & \vdots & \vdots & \vdots \\ X_{N1} & X_{N1} & \cdots & X_{NK} \end{bmatrix} \tag{2}$$

In equation (2), for a given column, the row to row phase difference, due to SOI, is $z_s = \exp(j2\pi d\sin(\theta_s)/\lambda)$ and column to column phase difference for a given row is $z_t = \exp(j2\pi f_d/f_r)$. Using z_s and z_t the SOI can be removed i.e. leaving just the interference, utilizing difference equations performed with elements offset in adjacent space, time and space-time samples as following

$$\mathbf{X}(n,k) - \mathbf{X}(n+1,k)\cdot z_s^{-1} \tag{3}$$

$$\mathbf{X}(n,k) - \mathbf{X}(n,k+1)\cdot z_t^{-1} \tag{4}$$

$$\mathbf{X}(n,k) - \mathbf{X}(n+1,k+1)\cdot z_s^{-1}\cdot z_t^{-1} \tag{5}$$

In order to minimize interference while keeping gains along the looking direction i.e. angle and Doppler frequency of SOI. A matrix equation can be constructed following figure1 and expressed as

$$\mathbf{FW} = \mathbf{Y} \tag{6}$$

Where $\mathbf{F}$ is system matrix and $\mathbf{W}$ is a vector of space-time weighs. The first element of $\mathbf{Y}$ consists of complex gain and the remaining elements are set to zeros to complete the cancellation equations.

The first row of $\mathbf{F}$ is used to set gain along the looking direction which is

$$\mathbf{F}(1,:)=\left[1,z_s,z_s^2,\cdots,z_s^{N_s-1},z_t,z_sz_t,\cdots,z_s^{N_s-1}z_t,\cdots,z_t^{N_t-1},z_sz_t^{N_t-1},\cdots,z_s^{N_s-1}z_t^{N_t-1}\right]$$

Where N_a and N_t is the spatial and temporal dimension of the window.

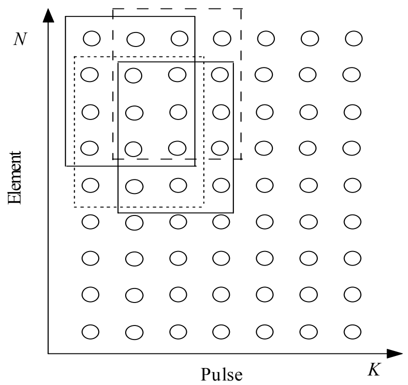


Figure1. A space-time data snapshot

When using a space-time window passes over the received data, for each location of the windows location, three rows in matrix $\mathbf{F}$ are formed by implementing equation (3)~(5) respectively. The rows are created by performing an element by element subtraction using value using the values contained within the offset windows and then arranging the resulting data into a row vector. The window is then displaced one space to the right and three more rows are generated, and this procedure is continued along the space and time axis. After this window has reached the second column from the far right, the windows is lowered and shifted to the left side of the data array. The generation of rows continues until $Q-1$ cancellation rows have been formed, where $Q=N_s\cdot N_t$, where N_s is the spatial dimension and N_t is the temporal dimension of the window.

Backward and Forward-backward method can be deduced similarly in this way[6].

Using the conjugant gradient method outlined in [7] equation (6) can be solved and amplitude of the SOI can now be estimated by

$$\hat{\alpha}=\frac{1}{C}\sum_{k=1}^{N_t}\sum_{n=1}^{N_s}\mathbf{W}((k-1)N_s+n)\mathbf{X}(n,k) \tag{7}$$

III. A Multiple Constraints Direct Data Domain Least Square Approach with Real Weighs

A. *Forward method*

In traditional D^3LS algorithm, adaptive space-time processing performs by changing both amplitude and phased associated with each array element, which is shown in equation (6), where $\mathbf{F}$ and $\mathbf{Y}$ are complex vectors which resulting in the adaptive weight $\mathbf{W}$ is also a complex one. This offers great control over array response at the expense of system complexity. At the same time, a fact that in equation (6) the gain of SOI is maintained based on the fact that the angel and Doppler frequency of SOI align with looking direction (center of radar's antenna beam and Doppler filter) which is known in advance. When SOI is slightly out of alignment with looking direction, it would be restrained as interferences. To solve these problems, a multiple constraints Direct Data Domain Least Square Approach with real weighs is presented here.

Use of real weighs will produce a symmetric pattern, it can have a ambiguity if SOI comes from θ_S while interference comes from $-\theta_S$, so before matrix $\mathbf{F}$ is formed, the incoming data x_{nk} should be shifted to 0° in angle domain, so there is no symmetric angle for -90° to 90°, the same dose in Doppler domain. A transformation matrix can be written as

$$\mathbf{Tr}(n,k)=\frac{\exp(j2\pi((n-1)d\sin\theta_0/\lambda+(k-1)f_0/f_r))}{\exp(j2\pi((n-1)d\sin\theta_S/\lambda+(k-1)f_d/f_r))} \tag{8}$$

where $\theta_0=0, f_0=0$, $n=1,2,\cdots N, k=1,2,\cdots K$

The transformed matrix can be formed by using element by element multiplication

$$\overline{\mathbf{X}}(n,k)=\mathbf{X}(n,k)\mathbf{Tr}(n,k) \tag{9}$$

Similar to section 2, a matrix equation can be constructed as

$$\overline{\mathbf{F}}\overline{\mathbf{W}}=\overline{\mathbf{Y}} \tag{10}$$

When the SOI arrives at the antenna array slightly off set in angle, Doppler or both. In order to prevent the processor from nulling the SOI, multiple constraints are used to help maintain system gain over the antenna beam width and Doppler filter width. Additional control can be implemented by replacing cancellation rows with constraints rows in system matrices $\mathbf{F}$. The *lth* row constraint can be constructed as

$$\overline{\mathbf{F}}(l,:)=\left[1,\bar{z}_{sl},\bar{z}_{sl}^2,\cdots,\bar{z}_{sl}^{N_a-1},\bar{z}_{tl},\bar{z}_{sl}\bar{z}_{tl},\cdots,\bar{z}_{sl}^{N_a-1}\bar{z}_{tl},\cdots,\bar{z}_{tl}^{N_t-1},\bar{z}_{sl}\bar{z}_{tl}^{N_t-1},\cdots,\bar{z}_{sl}^{N_a-1}\bar{z}_{tl}^{N_t-1}\right]$$

where $\bar{z}_{lt}=\exp(j2\pi f_l/f_r)$, $\bar{z}_{ls}=\exp(j2\pi d\sin(\theta_l)/\lambda)$, $\mathbf{Y}=[C_1,C_2,\cdots,C_L,0,\cdots,0]^T$, where $C_l(l=1,2,\cdots,L)$ is the complex gain in the *lth* direction of constraints.

In order to compute weighting vector containing real numbers, matrix equation (10) is separate into real and imaginary parts. Then the equation can be modified as

$$\mathbf{F}_R\mathbf{W}_R=\mathbf{Y}_R \tag{11}$$

where

$$\mathbf{F}_R=\begin{bmatrix}\mathrm{Re}(\bar{F}_{1,1}) & \mathrm{Re}(\bar{F}_{1,2}) & \cdots & \mathrm{Re}(\bar{F}_{1,Q})\\ \mathrm{Re}(\bar{F}_{2,1}) & \mathrm{Re}(\bar{F}_{2,2}) & \cdots & \mathrm{Re}(\bar{F}_{2,Q})\\ \vdots & \vdots & \cdots & \vdots\\ \mathrm{Re}(\bar{F}_{Q,1}) & \mathrm{Re}(\bar{F}_{Q+1,2}) & \cdots & \mathrm{Re}(\bar{F}_{Q,Q})\\ \mathrm{Im}(\bar{F}_{1,1}) & \mathrm{Im}(\bar{F}_{1,2}) & \cdots & \mathrm{Im}(\bar{F}_{1,Q})\\ \mathrm{Im}(\bar{F}_{2,1}) & \mathrm{Im}(\bar{F}_{2,2}) & \cdots & \mathrm{Im}(\bar{F}_{2,Q})\\ \vdots & \vdots & \cdots & \vdots\\ \mathrm{Im}(\bar{F}_{Q,1}) & \mathrm{Im}(\bar{F}_{Q,2}) & \cdots & \mathrm{Im}(\bar{F}_{Q,Q})\end{bmatrix}$$

Where $\mathrm{Re}()$ and $\mathrm{Im}()$ are real part and imaginary part respectively. $\mathbf{Y}_R = [\mathrm{Re}(C_1),\cdots,\mathrm{Re}(C_L),0,\cdots,0,\mathrm{Im}(C_1),\cdots,\mathrm{Im}(C_L),0,\cdots,0,]^T$.

Adaptive weights vector of real value $\bar{\mathbf{w}}$ in equation (11) can also be solved with conjugant gradient method.

The window size along the elements dimension is N_s and N_t along the pulse dimension. Selection of N_s determines the number of spatial degrees of freedom while N_t determines the temporal degree of freedom. In real value equation (11), the maximum of special dim and temporal dim are $N_s = (N + 0.5)/1.5$ and $N_t = (K + 0.5)/1.5$ respectively, while in traditional method the maximum are $N_s = (N+1)/2$ and $N_t = (K+1)/2$, increases about 20% respectively, which means more degree of freedom can be used in a real-weight D^{3}LS algorithm. Since the total number of freedom is $Q = N_s \times N_t$, one can exchange spatial degrees of freedom with the temporal degrees of freedom, so it is possible to cancel a number of interferers, which is greater than the number of antennas elements in a joint domain processing. At the same time, computation load for solving complex value equation (10) with conjugant gradient method is proportional to $4o[(N_s N_t)^2]$ while real value equation (11) is proportional to $2o[(N_s N_t)^2]$ [7].

B. Backward method

The backward method is implemented by conjugating the element pulse data in matrix $\mathbf{x}$ and processing this data in reverse direction as forward method. The form of this matrix equation is similar to that of the forward algorithm, which is

$$\mathbf{B}_R \mathbf{W}_R = \mathbf{Y}_R \tag{12}$$

and amplitude of the SOI can be estimated by

$$\hat{\alpha} = \sum_{k=1}^{N_t} \sum_{n=1}^{N_S} \mathbf{W}((k-1)N_S + n)\bar{\mathbf{X}}^*(N+1-n, K+1-k) \tag{13}$$

C. Forward-backward method

In the forward-backward model, double the amount of data by not only considering the data in the forward direction, but also conjugating it and reversing the direction of increment of the independent variable. Similar to 3.1 and 3.2, the matrix equation is formed as

$$[\mathbf{FB}]_R \mathbf{W}_R = \mathbf{Y}_R \tag{14}$$

Under this condition, the maximum number of spatial dim and temporal dim are $N_s = (N+0.5)/1.25$ and $N_t = (K + 0.5)/1.25$ respectively, which also increases about 20% compared with traditional forward-backward method.

IV. Simulation Results

Consider a 10-elements linear array with an element spacing of $\lambda/2$ and 16 pulses in a coherent processing interval. A simulated SOI arrives from 0° in azimuth with normalized Doppler 0.2 . A $40\mathrm{dB}$ discrete interference located at the same azimuth and at normalized Doppler of 0.5 , three $30\mathrm{dB}$ jammers arrive form -50°, -20° and 20° respectively and cover all Doppler frequency. The average input signal to interference and noise ratio (SINR) is about $-41\mathrm{dB}$.

A. Single constraint real weight D^{3}LS method s

For the first case, the forward and the backward method utilizes 7 spatial degrees of freedom $(N_s = 7)$ and 11 temporal degrees $(N_t = 11)$ while forward-backward method employs 8 spatial and 12 temporal degrees of freedom. The amplitude of SOI is varied from $1\,\mathrm{v/m}$ to $10\,\mathrm{v/m}$ in steps of $1\,\mathrm{v/m}$. Figure 2 illustrates the recovered amplitude with forward, backward and forward-backward method and figure 3 shows antenna pattern along azimuth and normalized Doppler with these three methods.

As seen in figure 2 and figure 3, each of the processors performs well in mitigating discrete interference and recovery the amplitude of signal effectively. The processors also form deep notches along the location of jammers and system gain is maintained in the direction of SOI.

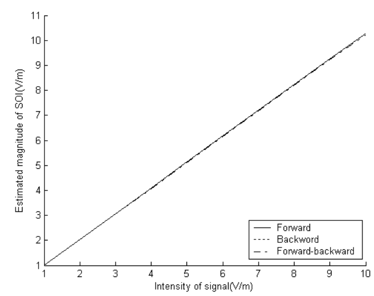
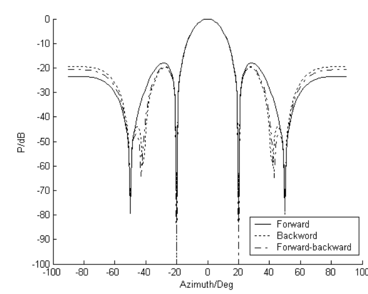
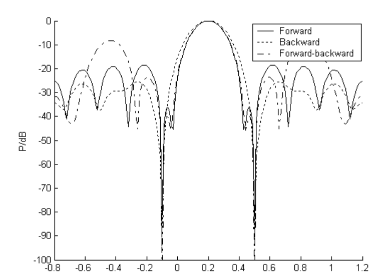


Figure 2. Estimated magnitude of SOI, single constraint, aligned



(a) Azimuth



(b) Normalized Doppler

Figure 3. Beam pattern of single constraint, aligned

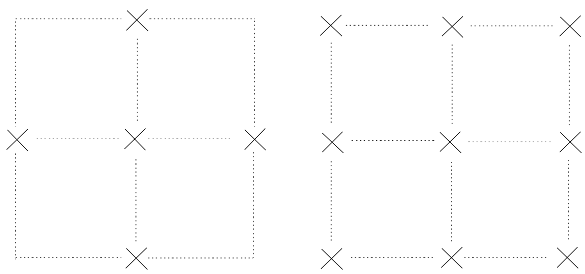
According to the definition of signal to interference and noise ratio

$$\mathrm{SINR}_{out} = 20\lg[\alpha_s/(\alpha_s - \hat{\alpha}_s)] \tag{15}$$

where α_s is the amplitude of SOI and $\hat{\alpha}_s$ is the estimated value. The output signal to interference and noise ratio of forward, backward and forward-backward method is about 24.7 , 24.8 and 25.4 dB respectively over 100 independent realizations.

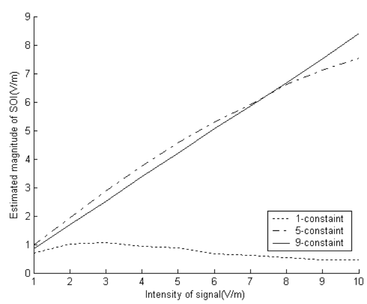
B. Multiple constraints real weight D^3LS method

In this case, SOI is located at 3° in azimuth with normalized Doppler of 0.25 while system constraint is set to 3° and 0.2, In order to prevent the processor from nulling the SOI, five and nine constraints are used to maintain system gain over the antenna beam width and Doppler filter width. The location of constraint is shown in figure4.

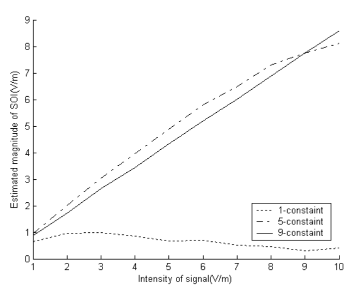


(a) 5-constraint (b) 9-constraint

Figure 4. Space-time constraint configurations



(a) Forward method



(b) Forward-backward method

figure5. Estimated magnitude of SOI, multiple constraint, misaligned

As the amplitude of SOI varies from 1 v/m to 10 v/m , one, five and nine constraints are used respectively to protect system gain in the looking direction. The recovered value is shown in figure 5 and output SINR in table1.

It can be seen when SOI is misaligned with looking direction and only one constraint is utilized, both processor treat SOI as interference and try to null it. Additional four constraint help to keep processor from placing a null in SOI through the main beam and Doppler filter, it is protected. This situation can be improved further with nine constraints.

TABLE I. OUTPUT SINR OF MULTIPLE CONSTRAINTS

constraint / output(dB) \ Method	1-constaint	5-constaint	9-constaint
Forward	-3.1	4.97	5.46
Forw ard-backw ard	-1.2	4.73	5.80

V. CONCLUSION

An improved direct data domain least square approach in space-time adaptive processing is present here. As shown through numeral examples, with amplitude-only weights, this method can cancel strong interference and recover SOI effectively at the same time degree of freedom increase about 20% in spatial and temporal domain respectively. With additional constraint, SOI is protected and system gain is maintained although antenna looking direction is misaligned with the location of SOI. The improved algorithm reduces complexity and computation and maybe useful for real time realization.

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